

THE FOUNDATIONS  
OF MATHEMATICS  
and other Logical Essays

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## APPENDIX

### CRITICAL NOTICE (1923)

*Tractatus Logico-Philosophicus*, by LUDWIG WITTGENSTEIN, with an Introduction by BERTRAND RUSSELL. (International Library of Psychology, Philosophy and Scientific Method.) 1922.

This is a most important book containing original ideas on a large range of topics, forming a coherent system, which whether or not it be, as the author claims, in essentials the final solution of the problems dealt with, is of extraordinary interest and deserves the attention of all philosophers. And even if the system be altogether unsound the book contains a large number of profound *obiter dicta* and criticisms of other theories. It is, however, very difficult to understand, in spite of the fact that it is printed with the German text and an English translation on opposite pages. Mr Wittgenstein writes, not consecutive prose, but short propositions numbered so as to show the emphasis laid upon them in his exposition. This gives his work an attractive epigrammatic flavour, and perhaps makes it more accurate in detail, as each sentence must have received separate consideration; but it seems to have prevented him from giving adequate explanations of many of his technical terms and theories, perhaps because explanations require some sacrifice of accuracy.

This deficiency is partly made up by Mr Russell's Introduction; but it is possible that he is not an infallible guide to Mr Wittgenstein's meaning. "In order to understand Mr Wittgenstein's book," says Mr Russell, "it is necessary to realize what is the problem with which he is concerned. In the part of his theory which deals with Symbolism he is concerned with the conditions that would have to be fulfilled by a logically perfect language." This seems to be a very doubtful generalization; there are, indeed, passages in which Mr Wittgenstein is explicitly concerned with a logically perfect, and not with any language, e.g. the discussion of 'logical syntax' in 3.325 ff.; but in general he seems to maintain that his doctrines apply to ordinary languages in spite of the appearance of the contrary (see especially 4.002 ff.). This is obviously an important point, for this wider

application greatly increases the interest and diminishes the plausibility of any thesis such as that which Mr Russell declares to be perhaps the most fundamental in Mr Wittgenstein's theory; that "in order that a certain sentence should assert a certain fact there must, however the language may be constructed, be something in common between the structure of the sentence and the structure of the fact".

This doctrine appears to depend on the difficult notions of a picture' and its 'form of representation', which I shall now try to explain and criticize.

A picture is a fact, the fact that its elements are combined with one another in a definite way. These elements are co-ordinated with certain objects (the constituents of the fact of which the picture is a picture). These co-ordinations constitute the representing relation which makes the picture a picture. This representing relation "belongs to the picture" (2.1513); this, I think, means that whenever we talk of a picture we have in mind some representing relation in virtue of which it is a picture. Under these circumstances we say that the picture represents that the objects are so combined with one another as are the elements of the picture, and this is the sense of the picture. And I think this must be taken to be the definition of 'represents' and of 'sense'; that is to say, that when we say that a picture represents that certain objects are combined in a certain way, we mean merely that the elements of the picture are combined in that way, and are co-ordinated with the objects by the representing relation which belongs to the picture. (That this is a definition follows, I think, from 5.542.)

Light may be thrown on the 'form of representation' by the following remarks made earlier in the book on the structure and form of facts. "The way in which objects hang together in the atomic fact is the structure of the atomic fact. The form is the possibility of the structure. The structure of the fact consists of the structures of the atomic facts" (2.032, 2.033, 2.034). The only point which I can see in the distinction between-structure and form, is that the insertion of 'possibility' may include the case in which the alleged fact whose form we are considering is not a fact, so that we can talk of the form of the fact  $aRb$ , whether or no  $aRb$  is true, provided it is logically possible. It is to be regretted that the above definitions do not make it clear whether two facts can ever have the same structure or the same form; it looks as if two atomic facts might well have the same structure, because



objects hung together in the same way in each of them. But it seems from remarks later in the book that the structure of the fact is not merely the way in which the objects hang together but depends also on what objects they are, so that two different facts never have the same structure.

A picture is a fact and as such has a structure and a form; we are, however, given the following new definitions of its 'structure' and its 'form of representation' in 2·15, 2·151. "That the elements of the picture are combined with one another in a definite way, represents that the things are so combined with one another. This connection of the elements of the picture is called its structure, and the possibility of this structure is called the form of representation of the picture. The form of representation is the possibility that the things are so combined with one another as are the elements of the picture". This passage is puzzling; firstly, because we have here two different definitions of the form of representation, and secondly, because it is not obvious how to interpret "this connection" in the first of the two definitions; it may refer to the definite way in which the elements are combined, or to the whole of the preceding sentence, i.e. "this connection of the elements" may be that their combination represents a similar combination of the things. On neither interpretation does the first definition seem to coincide with the second. We can only hope to decide between these possible meanings of 'form of representation' by considering the things which Mr Wittgenstein says about it. Its chief property, which makes it of fundamental importance in his theory, is that asserted in 2·17: "What the picture must have in common with reality in order to be able to represent it after its manner—rightly or falsely—is its form of representation." Further, "what every picture, of whatever form, must have in common with reality in order to be able to represent it at all—rightly or falsely—is the logical form, that is, the form of reality. If the form of representation is the logical form, then the picture is called a logical picture. Every picture is *also* a logical picture. (On the other hand, for example, not every picture is spatial.)" (2·18, 2·181, 2·182.) It appears, then, that a picture may have several forms of representation, but one of these must be the logical form; and that it is not asserted that the picture must have the same logical form as what it pictures, but that all pictures must have the logical form. This also makes more plausible the deduction that the logical form of representation cannot be represented; for that it was common

to one picture and reality could afford no ground for supposing that it could not be represented in another picture.

Now it is easy to see a sense in which a picture may have the spatial and must also have the logical form, namely, by taking the form to be the (possibility of the) way in which the elements of the picture are combined. (One of the interpretations of the first definition given above.) This may be logical, as when the colour of a patch on a map represents the height above sea-level of the corresponding patch of country; the elements of the picture are combined as predicate and subject, and this represents that the corresponding things are also combined as predicate and subject. On the other hand, the form may be spatial, as when one dot being between two others represents that a certain town is between two others; but in this case we can also regard betweenness not as the way in which the dots are combined but as another element in the picture, which corresponds with itself. Then since betweenness and the dots are combined, not spatially, but as triple relation and its relata, that is logically, the form is logical. (Here then we have something which may be spatial and must also be logical; but it does not follow that this is the form of representation, for the form of representation may be some more complicated entity involving this and so derivatively spatial or logical.) If, indeed, the above were what were meant by the form of representation, then in saying that a picture must have the logical form Mr Wittgenstein would be saying no more than that it must be a fact; and in saying that we cannot represent or speak about the logical form of representation, no more than that we cannot talk about what makes a fact a fact, nor ultimately about facts at all, because every statement apparently about facts is really about their constituents. These things he certainly believes, but it seems to me unlikely that his complicated propositions about the form of representation amount to no more than this. Probably he is confused and does not use the term consistently; and if we revert to the second of the definitions given above, "The form of representation is the possibility that the things are so combined with one another as are the elements of the picture," we may discover another sense in which the picture has the form of representation in common with the pictured, namely, that the things with which its elements are co-ordinated by the representing relation are of such types that they *can* be combined in the same way as the elements of the picture; and so we arrive at the important principle that "the picture contains the possibility of the state of affairs which it



represents" (2.203). It seems to me, for reasons explained later, that the independent acceptance of this principle will justify almost all the non-mystical deductions which Mr Wittgenstein makes from the necessity of something in common between the picture and the world, which cannot itself be represented; and that these deductions can so be given a firmer basis than is provided by the nature of this elusive entity, the form of representation, which is intrinsically impossible to discuss.

In order to obtain any further comprehension of what Mr Wittgenstein thinks a sentence must have in common with the fact which it asserts, or, indeed, of most of his book, it is necessary to understand his use of the word 'proposition'. This is, I think, made easier by the introduction of two words used by C. S. Peirce. A word, in the sense in which there are a dozen words 'the' on a page, he called a *token*; and these dozen tokens are all instances of one *type*, the word 'the'. Besides 'word' there are other words which have this type-token ambiguity; thus a sensation, a thought, an emotion or an idea may be either a type or a token. And in Mr Wittgenstein's usage, in contrast, for instance, to Mr Russell's in *The Principles of Mathematics*, 'proposition' also has type-token ambiguity.

A *propositional sign* is a sentence; but this statement must be qualified, for by 'sentence' may be meant something of the same nature as the words of which it is composed. But a propositional sign differs essentially from a word because it is not an object or class of objects, but a fact, "the fact that its elements, the words, are combined in it in a definite way" (3.14). Thus 'propositional sign' has type-token ambiguity; the tokens (like those of any sign) are grouped into types by physical similarity (and by conventions associating certain noises with certain shapes) just as are the instances of a word. But a *proposition* is a type whose instances consist of all propositional sign tokens which have in common, not a certain appearance, but a certain *sense*.

As to the relation between a proposition and a thought Mr Wittgenstein is rather obscure; but I think his meaning is that a thought is a type whose tokens have in common a certain sense, and include the tokens of the corresponding proposition, but include also other non-verbal tokens; these, however, are not relevantly different from the verbal ones, so that it is sufficient to consider the latter. He says "It is clear that 'A believes that *p*', 'A thinks *p*', 'A says *p*', are of the form "'*p*' says *p*'" (5.542), and so explicitly reduces the question as to the analysis of judgment, to

which Mr Russell has at various times given different answers, to the question "What is it for a proposition token to have a certain sense?" This reduction seems to me an important advance, and as the question to which it leads is of fundamental importance, I propose to examine carefully what Mr Wittgenstein says by way of answering it.

First, it may be remarked that if we can answer our question we incidentally solve the problem of truth; or rather, it is already evident that there is no such problem. For if a thought or proposition token '*p*' says *p*, then it is called true if *p*, and false if  $\sim p$ . We can say that it is true if its sense agrees with reality, or if the possible state of affairs which it represents is the actual one, but these formulations only express the above definition in other words.

According to Mr Wittgenstein a proposition token is a logical picture; and so its sense should be given by the definition of the sense of a picture; accordingly the sense of a proposition is that the things meant by its elements (the words) are combined with one another in the same way as are the elements themselves, that is, logically. But it is evident that, to say the least, this definition is very incomplete; it can be applied literally only in one case, that of the completely analysed elementary proposition. (It may be explained that an elementary proposition is one which asserts the existence of an atomic fact, and that a proposition token is completely analysed if there is an element in it corresponding to each object occurring in its sense.) Thus if '*a*' means *a*, '*b*' *b*, and '*R*', or more accurately the relation we establish between '*a*' and '*b*' by writing '*aRb*', means *R*, then that '*a*' stands in this relation to '*b*' says that *aRb*, and this is its sense. But this simple scheme must evidently be modified, if, for example, one word is used for 'having *R* to *b*' so that the proposition is not completely analysed; or if we have to deal with a more complicated proposition which contains logical constants such as 'not' or 'if', which do not represent objects as names do. Mr Wittgenstein does not make it quite clear how he proposes to deal with either of these difficulties. As regards the first, which he almost ignores, he may reasonably plead that it results from the enormous complication of colloquial language, which cannot be disentangled *a priori*; for in a perfect language all propositions would be completely analysed except when we defined a sign to take the place of a string of simple signs; then, as he says, the defined sign would signify *via* the signs by which it is defined.



But the other difficulty must be faced, since we cannot be satisfied with a theory which deals only with elementary propositions.

The sense of propositions in general is explained by reference to elementary propositions. With regard to  $n$  elementary propositions there are  $2^n$  possibilities of their truth and falsehood, which are called the truth-possibilities of the elementary propositions; similarly there are  $2^n$  possibilities of existence and non-existence of the corresponding atomic facts. Mr Wittgenstein says that any proposition is the expression of agreement and disagreement with the truth-possibilities of certain elementary propositions, and its sense is its agreement and disagreement with the possibilities of existence and non-existence of the corresponding atomic facts. (4.4, 4.2.)

This is illustrated by the following symbolism for truth-functions.  $T$  stands for true,  $F$  for false, and we write the four possibilities for two elementary propositions thus:—

$p$	$q$
$T$	$T$
$F$	$T$
$T$	$F$
$F$	$F$

Now by setting a  $T$  against a possibility for agreement and leaving a blank for disagreement we can express, for example,  $p \supset q$ , thus:

$p$	$q$	
$T$	$T$	$T$
$F$	$T$	$T$
$T$	$F$	
$F$	$F$	$T$

Or, adopting a conventional order for the possibilities, ( $TT-T$ ) ( $p, q$ ). Evidently this notation does not in any way require  $p, q$  to be elementary propositions; and it can be extended to include propositions containing apparent variables. Thus  $p, q$  may be given not by enumeration but as all values of a propositional function,

i.e. all propositions containing a certain expression (defined as "any part of a proposition which characterizes its sense" (3.31)); and  $(-----T)(\xi)$ , where the solitary  $T$  expresses agreement only with the possibility that all the arguments are false, and  $\xi^1$  is the set of values of  $f\xi$ , is what is written ordinarily as  $\sim : (\exists x) . f x$ . So every proposition is a truth-function of elementary propositions, and many differently constructed propositional signs are the same proposition, because, expressing agreement and disagreement with the same truth-possibilities, they have the same sense and are the same truth-function of elementary propositions. Thus

$q \supset p : \sim q \supset p$  and  $\sim (\sim p \vee \sim p)$  are the same as  $p$ .

This leads to an extremely simple theory of inference; if we call those truth-possibilities with which a proposition agrees, its truth-grounds, then  $q$  follows from  $p$  if the truth-grounds of  $p$  are contained among those of  $q$ . In this case Mr Wittgenstein also says that the sense of  $q$  is contained in that of  $p$ , that in asserting  $p$  we are incidentally asserting  $q$ . I think this statement is really a definition of containing as regards senses, and an extension of the meaning of assert partly in conformity with ordinary usage, which probably agrees as regards  $p . q$  and  $p$ , or  $(x) . f x$  and  $f a$  but not otherwise.

There are two extreme cases of great importance; if we express disagreement with all the truth-possibilities we get a *contradiction*, if agreement with them all, a *tautology*, which says nothing. The propositions of logic are tautologies; and to have made clear this, their essential characteristic, is a remarkable achievement.

We have now to consider whether the above is an adequate account of what it is for a proposition to have a certain sense; and it seems to me that it certainly is not. For it is really only an account of what senses there are, not of what propositional signs have what sense. It enables us to substitute for " $p$ " says  $p$ ', " $p$ " expresses agreement with these truth-possibilities and disagreement with these others'; but the latter formulation cannot be regarded as an ultimate analysis of the former, and it is not at all clear how its further analysis proceeds. We have therefore to look elsewhere for the answer to our question. Towards this answer Mr Wittgenstein does make a clear contribution; in 5.542, he says that in " $p$ " says  $p$ ' we have a co-ordination of facts by means of a co-ordination of their objects. But this

<sup>1</sup> [Wittgenstein's use of the bar is quite different from that of other places of this book.—ED.]



account is incomplete because the sense is not completely determined by the objects which occur in it; nor is the propositional sign completely constituted by the names which occur in it, for in it there may also be logical constants which are not co-ordinated with objects and complete the determination of the sense in a way which is left obscure.

If we had only to deal with one logical symbolism I do not think there would be any difficulty. For, apart from variation in the names used, there would be a rule giving all propositional signs which, in that symbolism, had a certain sense, and we could complete the definition of 'sense' by adding to it these rules. Thus "' $p$ '" says that  $\sim aRb$  would, supposing us to be dealing with the symbolism of *Principia Mathematica*, be analysed as follows: call anything meaning  $a$ , ' $a$ ', and so on, and call ' $a$ ' ' $R$ ' ' $b$ ', ' $q$ '; then ' $p$ ' is ' $\sim q$ ' or ' $\sim \sim \sim q$ ' or ' $\sim q \vee \sim q$ ' or any of the other symbols constructed according to a definite rule. (It may, of course, be doubted whether it is possible to formulate this rule, as it seems to presuppose the whole of symbolic logic; but in any perfect notation it might be possible; for example in Mr Wittgenstein's notation with  $T$ 's and  $F$ 's there would be no difficulty.) But it is obvious that this is not enough; it will not give an analysis of ' $A$  asserts  $p$ ', but only of ' $A$  asserts  $p$  using such-and-such a logical notation'. But we may well know that a Chinaman has a certain opinion without having an idea of the logical notation he uses. Also the evidently significant statement that Germans use ' $nicht$ ' for not becomes part of the definition of such words as 'believe', 'think' when used of Germans.

It is very hard to see a way out of this difficulty; one may perhaps be found in Mr Russell's suggestion in *The Analysis of Mind* (p. 250) that there may be special belief feelings occurring in disjunction and implication. Logical constants might then be significant as substitutes for these feelings, which would form the basis of a universal logical symbolism of human thought. But it looks as if Mr Wittgenstein believes in another kind of solution, going back to his earlier statement that the sense of a picture is that the things are so combined with one another as are the elements of the picture. The natural interpretation of this in our present context is that we can only represent that  $a$  does not have a certain relation to  $b$ , by making ' $a$ ' not have a certain relation to ' $b$ ', or in general that only a negative fact can assert a negative fact, only an implicative fact an implicative fact, and so on. This is absurd and evidently not what he means; but he does seem to

hold that a proposition token resembles its sense somehow in this sort of way. Thus he says (5.512), "That which denies in ' $\sim p$ ' is not ' $\sim$ ', but that which all signs of this notation, which deny  $p$ , have in common. Hence the common rule according to which ' $\sim p$ ', ' $\sim \sim \sim p$ ', ' $\sim p \vee \sim p$ ', ' $\sim p . \sim p$ ', etc., etc. (to infinity) are constructed. And this which is common to them all mirrors denial". I cannot understand how it mirrors denial. It certainly does not do so in the simple way in which the conjunction of two propositions mirrors the conjunction of their senses. This difference between conjunction and the other truth-functions can be seen in the fact that to believe  $p$  and  $q$  is to believe  $p$  and to believe  $q$ ; but to believe  $p$  or  $q$  is not the same as to believe  $p$  or to believe  $q$ , nor to believe not- $p$  as not to believe  $p$ .

We must now turn to one of the most interesting of Mr Wittgenstein's theories, that there are certain things which cannot be said but only shown, and these constitute the Mystical. The reason why they cannot be said is that they have to do with the logical form, which propositions have in common with reality. What sort of things they are is explained in 4.122. "We can speak in a certain sense of formal properties of objects and atomic facts, or of properties of the structure of facts, and in the same sense of formal relations and relations of structures. [Instead of property of the structure I also say 'internal property'; instead of relation of structures 'internal relation'. I introduce these expressions in order to show the reason for the confusion, very widespread among philosophers, between internal relations and proper (external) relations.] The holding of such internal properties and relations cannot, however, be asserted by propositions, but shows itself in the propositions, which present the atomic facts and treat of the objects in question." As I have already said, it does not seem to me that the nature of the logical form is sufficiently clear to provide any cogent arguments in favour of such conclusions; and I think that a better approach to the treatment of internal properties may be given by the following criterion: "A property is internal if it is unthinkable that its object does not possess it" (4.123).

It is a principle of Mr Wittgenstein's, and, if true, is a very important discovery, that every genuine proposition asserts something possible, but not necessary. This follows from his account of a proposition as the expression of agreement and disagreement with truth-possibilities of independent elementary propositions, so that the only necessity is that of tautology, the only impossibility



that of contradiction. There is great difficulty in holding this; for Mr Wittgenstein admits that a point in the visual field *cannot* be both red and blue; and, indeed, otherwise, since he thinks induction has no logical basis, we should have no reason for thinking that we may not come upon a visual point which is both red and blue. Hence he says that 'This is both red and blue' is a contradiction. This implies that the apparently simple concepts red, blue (supposing us to mean by those words absolutely specific shades) are really complex and formally incompatible. He tries to show how this may be, by analysing them in terms of vibrations. But even supposing that the physicist thus provides an analysis of what we mean by 'red', Mr Wittgenstein is only reducing the difficulty to that of the *necessary* properties of space, time, and matter or the ether. He explicitly makes it depend on the *impossibility* of a particle being in two places at the same time. These necessary properties of space and time are hardly capable of a further reduction of this kind. For example, considering between in point of time as regards my experiences; if *B* is between *A* and *D*, and *C* between *B* and *D*, then *C* must be between *A* and *D*; but it is hard to see how this can be a formal tautology.

But not all apparently necessary truths can be supposed, or are by Mr Wittgenstein supposed, to be tautologies. There are also the internal properties of which it is unthinkable that their objects do not possess them. Sentences apparently asserting such properties of objects are held by Mr Wittgenstein to be nonsense, but to stand in some obscure relation to something inexpressible. This last seems to be involved by his reason for thinking that they are nonsense, which is that what they are meant to assert cannot be asserted. But it seems to me possible to give reasons why these sentences are nonsense and a general account of their origin and apparent significance, which have no mystical implications.

Sentences of this kind, which we call 'pseudo-propositions', arise in various ways depending on our language. One source is the grammatical necessity for such nouns as 'object' and 'thing', which do not like ordinary common nouns correspond to propositional functions. Thus from 'This is a red object' appears to follow the pseudo-proposition 'This is an object', which in the symbolism of *Principia Mathematica* could not be written at all. But the commonest and most important source is the substitution of names or relative names for descriptions. (I use 'relative names' to include 'p', the expression for a given sense *p*; in

contrast to a description of that sense, such as 'what I said'.) Usually this is legitimate; for, if we have a propositional schema containing blanks, the significance of the schema when the blanks are filled by descriptions presupposes, in general, its significance when they are filled by the names of things answering to the descriptions. Thus the analysis of 'The  $\phi$  is red' is 'There is one and only one thing which is  $\phi$ ; and it is red'; and the occurrence in this of 'It is red' shows that the significance of our proposition presupposes the significance of '*a* is red', where *a* is of the type of the  $\phi$ . But sometimes this is not the case because the proposition containing the description must be analysed a little differently. Thus 'The  $\phi$  exists' is not 'There is one and only one thing which is  $\phi$ ; and it exists', but simply 'There is one and only one thing which is  $\phi$ '; so that its significance does not presuppose that of '*a* exists', which is nonsense, for its truth could be seen by mere inspection without comparison with reality, as is never the case with a genuine proposition. But partly because we sometimes fail to distinguish '*a* exists' from 'The object meant by "*a*" exists', and partly because '— exists' is always significant when the blank is filled by a description, and we are not sufficiently sensitive to the difference between descriptions and names; '*a* exists' sometimes feels as if it were significant. Mr Wittgenstein gives in to this deceptive feeling so far as to hold that the existence of the name '*a*' shows that *a* exists, but that this cannot be asserted; it seems, however, to be a principal component in the mystical: "Not *how* the world is, is the mystical, but *that* it is" (6.44).

Our next example is provided by identity, of which Mr Wittgenstein gives an important destructive criticism: "Russell's definition of '=' won't do; because according to it one cannot say that two objects have all their properties in common. (Even if this proposition is never true, it is nevertheless *significant*)" (5.5302). And '*a* = *b*' must be a pseudo-proposition since it is true or false *a priori* according as '*a*', '*b*' are names for the same or different things. If now we adopt the new convention that two different signs in one proposition must have different meanings, we get a new analysis of descriptions not involving identity. For  $f(x)$  ( $\phi x$ ), instead of

$$(\exists c) : \phi x \supset_x x = c . fc,$$

we have  $(\exists x) . \phi x . fx : \sim (\exists x, y) . \phi x . \phi y$ .

And since  $(\lambda x) (\phi x) = c$  is analysed as  $\phi c : \sim (\exists x, y) . \phi x . \phi y$  we



see that '— = —' is only significant when one blank at least is filled by a description. Incidentally this rejection of identity may have serious consequences in the theory of aggregates and cardinal number; it is, for example, hardly plausible to say that two classes are only of equal number when there is a one-one relation whose domain is the one and converse domain the other, unless such relations can be constructed by means of identity.

Next I shall show how this account applies to internal properties of the senses of propositions, or, if they are true propositions, the corresponding facts. '*p* is about *a*' is an example; its significance might be thought to follow from that of 'He said something about *a*'; but if we reflect on the analysis of the latter proposition we shall see that this is not the case; for it evidently reduces not to 'There is a *p* which he asserted and which is about *a*' but to 'There is a function  $\phi$  such that he asserted  $\phi a$ ', which does not involve the pseudo-proposition '*p* is about *a*'. Similarly '*p* is contradictory to *q*' might be thought to be involved in 'He contradicted me'; but it is seen to be a pseudo-proposition when we analyse the latter as 'There is a *p* such that I asserted *p*, he  $\sim p$ '. Of course this is not a complete analysis, but it is the first step and sufficient for our present purpose, and shows how '—is contradictory to—' is only significant when one blank at least is filled by a description.

Other pseudo-propositions are those of mathematics, which, according to Mr Wittgenstein, are equations obtained by writing ' $=$ ' between two propositions which can be substituted for one another. I do not see how this account can be supposed to cover the whole of mathematics, and it is evidently incomplete since there are also inequalities, which are more difficult to explain. It is, however, easy to see that 'I have more than two fingers' does not presuppose the significance of ' $10 > 2$ '; for, remembering that different signs must have different meanings, it is simply ' $(\exists x, y, z) : x, y, z \text{ are fingers of mine.}$ '

Just as the explanation of some apparently necessary truths as tautologies met with difficulty in the field of colour, so does the explanation of the remainder as pseudo-propositions. "This blue colour and that," says Mr Wittgenstein, "stand in the internal relation of brighter and darker *eo ipso*. It is unthinkable that *these* two objects should not stand in this relation" (4.123). Accordingly a sentence apparently asserting that one named colour is brighter than another named colour must be a pseudo-proposition; but it is hard to see how this can be reconciled

with the indubitable significance of a sentence asserting that a described colour is brighter than another, such as 'My cushion at home is brighter than my carpet'. But in this case the difficulty could be completely removed by the supposition that the physicist is really analysing the meaning of 'red'; for his analysis of a colour comes eventually to a number, such as the length of a wave or what not, and the difficulty is reduced to that of reconciling the non-significance of an inequality between two given numbers with the significance of an inequality between two described numbers, which is evidently somehow possible on the lines suggested for 'I have more than two fingers' above.

Let us now pass to Mr Wittgenstein's account of philosophy. "The object of philosophy," he says, "is the logical clarification of thoughts. Philosophy is not a theory but an activity. A philosophical work consists essentially of elucidations. The result of philosophy is not a number of 'philosophical propositions', but to make propositions clear. Philosophy should make clear and delimit sharply the thoughts which otherwise are, as it were, opaque and blurred" (4.112). It seems to me that we cannot be satisfied with this account without some further explanation of 'clarity', and I shall try to give an explanation in harmony with Mr Wittgenstein's system. I think that a written sentence is 'clear' in so far as it has *visible* properties correlated with or showing the internal properties of its sense. According to Mr Wittgenstein the latter always show themselves in internal properties of the proposition; but owing to the type-token ambiguity of 'proposition' it is not immediately clear what this means. Properties of a proposition must, I think, mean properties of all its tokens; but the internal properties of a proposition are those properties of the tokens which are, so to speak, internal not to the tokens but to the type; that is, those which one of the tokens must have if it is to be a token of that type, not those which it is unthinkable that it should not have anyhow. We must remember that there is no *necessity* for a sentence to have the sense it does in fact have; so that if a sentence says *fa*, it is not an internal property of the sentence that there is something in it somehow connected with *a*; but this is an internal property of the proposition, because the sentence could not otherwise belong to that proposition type, i.e. have that sense. So we see that the internal properties of a proposition which show those of its sense are not, in general, visible ones, but complicated ones involving the notion of meaning. But in a perfect language



in which each thing had its own one name, that in the sense of a sentence a certain object occurred, would be also shown visibly by the occurrence in the sentence of the name of that object; and this might be expected to happen with regard to all internal properties of senses; that one sense, for example, is contained in another (i.e. one proposition follows from another) might always appear visibly in the sentences expressing them. (This is nearly achieved in Mr Wittgenstein's *T* notation.) Thus in a perfect language all sentences or thoughts would be perfectly clear. To give a general definition of 'clear' we must replace 'visible property of the sentence' by 'internal property of the propositional sign', which we interpret analogously to 'internal property of the proposition' as a property which a token must have if it is to be that sign, which, if the token is written, is the same as a visible property. We say then that a propositional sign is clear in so far as the internal properties of its sense are shown not only by internal properties of the proposition but also by internal properties of the propositional sign.

(It may perhaps be confusion between the internal properties of the proposition and those of the propositional sign which gives rise to the idea that Mr Wittgenstein's doctrines are, in general, only asserted of a perfect language.)

We can easily interpret this idea of philosophy in terms of the non-mystical account of internal properties given above. First, we notice and explain the fact that we often apparently do or do not recognize that something has an internal property, although this is a pseudo-proposition and so cannot be recognized. What we really recognize is that 'The object or sense meant or asserted by the words before us has this property', which is significant because we have substituted a description for a name. Thus as the result of logical proof we recognize, not that  $p$  is a tautology which is a pseudo-proposition, but that ' $p$ ' says nothing. To make propositions clear is to facilitate the recognition of their logical properties by expressing them in language such that these properties are associated with visible properties of the sentence.

But I think this activity will result in philosophical propositions whenever we discover anything new about the logical form of the senses of any interesting body of sentences, such as those expressing the facts of perception and thought. We must agree with Mr Wittgenstein that ' $p$  is of such-and-such a form' is nonsense, but " $p$ " has a sense of such-and-such a form' may nevertheless not be nonsense. Whether it is or not depends on the analysis of

" $p$ " is significant', which seems to me probably a disjunctive proposition, whose alternatives arise partly from the different possible forms of the sense of ' $p$ '. If this is so, we can by excluding some of these alternatives make a proposition as to the form of the sense of ' $p$ '. And this in certain cases, such as when ' $p$ ' is 'He thinks  $q$ ' or 'He sees  $a$ ', could be appropriately called a philosophical proposition. Nor would this be incompatible with Mr Wittgenstein's more moderate assertion that "Most propositions and questions, that have been written about philosophical matters, are not false, but senseless. We cannot, therefore, answer questions of this kind at all, but only state their senselessness. Most questions and propositions of the philosophers result from the fact that we do not understand the logic of our language" (4.003).

Lastly I wish to touch on Mr Wittgenstein's general view of the world. "The world," he says, "is the totality of facts, not of things" (1.1), and "it is clear that however different from the real one an imagined world may be, it must have something—a form—in common with the real world. This fixed form consists of the objects" (2.022, 2.023). It is an unusual view that any imaginable world must contain all the objects of the real one; but it seems to follow from his principles, for if ' $a$  exists' is nonsense, we cannot imagine that it does not exist, but only that it does or does not have some property.

Mr Russell in his Introduction finds an acute difficulty in the fact that  $(x) . \phi x$  involves the totality of values of  $\phi x$  and so, apparently, that of the values of  $x$ , which according to Mr Wittgenstein cannot be spoken of; for it is one of his fundamental theses "that it is impossible to say anything about the world as a whole, and that whatever can be said, has to be about bounded portions of the world". It seems doubtful, however, whether this is a fair expression of Mr Wittgenstein's view; for one thing, it suggests that it is impossible to say  $(x) . \phi x$ , but only perhaps 'All  $S$ 's are  $P$ ' taken as asserting nothing about the non- $S$ 's, which he certainly does not maintain. It may, then, be interesting to consider what he says which gives plausibility to Mr Russell's interpretation. He does undoubtedly deny that we can speak of the number of all objects (4.1272). But this is not because all objects form an illegitimate totality, but because 'object' is a pseudo-concept expressed not by a function but by the variable  $x$ . (Incidentally I do not see why the number of all objects should not be defined as the sum of the number of things having any specified property and the number of things not having



that property.) Also he says that "The feeling of the world as a limited whole is the mystical feeling" (6.45). But I do not think we can follow Mr Russell in deducing from this that the totality of values of  $x$  is mystical, if only because "The world is the totality of facts, not of things" (1.1). And I think that 'limited' gives the key to the sentence quoted above. The mystical feeling is the feeling that the world is not everything, that there is something outside it, its 'sense' or 'meaning'.

It must not be thought that the topics I have discussed nearly exhaust the interest of the book; Mr Wittgenstein makes remarks, always interesting, sometimes extremely penetrating, on many other subjects, such as the Theory of Types, Ancestral Relations, Probability, the Philosophy of Physics, and Ethics.

## EPILOGUE

Having to write a paper for the Society I was as usual at a loss for a subject; and I flattered myself that this was not merely my personal deficiency, but arose from the fact that there really was no subject suitable for discussion. But, happening to have recently lectured on the Theory of Types, I reflected that in such a sentence the word 'subject' must be limited to mean subject of the first order and that perhaps there might be a subject of the second order which would be possible. And then I saw that it lay ready before me, namely, that I should put forward the thesis that there is no discussable subject (of the first order).

A serious matter this if it is true. For for what does the Society exist but discussion? And if there is nothing to discuss—but that can be left till afterwards.

I do not wish to maintain that there never has been anything to discuss, but only that there is no longer; that we have really settled everything by realizing that there is nothing to know except science. And that we are most of us ignorant of most sciences so that, while we can exchange information, we cannot usefully discuss them, as we are just learners.

Let us review the possible subjects of discussion. They fall, as far as I can see, under the heads of science, philosophy, history and politics, psychology and æsthetics; where, not to beg any question, I am separating psychology from the other sciences.

Science, history, and politics are not suited for discussion except by experts. Others are simply in the position of requiring more information; and, till they have acquired all available information, cannot do anything but accept on