Gustav Bergmann
(dates unknown)

Supplement to old introduction

A better organization of the collapse of my world

contrasting then-and now. Then, when the foil collapsed, now, after reconstruction.

1.I then believed, as I still believe that cardinal numbers must be given ontolggical status and that the proper way to do ~~is~~ that is the one which, improving on Frege's design by introducing his types, is to assay them as classes of classes. ~~Hence x of x must have ontological status~~

2. I ~~s~~hen believed, as I still believe now, what ~~is~~ obvious, should be that fo this program to be realizable classes must be so assayed that they have ontological status.

3. I then believed , as I still believe that there are entities which I called in the foil derived characters, ~~x x x~~ such as e.g., the property of being both green and square, which zre of the same type as universals. ~~x green and square x~~ , being-green-and square, e.g., being of the first nonrelational type and in this respect like the universals green and square which in some sense are in them, though surely not, as already clearly brought out in the foil in the same sense in which green is in "a is green." Thus, even though falling in the type hierarchy and in this sense"properties " certainly a realm of their own. as shown by the ontological hypothesis which I called the Principle of Exemplification, does not apply to them. E.g., for winged horse to have this ~~weak status~~ "weak" status, it is not re quired,(as by that hypothesis for the ~~true universal~~ nonderived (simple) universal) that there be one,(that, as one says, it be exemplified. I then and now believe that, though I can now improve the dialectical argument

the dialectical argument for this belief by basing it upon what I consider a fundamental principle of ontology. another

4a. I then believed that classes are a very special kind of derived characters "built from", in a sense I shall not now specify, so-called unit classes.

4b. I now no longer believe that classes are of the same type as universals, either derived or underived, but entities entirely sui generis; although a certain analogy exists in three respects. (a) They are, inthe same as yet unspecieif sense of build', built from unit classes. (b) They fall into a type hierarchy of their own, and Ic) there are certain important correspondences between the two hierarchies, of types and classes, respectively.

I always also knew that there were worries about some inadequacies, or gaps, or difficulties, which R., bent as he was on"somehow founding mathematics" "overcame" by certain"axioms " .(I say "somehow found" because the ontological adequacy or even import seemed always questionable to me, if only because of the nominalism which is implicit in the first and all but explicit Ias I shall show) in the second PM, abd because his unfortunate tendency of confusing words and things, e.g., the property green with a word that may or may not stand for it.) I also had my troubles with these "axioms " Perhaps it will help if I mention them all three, even

substantive.
say something about only one.

a. Infinity. The two ~~properties~~ worries: truth, propriety, see ~~notes~~ these notes.
while I shall
b. Reducibility I~~x~~shall not even explain now what that is, there was something peculiar. see ~~notes~~ these notes.

c. The axiom of choice ~~seems~~ aleays seemed to me the opposite of Infinity, in being an ontological truism. I shall indeed later state as an ontological truism, ~~in every case the sort of thing the denial~~ of a stronger proposition, stronger in the sense that once classes are gfounded, the axiom of choice follows from it but not conversely. ~~Thus~~ yet the mathematicians needed it for the proof of well-order, a central notion from whhich I shall start (I do start, in a specifiable sense, in which R. starts from cardinality, from ordinality.) Zermelo's proof.

d. I ~~knew that Russell~~ have felt dimly for a long time, but have convinced my self now that R's did not really solve both of the two root paradoxes (perhaps that should be first point when discussing Russell's contribution..... explanation of the Cantor paradox, statement of V.C. ..... explaining well-prder.... Burali-Forti paradox..... Russell"s answer was inadequate. I knew all that. ~~Indeed~~ Yet, as firmly bent on other issues as ~~Russell~~ R ssell was on somehow founding mathematics, ~~AND MOST IMPORT~~ I trusted that the blemishes (if I may so call them) could be removed) so that th Russell-approach .... ~~type~~ un unramified type theory
cardinals
, with ~~ordinals~~ cllasses of classes and ~~cardinals clax~~
~~of~~ enumerous

ordin als classes of similar relations (check.. relation numbers)

. Or, tbriefly, I l ft well enough alone, trusting that a
that
solution lay in the direction of the R. would eventually emerge,
disposed to this belief
IN THIS IWAS POWERFULLY SUPPORTED BY MY ONTOLOGICAL PREOCCUPATION,

WHICH GRIPPED ME AS FIRMLY AS HE WAS GRIPPED BY HIS SOMEHOW FOUNDING
AND WITHIN THIS PREOCCUPATION
PREOCCUPATION, MY ANTINOMINALISTIC CONVICTION WHICH REQUIRED

A TYPE DIVISION OF UNIVERSALS..... irrespective of Russell's own

implicit and in the second edition virtually explicit, see these

notes.

QUestion that arose when my world collapsed: whether or not
data is in principle adequate, is it not in fact? That is a
question about our world which , like infinity axiom, raises

another, whether or not it is the case (it raises a question

of propriety. Isn't, if only relatively speaking, the feature

so contingent that it would not be ontologically appropiate to

ground arithmetic on it. The negative answ er to the second

question seems to me as obvious as in case of infinity; though I

shall in this Introduction say no more about the first, even

though there will a good deal about it later chapters.

I knew very well , long before my world had collapsed, that

the mathematical logicians were not only worrying about

the flaws, and irritated about what they considered inelegancies,

that would remain even if the flaws could be eliminated , but that

they, as is there way , they also tried to do something

This they developed what is known as axiomatic set theory. Not preoccupied as I was with the ontological issue, uninterested in mathematics as such, I did not follow their work at all as soon as I had convinced myself, to my own satisfaction that, for all its profoundity of some of it (the most distinguished practioners, in chronological order, probably Zermelo, von Neuman, Bernays, and Goedel (with his 1940 paper)... as mathematics, it was philosophically irrelevant. Yet, when my world collapsed, I did the obvious thing and turned once more to it, with a triple result.

First, I confirmed my convixtion of its philosophical irrelevance.

Second, I did, if only indirectly and tangentially, find myself and, in one respect of which more presently, encouraged stimulated by whatever I could understand of it (the books I found most helpful: Fraenkel, Bernays, and, really not quite so elementary, even though he calls it though

Third . I saw that while I still was not at all interested in their (mathematical subtleties and the connection between their math. and my ontological ones was only indirect, I could yet do no less than establish and make clear whatever strcutural similarities and connwctions there may be. And we shall see that, at least from an ontologist's view point, (I cannot speak from a mathmeaticians, there are some. So I shall make a point of bringing them out in the second half of the book.

see p. 7

That suggests an order of exposition.

I shall first state carefully the Principle which always has and still has been the cause of my belief that what they did was not irrelevant, at least as it stood, of no use to theontologist. This is the

COLLECTION PRINCIPLE,

first stated clearly in REALISM.

Restate it carefully. Add that it will have to be"qualified,"but that the qualification is more aparent than real, (order) and natural in view of a new category of entities of which more presently.

Then explain the encouragement by what, nontechnicallyvspeaking is von N. basic idea. He distinguished between sets and collectionsxxx classes. I between collex classes and collections. hexisxax Ontologically, if I may so exprrss myself, his is half-way house ifxIxmayxsayxsoyx on the way to mine. Are not elements..... do not exist.... hisxmotivatx brings out the different motivation.....xexcludesxparadoxes.. classesyxunlikexsets his inspiration c,early comes from the paradoxes: classes, since they cannot Be members of anything, cannot be members of themselves. My inx stubbornness is the One-Many issue in a contemporary version, my incapacity how the n as such make a n=1st.

bring in or merely mention? Perhaps there is better place later?

Then bringxin Bernays, who comes in naturally here; he too makes the distinction, though for a different and as he seems to think merely expository purpose ... Yes, that is better later, after explained what they do....... whatxthe ... showing that Bernays for convenience-sake use raises the one-or two and the one or two

ctoccrrecquestcoccr closure question

I shall second describe what they are doing

Describe axiomatics.... gently... elementary.....
with thenonformalized reasoning.... interpretation
issue..... in the most clear cut versions.
one basic undefined character.... epsilon.....
best stated formalizing the reasoning.....
all individuals sets..... single
basic character (fn. about LFC with
without equality..... those show more sense who
like Bernays, identify equality with sameness, thus
leave it to the formalized reasoning..

I repeat: formalizing the reasoning is helpful by permitting both to state the nature of what they are doing and providing cue for two questions.

Bernays does it.....also enough sense to introduce same into logic.....

Makes clear why I am unhappy. what is the interpretation. What are these entities? what is membership relation. If a collection is nothing how can something being a member of it be anything? (You see the source of the half-way house met aphor of von N.

BERNAYS WAY RAISES TWO OTHER QUESTIONS THAT WILL OCCUPY US...

Closurequestion and One-two question.

acknowledge and other memoranda p. 2

INTRODUCTION : STRUCTURAL HINT

Within the transition to , or near beginning of, part 3, or, equally plausibly, at end of part 2, when talking about the "minor" flaws of PM, introduce the dichotomy

FINITE - INFINITE

as follows. But first for a comment about

THE "MINOR" AND THE "MAJOR" FLAWS OF PM.

I presume no evaluation. Nor does it now make sense to score against him, except in the context of a historical analysis more rigorous than I am either capable or willing.

It is thus to be taken "biographically", as I hope has come out clearly, what was for me the "major" flaw, i.e., what shook me when I turned to the task, i.e., the collapse of delta with R.-L.-identity and, thereby, the question, what is a class; for until then, since I had convinced myself that, as I still believe, though I see now the need of improving and hope that I shall improve the argument, cla the existence of what in the foil is called a derived character, classes, or at least finite classes, say, being identical with either a or b, have ontological status as a very special kind of derived character. I knew that there were problems, of course, but even though I saw that the logicians approach did not solve them to the ontologist's satisfaction, I took it on faith that they could somehow be solved, in a manner that would safeguard the on. status of classes and thereby of numbers.... nor did I find it hard pushing these problems aside, since I was neither by taste nor by aptitude attracted to them, so as long as the unit class had not collapsed for me , I left well enough alone and "took it on faith " But when the unit class collapsed, my world collapsed, as it were, I knew that radical rethinking was necessary,

see attachment for better statement

(a) fine
(b) appropriate (analytic) ... non
(c) the way I circumvent ... issue

Stress finite - infinite
limitation to finite. But use BR's Inf

not merely because my ontology of arith had collapsed, but because I was convinced that its collapse was but a symptoms of inadequacies which affected the whole construction.

In this conviction, if I am now more nearly right or at least, less radically wrong, I have been confirmed by the success, such as it may or may not be, of the reconstrcution. Because it is radical indeed in two ways and insists on

(1) It has reached what one may call, dramatically, the limits of ontology. Less dramatically, a fact as simple as this (tone) being louder than that has in addition to two "constituents" which are "existents" also other "ingredients", namely a basic "action" and order (a is louder than b, then not b than a.) Thus if ontology is a way of assay all the worlds entities, including of course its facts, as "constituted" of other entities, since these ingredients are not, entities, what is meant by "limits, is clear.

(2) still radically, but less so than (1) new entities have been added. New kinds so general that categories. the cue is dwlta..... internal relations.... unternal properties.... I have had before but neglected.....

ALL THIS IS BY THE WAY. I MERELY WANT TO EXPLAIN WHY I CALL ONE OF THE PM INADEQUACIES MAJOR (The explanation is biographical, yet I hope, as it should, useful in finding a way through the mace of and to the heart of my involved argument.)

One of the MINOR WHICH WILL PROVIDE A CUE: INFINITY AXIOM

BR saw that in order to ground arithmetic (and particularly the whole rich structure of classical mathematics which unlike the so-called intuitionists and later, following them, W., he was obviously not prepared to abandon, he had to postulate that (say, if only to be specific, without distorting anything which is at this point essential) the number of particulars in the world (or, as he would have said individuals) is infinite.

Be worried about that. In two ways, both justified. First, is the "postulate" true Second, is that the sort of thing on which, even if true, the ontology of arith.

ought to be made to depend? This worry is philosophical, of course. The math-log, the axiomatic set theorists all at some point make this postulation, without worrying about it, nor need they worry about ft, considering what they are about. This, though, shows what theh are about is of little, if any help to the ontologist, but something radically different.

Russell, though he worried about it, which does honor to the philosopher in him, was yet so bent on "founding" mathematics that, however uneasily, he accpeted the postulate. Or, at least, he did nothing about it.

This is characteristic of what he did also with at least

in the sense above) which I here

this or some other place in the

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speaking, technical. The first is

of the V.C. P., in which

after a fashion, as will come out

repeat

repeat only after a fashion, and

the math, worried so much. The technical ground was that he could not without it produce a viable "definition" of R.L identity.

Yet, in the second edition, he gave up the axiom and with it the safeguard against violation of principde, which he first thought and which now again think, lies so deep that it deserve the status, not, to be sure, of a postulate in the mathematicians sense, but of an ontological hypothesis (what I mean by that will be explained).

This, I gather, he did, at least in part, because he was wrongly reassured by a comment of Ramsey's which, if I am right, eliminates the worry only in the case of classes but not of properties. But, then, in this he

but something radically different. } apt comment which might be used later on this point in order to bring in both.

Russell, though he worried about it, which ~~shows~~ does honor to the philosopher in him, was yet so bent on "founding" mathematics that, however uneasily, he accpeted the postulate. Or, at least, he did nothing about it.

This is characteristic of what he did also with respect to other "minor" flaws ("minor" at least in the sense above) which I here mention (for ~~possible~~ consideration for inclusion at this or some other place in the Introd, because they will be dealt with later.

all this comes out in conrast between two editions (introductions

1. Another Postulate. Reducibility. ~~Postulated it~~ Postulated ~~it~~ it for two reasons, one of principle, or even philosophical; the other., relatively speaking, technical. The first is because he needed it in order to ~~avoid the V.C.~~ guard against violation of the ~~vicious~~ V.C. P., in which he thought he had identified the common root ~~which he thought was the common ground of all the~~ paradoxes (~~this~~ in this, after a fashion, as will come out, if I am right, he had the right flair, though, I repeat ~~repeat~~ only after a fashion, and very broadly), ~~which~~ about which both he and the math, worried so much. The technical ground was that he could not without it produce a viable "definition" of R.L identity.

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that is done better in new introd. will insert 4 to prefix (after infinity has been mentioned and used to introd. his point)

was at least consistent since, under pressures to whixh I will turn ~~in a moment~~, he had by then given up the distinction between properties and classes. (INCl.) Add quotation feom Introd.2!!! Fromxwherexlxx standxx My thought has developed in the opposite direction. Thexfex I always have distinguishedxbetweenxpropertiex, and certainly have in the foil, distinguished prop between simple properties and those I called (and still call) derived in general, on the one hand, and, on the other, those very special derived onesx(disjunctionxofx ones, namely, disjunctions (either finite or infinite of ones, that was the point I didxnotxwo deliberately did not worry) of R.L type unit classes By now however I believe that classes, say the class of all green think things,

(Incl.) Nor was he just consistent, he was right in that, as I shall show that, in view of what certain logicians would call my Platonism, not only with respect to properties but also qith respect to classes, which makes Ramsey,s comment eliminate the worry.

on the one hand, and the property green, though both existents, are not only two but not even of the same type!

Similarly with respect to another"minor"flaw. In the first edition, hexdefine introducing classes as what he calls one kind of incomplete symbol, (FN about the other kind; def. descritions, which indeed do not refer, even if successful, to any existent in addition to the one they describe, if the condition is fulfilled.(and... while, with a characteristic difference no such condition is required in the case of classes) he claims on this ground that classes do not exist. Thatxinxthexsecondxeditionxxfromxwhere That calls for two comments.

1. By the second edition, where the difference between classes and properties is deniedx denied, that makes the his 2x putsxhisxin?? nominalism (there are only individuals) quitexexplicitxasxnearl explicit, even if he himself does not draw the thisxlast,obvious obvious conclusion from these two premisses, i.e., that classes do not exist, and that there is not even a shadowy diff-

2. Classes are arbitrary. Explain. Yet the way classes are introduced as incomplete symbols they require that there be a property shared by all the individuals which are , as one says its extension. There seems to be a prima facie plausible defense against this criticism. Of it, or, rather of what it implies, without there being any indication that R. would have been prepared to accept the implication, presently.

General COMMENT: THIS ARBITRARYNESS BUSINESS IS ITSELF SO MAJOR A FLAW, THAT IT IS AT LEAST A PART OF THE "ONE"MAJOR FLAW. CONSIDER THAT IN THE TOTAL EXPOSITION.

Be that as it may, by the second edition he had realized that his device of introducing classes as incomplete symbols does not work, i.e., did not yield him smoothly the classes of classes, and so on, without which he could not "found" arithmetic in a way that resolved at least the Cantor paradox.

FN. NOTICE I SAY AT LEAST THE CANTOR PARADOX: FOR THE BURALI FORTY PARADOX HE HAS AS FAR AS I KNOW NEVER RESOLVED, OR, RATHER, NEVER APPRECIATED, I.E., THE SO_CALLED SOLUTION OF IT AT THE END OF 1908 PAPER, WHICH IS THE CLEAREST PRESENTATION OF HIS IDEAS (IT IS CERTAINLY THE CLEAREST I KNOW) is clearly inadequate, unless one assumption even less justifiable than that of the infinity axioms, More precisely, it is less justifiable because it is much stronger than the infinity postulate, infinitly much stronger, if I may put it this way, letting it go at that for the moment.

There is at least a glimpse of this roughness in the first edition (add Quoation). In the interval between the two edition it had apparently come home to him. As a result, preponderantly bent as he was on founding mathematics, at any ontological prize, if I may so express myself, he was in the second edition prepared to assume what in the first he had strenuously argued against, that our world as a whole is extensional.

FN. I say"our world as a whole, because of the way he kept introducing classes as insisting, at least verbally , that classes must be introduced, namely, as incomplete symbols. But perhaps this is not quite fair. For his argument in the first edition was rested essentially - though as we shall see, if I am right,

arbitrariness
Extensionality
and Intentionality
further VII classes, systematically — then in X with respects Russell

see [illegible] in ISM, Item 17

as here

inadequately or improperly, leave that until later, on the what has since be called the intentional contexts, which of which there are none in the trunk. Yet much would indeed be gained if arithmetic could be secured in the trunk. Hence: not fair. But there is again a twist, which permits me to anounce another result of the reconstruction. I have since come to belief that even if the trunk were in fact extentional, its intentionality would not be the proper ground on which to rest arithmetic; just as an infinity of individuals, even if there were on, would not be.

later

I return to and conclude briefly with what I started with

FINITE - INFINITE/

In my ontology, a certain distinction becomes fundamental

(1) finite or infinite with respect to things (say particulars, but also, if you wish, properties of any type, or relations, (simple, not derived)

(2) finite with respect to its facts.

My world, already foil and much earlier than RCBM, is such that a world may what our world quite probably is, finite (1) and yet infinite (2)..... that permits me to together with certain other of its features (exist of forms and potential facts) to get along without his unreasonable postulation ("Unreasonable in at least one and quite possibly 2 senses).... connected with actions.... facts conn. with actions, ctaions connected with order..... ther new ingredients more radical....more radical even than the new entities ..... no more now. see chapters III, V. VII, VIII, and, of course X..... only a minimum now, "verbally" if at all.