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AN EXAMINATION OF LOGICAL POSITIVISM

By

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NEW YORK

HARCOURT, BRACE AND COMPANY

LONDON: KEGAN PAUL, TRENCH, TRUBNER & CO., LTD.

1936

Dedicated to my Mother

PRINTED IN GREAT BRITAIN

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PART I

LOGICAL FOUNDATIONS

CHAPTER I

WITTGENSTEIN'S THEORY OF MEANING

The new logic, developed by Boole, Schroeder, Pierce, Peano, and Frege (to mention only the most important names) was made into a well-organized system by Russell and Whitehead in the *Principia Mathematica.* A definite logical theory underlies this work. The *Principia Mathematica* is, nevertheless, incomplete or erroneous in at least three respects. This incompleteness or erroneous character could be explained somewhat paradoxically by saying that (1) there is too much theory, (2) there is not enough theory, and (3) there is no theory whatsoever. There is too much theory in the sense that a purely symbolic system, purporting to be logically autonomous, should not require any verbal or non-formal instruction for its manipulation and should not require any theoretical basis not contained in the formal paraphernalia of the system, whereas *Principia* must be explained at every step by non-formal instruction and theories, etc. On the other hand, it is no objection, but rather a logical demand, that whatever theory can be formalized within a formal system should be so formalized, and that whatever cannot be formalized should not occur at all within the system. In this sense, *Principia* does not contain enough theory. Finally, whatever deserves the name of " theory of a formal system " should be organized in a completely articulate manner, such that no part of the theory does not have a well-defined connection

with the theory as an organized whole. In this sense *Principia* has no theory at all.

An axiom which is not formally expressed, but which is integral to *Principia*, is the so-called axiom of Extensionality. This axiom states that every function of functions is an extensional function. It is not necessary to inquire whether " function f is extensional " is an exception to this axiom. The important thing is that the axiom is apparently violated almost at the outset by the introduction of the proposition connecting real and apparent variables. For instance $(x)\ \phi x \supset \phi u$, which is roughly translated as " whatever holds of all, holds of any ", should be an extensional function by the axiom of extensionality. Now $(x)\ \phi x$ is evidently an extensional function of $\phi\hat{x}$ whereas ϕu is apparently not an extensional function of the propositional function, since the idea of " any " is equivalent neither to that of " all " nor to that of " this individual one ". The idea of " any ", therefore, has no place in the system, and its introduction indicates the absence of a theory in *Principia*.

Again, the theory of types may be considered. A type is the range of significance of all propositional functions which take the same objects as values of their arguments, i.e. of all equivalent functions. The theory, or, better, axiom of types, states : Arguments of a given function are all of the same type. This theory cannot be formulated within the system of *Principia* because the idea of " any " possible argument of a given function is not an extensional idea. There are other reasons why this idea cannot be formulated, one of which is that constant expressions occurring in mathematical logic are limited to the logical constants, and because " type " is a constant expression, but not a logical constant, it cannot occur in the formal system of *Principia*. This much for the theoretical difficulties of *Principia*, considering it solely from the formal point of view.

From the broader standpoint of general philosophy, other difficulties arise which are of greater interest here. The explicit purpose of the *Principia* was to demonstrate

that the concepts and assertions of mathematics are entirely derivable from the concepts and assertions of symbolic logic. From many essays of the authors (particularly Russell), as well as from the fact that certain sections have an especially philosophical interest, it is revealed that another equally important purpose also guided the construction of the *Principia*. The construction of an exact logical language is to serve in solving philosophical problems and in presenting a complete schematism for representing the structure of the world of science and experience.

This construction can occur in two ways, each of which stands in subtle opposition to the tenets of Logical Positivism, but seems none the less to be demanded by those tenets. The first method of construction is to introduce the set of forms of all elementary propositions as a complete group of primitive forms of all facts which can occur in science or experience. Thus, if $x, y, z \ldots$ represent the constituents of a fact and R_n represents the component, then the forms of all facts would be illustrated by the following schema :—

$R_1\ (x)$ " Quality-individual " form
$R_2\ (x, y)$, " binary " relational form
$R_3\ (x, y, z)$ " ternary " relational form.

In general :—

$R_n\ (x_1 \ldots x)$, " n-adic " relational form.

From the purely notational point of view this schema could be simplified by treating all forms as classes or as predicates.

$\phi\ (x)$
$\phi\ (\,(x),\ (x, y)\,)$
$\phi\ (\,(x),\ (x, y),\ (s, y, z)\,)$, etc.

Philosophically these schemata are indefensible because they neglect the fundamental distinction between classes and relations. But could not the same criticism be levelled against the treatment of an n-adic and an $n+1$-adic relation which neglects the fundamental distinction between n-ads and $n+1$-ads ? The difference between two relations with different numbers of terms is not simply the difference of

number. On the one hand the whole structure of a fact is different according to the number of constituents of the fact. A general sequence of propositional forms has, therefore, only a notational value, but tells us nothing about the world's structure. The attempt to set forth all possible forms of propositions is required by Logical Positivism. Its principal instrument of investigation is logic, for logic alone can express the syntax of language. The example above shows that the syntax of language cannot be formulated except from an arbitrary standpoint. On the other hand, this conclusion would seem to follow from the empiricism of Logical Positivism. Only those forms of propositions which have genuine counterparts in the empirical world are to be admitted in the logical schematism of admissible (i.e. significant) concepts.

The second method of construction involves no *a priori* decision concerning the possible forms of all propositions. The structure of propositions is related to the structure of facts ; if new kinds of facts cannot be foreseen, their possible forms cannot be anticipated in discourse. A strict and thorough-going empiricism would have to concede that new kinds of facts cannot be foreseen. The second method, therefore, is simply concerned with the logical treatment of propositional forms already known to have objective counterparts in the empirical world. Logical analysis can, therefore, be applied only to what is already known to be significant, because discovered in the empirical world. Here, logical analysis would, except in a few cases, be superfluous because the genuine value of logical analysis consists in its application to those assertions which have hitherto not been examined. Elimination of pseudo-concepts of science by means of logical analysis seems to demand a schematism of admissible conceptual forms. An empirical criterion of meaning and verity seems to make the construction of such a schematism impossible. This is the problem inherited by Logical Positivism from its empiristic and logistic forbears. The *Principia Mathematica* seems to favour the construction of

a schematism such as has been set out above. Its logical theory, which is only implicit as we have seen, must be altered for the purposes of analysis in the positivistic sense. Wittgenstein's logical theory may, from this point of view, be conceived as a criticism and alteration of the logical language of *Principia Mathematica*.

With these remarks I shall proceed to develop the logical theory of Wittgenstein.

I

The fundamental characteristic of Wittgenstein's philosophy is the relationship which he attempts to establish between language and the world. By language, in this usage, is meant the totality of significant assertions (as contrasted, e.g., with language as used in the emotive sense). The totality of significant propositions is related to the totality of objectives of those assertions, and this is the world. " The world " is thus a phrase with a denotation but without connotation.

Wittgenstein calls the objectives of significant assertions " facts ". Facts are what make propositions true, or, alternatively, propositions assert the existence or non-existence of facts. Since facts are the fundamental parts of the world, it would be impossible to define " fact " without circularity. A fact may be described as a combination of objects. This differs from the Aristotelian conception of fact only in so far as a fact may be of any conceivable structure in Wittgenstein's philosophy, whereas any Aristotelian philosophy (for metaphysical reasons) limits facts to the " inherence of something in something else ". An object is whatever can occur as the constituent of a fact. Now, if facts are taken as fundamental, and hence indefinable, an object could be variously defined. (*a*) It may be defined as the set of facts in which it occurs, i.e. as the set of facts which possess at least one feature of absolute similarity to one another. For example, the facts of " blue

colouring the sky at time t_o" and of "blue colouring this book at time t_n" have one feature, "blue," of absolute similarity. (*b*) Or an object can be defined as whatever is a distinguishable element of a fact. Thus, by exhaustively enumerating all the distinguishable elements constituting a fact, it is possible to isolate all the objects composing the fact in question.

The fact is an independent entity, for whatever dependence may mean in the strictly logical sense, it is reserved for objects, i.e. for entities obviously requiring completion. Facts, being self-sufficient, require no completion, and so are, in the logical sense, independent of one another. Objects are independent, too, in the sense that an object is not restricted to occurrence in one fact rather than another, but they are dependent in the sense that they must occur in some fact or other.

For the purpose of clarity and without doing too much violence to Wittgenstein's theory, examples of facts may be taken from the perceptual experience of an individual. Thus "coloured-spots," "relations among spatial figures such as black spots on white paper," etc., may be used as illustrations.

The relation between the colour and the object coloured, between two tones in order of temporal succession—in general, among objects which go to make up a fact—is not a further element of the fact over and above the objects related. The relation is the structural, the articulate, feature of the fact. But it is nothing beyond the objects which are related in a specific manner. In other words, the objects combining to form a fact do so by internally combining with one another. In short, the structure of the fact is not an element of the fact; rather objects are only elements of given fact in so far as they enter into the fact in some specific mode of combining with one another. Structure, accordingly, cannot be isolated and designated by a single term of discourse, that is, structure cannot be named.

In my opinion, this part of Wittgenstein's analysis is his

first important contribution to philosophical logic. The conclusions which are drawn from it are not unquestionable, but the importance of the emphasis on structure must not be overlooked. Those problems which depend on the nature of *relation* should find the key to their solution in the preliminary recognition that the structure of things and facts is represented in language and thought in an entirely different way from the way in which things themselves are represented. The notion of structure, which will presently be further elucidated, is of fundamental importance. I shall return to it frequently, for it is the basis both of the valid and the invalid deductions that are made by Logical Positivism.

In current usage, "fact" has a very vague significance. "Facts" are spoken of and the expression "it is a fact that . . ." is frequently used. It cannot be over-emphasized that "fact" is used by Wittgenstein in a technical sense which is somewhat uncommon among other philosophies, and which does not normally occur in discourse at all. At the same time, it is fairly clear that the philosophers who use "fact" in this way mean that facts are what make propositions significant and true. Now, this is somewhat paradoxical, because, while we can use "fact" correctly in sentences, and while, if we thought about it at all, all of us probably would come to explain "fact" in approximately similar ways, it is, nevertheless, true that we ordinarily use "fact" without knowing exactly what we mean. I believe this difficulty has two sources. In the first place, we speak of true and false propositions, and of the existence and non-existence of facts. These terms seem to be correlative, but since a "non-existent fact" is obvious nonsense, it is clear that the correlation is only apparent. There is not in ordinary language an adequate terminology to explain the relationship of propositions and facts. The second source of confusion results from the first. The introduction of the required terminology can occur only by way of a complicated theory, and none such is at our disposal. Hence, when an attempt

is made to understand Wittgenstein's conception of fact, the following must be borne in mind: (1) it is a technical idea; (2) it is, nevertheless, connected with ordinary usage, if that usage be exactly fixed; (3) speaking of "facts" violates the rules of logical syntax in Wittgenstein's theory, and serves only an elucidatory purpose, to be dispensed with as soon as the ideas have become clarified.

As a technical idea, a fact is simply a combination of formal entities; that is, a combination of objects. As indicated above, this is not a definition, because it would then be circular, since facts are the indefinables. Objects are distinguishable elements of facts. This again is not a definition, for it entails the definition of "being the same object as" and "sameness" in Wittgenstein's theory is likewise indefinable. Moreover, objects are named or described rather than defined. At the very best, one can indicate only what constitutes being an object by pointing it out or by enumerating the facts of which it is a possible constituent.

With these preliminary remarks I shall proceed to develop the fundamentals of Wittgenstein's theory.

The world is the totality of independent atomic facts. An atomic fact is a fact which is not compounded out of other facts. Since facts are ultimately independent of one another all compound facts are reducible to atomic facts. Which facts are atomic and which are not cannot be determined *a priori*, but must, in any case, be discovered by direct inspection. That atomic facts must exist as a demand of logical theory is, according to Wittgenstein, an abstract necessity capable of demonstration. This supposed demonstration will later be shown to be fallacious.

Atomic facts are composed of objects in immediate combinations. The way in which the objects are combined is called the structure of the fact. The possibility that objects may combine together in a definite way so as to constitute an atomic fact is called the form of the object. The form of the object is, therefore, the possibility of the structure of

the fact. Since "possibility" is a logical concept and not an ontological one, "object" is a formal reality requiring completion in order to exist, and only conceivable as being completable in a given set of ways. The totality of ways in which an object may acquire completeness or materiality is, as noted above, the form of the object.[1]

Wittgenstein also believes that the object must be an absolutely simple entity. This belief is supposed to be capable of logical demonstration. The fallacy which can be found in the demonstration of atomic facts occurs in this one also. It is advisable to present these proofs in their rightful place.

II

The realm of propositions is far richer, perhaps infinitely so, than the realm of objects and facts. Instead of the simple totality of atomic facts, there are positive and negative propositions, conjunctions, and disjunctions of propositions. There must also be added the different ways of expressing the same proposition. The complexity of the world is outdone by the greater complexity of symbolism. How, then, can a relationship be established between them? Wittgenstein attempts to do this in a simple, but amazingly forceful, analysis. There are two essential parts to this analysis, the explanation of the relation between propositions and facts, and second, the explanation of the interconnection of propositions among themselves.

One of the most commonly expressed opinions of the first mentioned relationship is: the relation between propositions and facts is an agreement when the propositions are true and a disagreement when they are false. "Agreement" is a somewhat vague term, and requires a precise definition in order to serve the purposes of exact analysis.

Wittgenstein conceives agreement between propositions and facts to consist of a pictorial relation. The proposition is a picture of the fact. If the pictorial relationship is

[1] I have explained "possibility" more fully in Chapters II and VI.

concealed, this is because of the use of an arbitrary system of notation, the rules of which are not known. The pictorial character of the proposition consists in the circumstance that it is itself a fact which possesses certain features in common with the other fact which it pictures. These common features between the two facts by virtue of which one can picture the other are : (*a*) a common logical form, (*b*) a one-one correlation between the objects comprising the respective facts. Wittgenstein believes that this relationship cannot be further explained. In Russell's logic, however, it is possible to express the community of structure of two facts. Let x, y and z,w be two pairs of objects, the first united in an R-way and the second in an S-way, thus xRy and zSw. Now the necessary and sufficient conditions that R and S possess the same structure are : There is a correlator P whose domain is the field of R and whose converse domain is the field of S, such that the relative product $P \mid S \mid \breve{P} = R$. Thus the structure of a fact, for Russell, would be the class of all facts structurally similar to a given fact. For Wittgenstein, however, structure is not further analysable. It must be presupposed. I introduced the definition, nevertheless, in order to enable the reader to understand what Wittgenstein intends to convey by the terms " structure " and " having a common structure ".

The necessary and sufficient condition for picturing one fact by another is community of structure. This does not mean that of any two facts possessing the same structure, one will be a picture of the other, for " being a picture of a fact " involves that which, by arbitrary convention, is decided upon to be so used. Nevertheless, it is not the arbitrary decision, but the possession of a certain structure, which makes the picturing of facts possible.

Propositions, then, are pictures of facts. The proposition which pictures a given fact is a fact in its own right. It may be a psychological process, a set of marks, a vocal utterance, or the like. It is noteworthy that two or more facts may be used to represent the same objective fact. In

this case, another complication arises. When two or more facts are so used they must all have something in common, for they all express the same sense. It is then necessary to distinguish between the essential and the unessential features of the propositions. The unessential features of a proposition are those characteristics of a particular language or mode of expression which disappear when the proposition is translated from one language to another, or when the proposition is expressed vocally and then written. The essential features are those which remain constant throughout all transformations of this kind. Wittgenstein claims that the logical form is the common invariant feature of the various modes used to express a proposition. Therefore, several facts differing in many particulars may be used to express the same sense, i.e. to picture the same objective fact, if they enjoy community of form. Thus it is necessary to amend the definition of a proposition. A proposition is a fact used to picture another fact, or a class of facts used to picture another fact. In the first case the structure of the propositional fact is identical with the structure of the other fact. In the second case the class of facts enjoys a common structure, and this structure is identical with the structure of the fact represented.

This emendation introduces a further property of the proposition. We may speak of the proposition as sign and of the proposition as symbol, or alternatively, of the external and of the internal features of the proposition, or, again (as above), of the unessential and the essential features of the proposition. Regarded as a sign, or externally considered, the proposition is a fact in its own right. As symbol or internally considered, it remains a fact in its own right, but is not so regarded. It is used, to represent (i.e. to picture) another fact. When so used it exists solely for representative purposes, and it is no longer an object of contemplation. As Russell, in explaining Wittgenstein's theory, has said, no mention is made of it, but by means of it mention is made of something else. The external feature of the proposition

disappears from view and the internal symbolic properties alone are contemplated in so far as they represent another fact. Again, the symbolic internal properties are the characteristics of structure.

Wittgenstein calls the structural properties of the proposition its form of representation. It is in virtue of this form that the proposition expresses a sense. *The sense of the proposition is the possibility of the fact which it represents.* It is necessary to distinguish between the sense of a proposition, the truth of a proposition, and the proposition itself. The delineation of the concept " sense " shows this very clearly. That which the picture represents is its sense. The proposition neither is identical with its sense nor does it contain its sense. The possibility of expressing the sense is all that is to be found in the proposition. The sense of a proposition is, therefore, distinct from the proposition as such. For two propositions may express the same sense, e.g. " Cæsar loves Brutus " and " Caesar amat Brutum ". Likewise " Brutus is loved by Cæsar " expresses the same sense as " Cæsar loves Brutus ", yet they are two different propositions. The truth and the sense are distinct, because the former consists of the actual agreement of sense with reality, whereas the latter is simply the possibility of agreement. Thus a proposition can be understood without knowing whether it is true.[1]

In this exposition I have tried as far as possible to allow the author to speak for himself without interrupting with criticisms. It is necessary to have a fuller comprehension of the theory before introducing critical considerations. I shall anticipate two points here which will perhaps aid in the understanding of the theory.

The first is that the ultimate reference of all propositions which have a sense for us is the empirical realm. The atomic facts, therefore, are experiential facts. The sense of propositions is to be found in experience and experience

[1] Wittgenstein, L., *Tractatus Logico-Philosophicus*, London, 1922, prop. 3.13, 4.031, 4.2.

alone. The second is that the ordinary conception of the proposition as written, spoken, or thought, leads to another view than the one outlined above. There are at least three reasons for this contrary opinion. The general proposition, the descriptive proposition, and many propositions which are neither general nor descriptive seem to be understandable without reference to any experience. Likewise, the propositions of logic and arithmetic seem to be independent of experience. Wittgenstein must show, therefore, that general propositions are reducible to elementary propositions, that descriptions are transformable into representations, that representations, i.e. elementary propositions, are simply and exclusively concerned with empirical reality, and finally that logic and mathematics can be treated in such a way as to avoid an a-prioristic interpretation. All of these considerations save one must await the treatment of general propositions. At present I shall simply discuss the question " If elementary propositions are not ostensibly pictures of facts of experience, how can they be shown actually to be pictures ? " [1]

The theory that elementary propositions, and thus thoughts of a certain kind, are pictures of reality is not new. In one sense it is a translation into logical language of Hume's theory of ideas as copies of impressions. The important difference lies in the fact that the nature of psychological processes does not enter into consideration in the present theory. None the less, certain objections to the former theory may be reiterated with regard to the present one.

Certainly the proposition in its external form does not resemble the fact for which it stands, save in rare instances, e.g. *in the case of a map*. The great majority of propositions does not appear to be remotely like the thought or the fact. Schlick, who now accepts Wittgenstein's theory, formerly

[1] For opposite views see : Frank, Phillipp, " Was bedeuten die gegenwärtigen physikalischen Theorien," usw., *Erkenntnis*, Bd. I. Gätschenberger, R., *Zeichen, Die Fundamente des Wissens*, Stuttgart, 1932. Schlick, M., *Allgemeine Erkenntnislehre*, 2 Auf. Springer, 1925.

criticized similar doctrines. Thus he wrote: "In ordinary language agreement simply means likeness. Two tones, two colours, two proportions, two opinions, agree if they are alike. The word is obviously not to be taken in this sense here, for the judgment is something completely different from that which is judged, . . . it is not like that which is judged, and this can only be contested from the standpoint of adventurous metaphysical systems which equate thought and being in general, and about which we should waste no words here."

"If agreement here does not mean likeness, perhaps it could mean similarity. In what sense are our judgments similar to facts? Similarity must mean at least partial likeness, hence it must be possible to find certain moments of the judgment which are revealed in the facts themselves. In purely conceptual truths where the object judged, as well as the judgment, consists of purely ideal forms, likeness might be found in both sides under certain circumstances, but that cannot be the essential requisite for truth, for propositions about real things also make claim to truth—here, indeed, the nature of truth first becomes a problem—but in both one will seek for such similar moments. For the concepts occurring in the judgments are certainly not of the same nature as the real objects which they designate, and the relations among concepts are not like the relation of things, for in the latter temporal moments always occur, and spatial ones usual do, whereas conceptual relations are non-spatial and non-temporal. In the judgment "The chair stands at the right of the table" the concept of "chair" is not placed at the right of the concept "table".

"Thus the concept of agreement melts away in the rays of analysis in so far as it is to mean sameness or similarity, and what remains of it is simply univocal arrangement. In the latter, the relation of the true judgment to reality consists, and all those naïve theories according to which our judgments and concepts could somehow 'picture' reality are fundamentally destroyed. . . . The judgment pictures

the nature of the judged as little as the note pictures the tone, or as the name of the man pictures his personality."[1]

This objection, which in essentials has been repeated by many critics, who see in the theory criticized a return to the "naïve" psychological theories of truth as correspondence, can easily be answered from Wittgenstein's point of view. The minimum requirements for picturing, in Wittgenstein's sense, are first that the fact pictured and the fact used to picture it possess the same number of distinguishable parts, and second, that the structure of the first fact be identical with the structure of the second fact. Now, it may happen that the two facts possess the first requirement, but not the second. The notes of a melody, to use Schlick's example, are equal in number to the tones of the instrument or voice, yet the relation among the notes in the score and the relation among the tones of the instrument or voice are not identical, for the former relation is spatial, where the latter is temporal. Likewise, when colours are used to represent altitudes in a map, the relation among the colours may be a difference of saturation, whereas the relationship among the altitudes is that of a spatial order. A final example is the use of Mercator's projection of the latitude and longitude of the glove on a plane surface. Here the relations are in both cases spatial, but the metrical properties have been so altered that the relations differ in essential respects. Nevertheless, there is a sense in which all these pairs of facts have the same forms, and it is this sense in which Wittgenstein understands the pictorial nature of the elementary propositions. I shall attempt to explain in what this formal identity consists, and to show wherein the difficulty lies.

Wittgenstein compares the relation of the proposition and its objective to geometrical projection. It would, of course, be theoretically possible to represent every fact by a fact of the same kind. Thus "the picture can represent every reality whose form it has. The spatial picture, everything

[1] Schlick, M., *Allgemeine Erkenntnislehre*, Berlin, 1925, pp. 56–7.

spatial, the coloured picture, everything coloured, etc."[1] Notation and spoken language is, nevertheless, arbitrarily usually chosen for obvious reasons of utility. It is, therefore, not possible to see immediately the inner formal connection between the proposition and the fact. Hence it is necessary to explain this connection by a further analysis. In geometrical projection a figure may be projected on a surface such that the result is visibly quite different from the original. The laws of geometrical projection, however, determine that certain properties remain invariant for all possible projections of a given figure. These formal properties which remain invariant are called the projective properties of the original figure. The projective properties of the original and those of all its possible projections are identical. A geometrical science could be developed such that facts from different sense-modalities could be used as projections of one another. The formal geometry of such a science has, in fact, been partially developed by Jean Nicod and Suzanne Langer. In short, the science of geometrical projection is by no means limited to spatial phenomena, first because the laws of projective geometry are propositional functions which may be satisfied by any one of several distinct groups of entities, and second because groups of non-spatial entities have actually been found to satisfy such laws.

The projective properties of a fact remain constant throughout all possible projections. Hence, the logical form of a fact may be identical with that of another fact even if the entities and relations of the one differ from those of the other. It is in this sense, that is in the sense of the formal identity of projective properties, that one must understand Wittgenstein's doctrine that there must be a formal identity between two facts if the one is to picture the other. The criticism of Schlick does not, therefore, seem to be justified. Moreover, his suggestion that bi-unique correspondence is the sole condition of agreement is likewise

[1] Wittgenstein, op. cit., *supra*, 2.171.

invalid. Bi-unique correspondence is a necessary, but not a sufficient, condition of agreement between propositions and fact. So far, Wittgenstein seems to be vindicated.

A difficulty remains. The proposition, as spoken or written, is a series of sounds or marks. It is, therefore, a group of facts and not a single fact, for evidently, if the connection between two sounds is a single fact, the connection between groups of sounds is a group of facts. In its significant use the proposition forms a unit. How, if facts are independent of one another, is it possible to derive one individual fact from a collection of facts? Wittgenstein insists that this must be done, for he says that " only facts can express a sense ; a class of names cannot ",[1] and again, that " the proposition is a fact ". It must be possible to have a fact composed of other facts, something apparently incompatible with the independence of facts. The only available explanation of this is that two distinct facts may be composed of the same objects, as for example in the case of the illusion of the reversible cube. Thus as an entity for consideration in its own right, the proposition as it stands on paper may be regarded as a series of facts (i.e. a series of marks or noises), whereas, as a symbol, the proposition is one fact. Take " Socrates loves Alcibiades ". As a factual objective in its own right, this sentence is a series of facts : " S-o-c-r-a-t-e-s l-o-v-e-s . . ." as a symbol it is not a series of facts, but one fact : " S-L-A ", i.e. a complex of three elements. But in either case it is composed of the same objects.

The essential connection between discourse and empirical reality is thus established by demonstrating the pictorial character of propositions which have empirical reference. The sense of a proposition is the method of its verification, that is to say what it represents if it is true. Sense and truth are distinct, for it is possible to understand a proposition without knowing whether it is true. The sense of

[1] Wittgenstein, op. cit., *supra*, 3.142.

empirical propositions is the possibility that the facts which they picture exist. This much is fairly clear.

But several problems present themselves. How do we know that all propositions are reducible to elementary propositions? Is it possible to show that elementary propositions are exclusively concerned with picturing empirical reality? The next step in Wittgenstein's philosophy consists in the demonstration that all propositions are reducible to elementary propositions, and that these latter are exclusively about empirical facts. This, obviously, limits discourse to the representation of empirical facts.

III

The first part of the demonstration is the proof that all propositions are truth-functions of the elementary proposition; the second part consists in proving that the elementary propositions exist which are not truth-functions, of any other propositions. Wittgenstein describes the truth of a proposition as the agreement of its sense with reality; disagreement of sense with reality is falsehood. Now, truth and falsity are neither properties nor relations, either of propositions or of facts. Thus, it is either redundant or nonsensical to say that "P" is true. For example, "Cæsar loves Brutus" and "It is true that Cæsar loves Brutus" say exactly the same thing. Here the addition of "It is true" is simply redundant. On the other hand, "$P = X$ is true" is nonsense, since "it is true" is not the predicate of X; in order that P be true X must already possess a predicate. The addition of "is true", "exists," etc., to the term X is nonsensical.[1] Truth and falsity are not properties of things, facts, names, or propositions.

Hence the words "true", "false," "exists," "does not exist," do not stand for any entities whatsoever. A

[1] See Kant, I., *Kritik der reinen Vernunft*, Leipzig, 1924. Reclam., pp. 650–8 (A. 592–602); Russell, B., and Whitehard, A. N., *Principia Mathematica*, vol. i, 2nd ed., Cambridge, 1925; Wittgenstein, op. cit., *supra*, 4.063.4.

proposition is true when it agrees with reality, otherwise it is false. This can only be established by comparison. Now truth is not the comparison, but its result. The result simply reveals agreement or disagreement with reality, and this must be seen or shown. It cannot be expressed. This point will be developed later.

When two propositions are asserted to be true, "$p.q$," the result of the double assertion does not represent two facts and a conjunctive relation between them. Facts are such as to have no external relations whatsoever. Moreover, "and" is not the proper name of any entity. Similarly, when a proposition is negated the negation-sign does not represent any object in the world. Truth and falsity are, therefore, not objective entities. The logical constants, "and," "not," etc., are simply a part of the linguistic apparatus necessary to represent the world. This is not peculiar to Wittgenstein's theory, and I shall not proceed further with it here. It is generally recognized by logicians that symbolism contains more than the names of objects, and that these other symbols do not represent anything.[1]

The logical constants do not represent anything in the world, but are simply a part of the apparatus used in the description of the world. This will be more fully discussed later. It is stated here, without proof, to be used presently as the premise of an argument.

As I have observed, in order to prove that all propositions are truth-functions of the elementary propositions, it is necessary to prove (1) that the truth-functions do not represent anything, (2) that one proposition is a part of a larger proposition only when the larger proposition is a truth function of the smaller one, and (3) that the elementary propositions are exclusively about empirical reality.

I have indicated a few of the reasons for the first proposition. I proceed to the second. A truth-function of a set of propositions, $p, q, r \ldots$ will be represented by

[1] Wittgenstein, op. cit., *supra*, 4.0312. Hahn, Hans, "Die Bedeutung der wissenschaftlichen Weltauffassung," *Erkenntnis*, i., pp. 98–9.

$f(p, q, r \ldots)$. An example would be "p or q or r or . . ." Every truth-function is also a proposition, if $p, q, r \ldots$ are propositions. In the following manner it can be shown that all the truth-functions can be defined in terms of a single primitive operation. Let p/q be the proposition which is true whenever p and q are both false, and false when either p or q is true. Then all the truth-functions of p, q can be defined in terms of p/q. In general all the truth-functions of $p, q, r, \ldots$ can be defined as repetitions of the operation " / " on some or all of this set of propositions. For two propositions the result can easily be shown. The number of ways in which some or all members of a group of n propositions can be affirmed or denied is 2^{2^n}, for there are 2^n ways in which $p, q, r, \ldots$ can be considered as true and false, and hence 2^{2^n} sub-classes of these. The number of sub-classes of a class of n members $= 2^n$, of $2^n = 2^{2^n}$. Hence for $n = 2$, i.e. (p, q) there will be $2^{2^2} = 16$ truth-functions to be defined. These are :—

$$p/p = df \text{ not } -p$$
$$q/q = df \text{ not } -q$$
$$p/q = df \text{ not } -p \text{ and not } -q$$
$$p/q \mid p/q = df \text{ not } -(\text{not } -p \text{ and not } -q) = p \text{ or } q$$
$$p/p/q \mid p/p/q = df \text{ not } -(p \text{ and not } -q) = p \text{ implies } q$$
$$q/q/p \mid q/q/p = df \text{ not } -(q \text{ and not } -p) = q \text{ implies } p$$
$$p/p \mid q/q = df\ p \text{ and } q$$
$$p/p \mid q/q \mid \bar{p}/p \mid q/q = df \text{ not } -p \text{ or not } -q$$
$$\bar{p}/p \mid \bar{q}/q = (p \text{ implies } p) \text{ and } (q \text{ implies } q)$$

(The line over p indicates negation and thus is an abbreviation for p/p.)

$$\overline{(p/p \mid q) \mid (q/q \mid p)} = (p \text{ and not } q) \text{ or } (q \text{ and not } -p)$$
$$q/q \mid q/q = \overline{q/q} = \bar{\bar{q}} = \mid^{0}{}' q = q$$
$$p/p \mid p/p = \overline{p/p} = \bar{\bar{p}} = \mid^{0}{}' p = p$$

(I.e. *no* application if the operation is the same as two applications on the same base which is the same as the original base considered as affirmed. The Law of Double Negation is introduced as a definition.)

$$\bar{p}/q \mid \bar{q}/p = df.\ p \text{ is equivalent to } q$$
$$q/q \mid p = df.\ q \text{ and not } -p$$
$$p/p \mid q = df.\ p \text{ and not } -q$$
$$\overline{p/p \mid q/q} = df.\ (p \text{ and not } -p) \text{ and } (q \text{ and not } -q)$$

Thus all the truth-functions of p and q are successive and diverse stroke-functions. In general, if ξ is any proposition and $[\xi]$ is the class of propositions, then any truth-function of $[\xi]$ will have the form $\{\mid^{n}{}'[\xi]\}$, where n represents the number of times the stroke is applied. Any proposition formed by the members of $[\xi]$ will be stroke-functions of the members of $[\xi]$. Therefore, so far as the construction of one proposition from other propositions is concerned, all propositions are constructed from other propositions out of stroke operations. Hence, all propositions are truth-functions of the propositions in which they occur. It might be the case, however, that the process of *analysis* of propositions should continue indefinitely. For example, a given proposition P might be a truth-function of p', p'', p''', etc. Each of these would in turn be truth-functions of others, and so on indefinitely. The truth-value of any compound proposition depends on the truth-values of its constituent propositions. The meaning of any compound proposition depends upon the meaning of its constituents. Now, if the process of analysis of this kind could be indefinitely continued, it would always be impossible to determine the truth or the meaning of any compound propositions. Hence there must be propositions which are not truth-functions of any lesser propositions, but out of which all compound propositions are developed. Now, since propositional compounds are truth-functions, all truth-functions will ultimately be truth-functions of the elementary propositions.

It remains to show why Wittgenstein believes that the

elementary propositions are exclusively about empirical reality. The importance of this demonstration for Positivism cannot be overlooked. If all propositions are truth-functions of the elementary propositions, and if the elementary propositions are exclusively concerned with empirical reality, then it will be impossible to say anything about the non-empirical. In other words, sentences about non-empirical things or facts will be simply nonsense, for it will be impossible to construct any legitimate proposition which could express such things or facts.

In order, then, to demonstrate the exclusively empirical content of elementary propositions, it is necessary to demonstrate the existence of logical simples. A logically simple object is an object which, according to its nature, makes any further analysis impossible. To find similar notions in the history of European thought would not be difficult and may serve to orient the reader. The " simple natures " of Descartes are such things as, relative to our understanding, are not susceptible of further reduction, and such as are the bases of all complex understanding. Some things are utterly simple, in respect of our understanding, but not necessarily simple in the ontological order.[1] These simple natures are not wholly parallel to the logically simple objects taken in Wittgenstein's sense, for the latter are regarded as simple without qualification. The monad of the Leibnizian metaphysics is an absolutely simple entity without qualification. However, since it is a non-empirical notion (being the idea of an existent substance with all its predicates) it does not quite compare with the logical simple of the present discussion. Perhaps the " simple impressions " of Hume would serve as better examples. But these are found in a discussion in which logical and psychological simplicity are not distinguished, and in which the impression is not asserted to be the ultimate basis of the real, but only of knowledge of the real. If, however, one translated Hume's

[1] Descartes, René, *Rules for the Direction of the Mind*, Rule XII.

" literary psychology " into the language of logic, and at the same time removed the naturalistic tendency from that system, whatever would correspond to the simple impressions, as a result of such a transformation, would be very much like the logically simple object of Wittgenstein's doctrine. But none of these comparisons is very exact.

The reasons for demonstrating the existence of simple objects as being the " ultimate furniture of the world ", to use Russell's phrase, are as follows : In order to demonstrate the exclusive concern of elementary propositions with empirical facts it is necessary to show that there is one and only one complete analysis of any proposition, that this analysis absolutely terminates in the elementary propositions, and finally that the elementary propositions are not capable of analysis save into the names which compose them. If many distinct analyses were possible, then even though a given analysis of propositions led to elementary propositions solely concerned with empirical reality, another possible analysis might very well lead to elementary propositions which were not solely concerned with empirical reality. Similarly, if there were no ultimate limit to analysis, then, although a given analysis led solely to elementary propositions of the kind in question, when carried out to any previously assigned limit, further analysis might reveal some non-empirical content in the propositions. Both of these possibilities must be excluded. The first is excluded by the fact that propositions are composed by truth-operations. The second can be excluded if the objectives of elementary propositions can be shown to be composed of logical simples.

The argument for logical simples is this. " If the world had no substance (i.e. simple objects) then whether a proposition had sense would depend on whether another proposition was true. It would then be impossible to form a picture of the world true or false." [1]

[1] Wittgenstein, op. cit., *supra*, prop. 2-0211-2.

As I understand it, this argument rests on two principles : first, that there is one and only one analysis of propositions about complexes, and, second, that the statement about a complex is completely analysable into a statement about its constituent parts.[1] If the argument is presented as formally as possible, this is brought out clearly. The reasoning is apagogic and must be transformed into a direct form. If the world does not consist of simple objects then : any proposition has sense implies another proposition is true ; this, in turn, implies that there are not pictures of the facts. Hence, if the world does not consist of simple objects there is no connection between discourse and reality. The argument in the direct form then is : If there are pictures of facts then some propositions have sense without being truth-functions of other propositions ; thus the existence of propositions with independent senses implies the existence of simple objects.

If, then, the world had no simples, propositions about complexes would be transformed into propositions about the constituents of the complex, and these into propositions asserting that the elements of the complexes are united in such and such ways. Inasmuch as the elements are complex, *ex hypothesi*, this process of translation continues *ad infinitum*. As propositions are pictures of facts, it would be impossible to establish any connection between propositions and reality, since the process of translation continues without end. Briefly the infinite process of analysis, or the ultimate complexity of the world, is incompatible with the pictorial nature of the propositions.

The argument allegedly demonstrates that there are simples, since it assumes that there are pictures of facts. But " pictures " in this usage means " absolutely unambiguous and direct representations ". Such pictures could exist only if there were simples. Hence, the existence of simples is implicitly assumed in the proof for simples. It

[1] Wittgenstein, op. cit., *supra*, prop. 2.

definitely begs the whole question. The *petitio principii* is evident from the definition of " sense of a proposition ". If propositions have " sense " in Wittgenstein's usage, then evidently the existence of simple objects upon which this sense depends is guaranteed. And, conversely, if simples exist, then correct propositions about the world will finally depend on propositions about simples. From the existence of propositions, however, we cannot infer the existence of simples, unless we either arbitrarily decide that propositions have " sense " in the restricted meaning in which Wittgenstein uses this term, or give some independent proof of the exclusive sovereignty of this meaning of " sense " by excluding the possibility of other meanings. From the existence of simples, we cannot infer that propositions exist which correctly represent the facts, since the difficulty of representing simples might be humanly insurmountable.

One might, nevertheless, attempt to argue the point in this way. If there are facts, the facts will have some structure or other. A structure without terms is inconceivable, so that the structure will be a structure of some things. A complex structure of simple things is inconceivable, and, similarly, a simple structure of complex things is inconceivable. In other words, the degree of complexity of the relations among things is relative to the degree of complexity of the things related. Simple terms are simply related and complex terms are related in a complex manner. Now, assertions are made ascribing a simple relation among things of a given kind. Hence, simple objects which are logically prior to these relations must exist if the ascription is true. This is as fallacious a proof as the former, because it assumes that there are true propositions asserting simple relations among things without questioning whether there are such propositions or such relations to be asserted.

Finally, it could simply be said that *no* conception of propositional meaning is compatible with the ultimate complexity of the world. Unless there is a final point of analysis, any analysis is foredoomed to failure. This assertion

also depends upon the unjustified assumption that the sense of propositions must either be conceived as a *univocal* expression of facts, or as devoid of meaning.

It is clear, then, that the existence of simple objects cannot be demonstrated, and that all arguments attempting to demonstrate it either beg the question or have simply a verbal force.

The doctrine that the ultimate referent of significant discourse is the realm of empirical facts cannot be supported by Wittgenstein's logical atomism. There may be other ways to show the empirical content of propositions, but these do not concern the present study. Within the frame of Wittgenstein's doctrine, the existence of simple objects and atomic facts cannot be demonstrated, yet the empiricism of the doctrine depends upon the assumption that the demonstration is possible. The ideas of logical simples and the atomic facts composed of them are thus the first serious flaws in Wittgenstein's logical theory.

In the remainder of my exposition I shall, nevertheless, retain the doctrine to be taken either as an assumption or as something intuitively evident. It is interesting, however, to anticipate the consequences of the contrary hypothesis, namely that the limits of analysis are determined not by the facts themselves, but rather by the sign-language which we use to describe the facts. If the limits of analysis are determined by language, there will be an ineradicable element of arbitrariness and convention in the symbolism used to describe the world. This alternative requires some detailed explanation.

On the assumption that the world is the totality of existent atomic facts, language is composed simply of univocal and immediate pictures of these facts. The meaning of propositions is wholly determined by the empirical facts. There is no element of really essential arbitrariness in the representations of the facts. We can say neither more nor less than what is given in the empirical world, because every meaning is uniquely determined thereby.

On the contrary hypothesis, all this becomes changed. We decide what, for the purpose of a given analysis, is to be the point of departure for constructing a symbolic system. This decision immediately entails some arbitrary standard of simplicity. It is not, therefore, possible to say " this and this are determined by the facts, whereas that is determined by the unessential character of our symbolism ". Truth and falsehood, and hence meaning, will become matters determined by the facts and the symbolic system. Within such a system it will be impossible to determine to what extent the facts are responsible for the truth of the propositions, etc. Outside such a system it will also be impossible to determine the roles which the facts and the symbols play in determining truth because (1) no super-system is postulated from which to make such a judgment, and (2) comparison between two different symbolic systems would yield no answer, because a basis for such a determination would not be present in the comparison.

Absolutism and relativism in logical theory are thus opposite doctrines between which no compromise can be effected. The relativistic doctrine is to be preferred, simply on the ground that it contains no unprovable assertions. I shall return to the theory of logical relativism in my concluding chapter. The theory derives, on the one hand, from the conventionalists such as Poincaré, Le Roy, *et al.*, and on the other from the American pragmatists.

IV

If the atomic facts and the simple objects are assumed as the ultimate objectives of analysis, it follows that the ultimate referent of propositions with existential import is the realm of absolutely simple facts of experience. Language, however, contains many more elements than are comprised in the pictures of facts. There are truth-connectives, such as " and ", " implies ", etc., which have to be accounted for by a theory which reduces all meanings to the empirical

content of thought. I have already dealt with the truth-connections. They are simply a part of the necessary apparatus of symbolism. The simultaneous assertion of two facts requires some sign, "and," to indicate that both are asserted. It does not follow from this that "and" stands for an entity in the objective world. If knowledge were complete there would be no false propositions, and hence no use for the negation sign. *A fortiori* there would be no use for the other logical constants. The incompleteness of knowledge requires the entertainment of propositions not known to be true, and thus the entertainment of groups of propositions not known to be true. It is, therefore, necessary to use logical constants. This presents one reason for the fact that there are more symbols in a language than objectives to which the language applies.

There are some difficulties which remain. Logical inference presents an instance of propositions which are *a priori* true. If propositions represent facts, if their truth or falsity depends upon the existence of the facts, it is clear that no proposition of logic could be *a priori* true. Yet there seem to be true propositions which do not depend on the facts for verification.

Wittgenstein's answer to this difficulty is one of his most valuable contributions to modern logical theory, and I have reserved a special chapter for it. Briefly stated, it amounts to denying that the propositions of logic express anything. They are merely formulæ which indicate the admissible transformations within a language. Given a complex proposition, the principles of logic indicate the different possible ways of expressing the same proposition or any of its constituents. For example: $p \supset q . \supset . \sim q \supset \sim p$ or $p . q . \supset . \sim (\sim p \vee \sim q)$ show how the same proposition can be differently expressed. The two parts of the principal implication express the same meaning. Similarly, $p . q . \supset . p$ shows how, given a complex proposition, we can express any one of its parts. The propositions of logic do not give expression to any fact, but rather present ways of

expressing the same meaning (partially or wholly) in different ways.

Some objections still remain. It seems to be possible to express the relation between propositions and facts on the one hand, and propositions and the individuals asserting them on the other. Finally, it seems possible to express internal relations among propositions which are not formal truths of logic. Wittgenstein attempts to eliminate all of these apparent possibilities. These possibilities must be considered.

The first apparent exception to the doctrine that the ultimate referent of discourse is empirical, is the fact that some connection must exist between propositions and facts. Such a connection is not empirical. The propositions asserting this connection would then be cases of a non-empirical assertion. Wittgenstein holds that it is impossible for such a proposition really to be formulated. For him, the proposition represents what it represents, namely some empirical fact, by virtue of its logical form. The proposition asserts that things are related in a certain way by presenting the relation in which they stand if the proposition be true. By possessing a logical form the proposition reveals the form of the fact represented. It cannot represent the connection between itself and its objective, for it represents simply the form of the objective. It might be supposed that another proposition could represent the connection, but for Wittgenstein this is not possible because all that such an attempted proposition could assert is that $f = p$, i.e. that the fact and the original proposition have the same structure. The last formula is either contradictory or asserts nothing; contradictory if f and p have different meanings, non-significant if they possess the same meaning. In general, the formal connection between propositions and facts cannot be expressed by any proposition. This connection is seen or shown by the comparison between propositions and facts, but it cannot be the subject-matter of a proposition. This may also be stated in another way. If propositions are

pictures of facts, and, as being pictures of facts, possess forms identical with the structure of the facts, this community of form will be a presupposition of the possibility of symbolization. But, as being such a presupposition, the community of form cannot itself be symbolized. It can, however, be shown by exhibiting the facts whose forms are represented in the proposition, or by exhibiting the propositions whose forms mirror the structures of the facts. Inasmuch as no proposition can assert the connection between discourse and the world, the apparent exception to Wittgenstein's theory that the ultimate referent of discourse is empirical reality alone is dismissed.

It seems possible, nevertheless, to represent the connection between the proposition and the person who asserts, thinks, or believes it. This is the second alleged exception. For example: " John thinks that Mary went to town." This apparently violates the doctrine that all propositions are truth-functions of the propositions which occur in them. It seems to present a case of a proposition which cannot be reduced to elementary (empirical) propositions. This alleged case of a non-extensional proposition is eliminated by Wittgenstein's analysis.

Let A represent any person and P any proposition. Then A says (believes, thinks, asserts, etc.) P is apparently an exception to the two theses, (1) that one proposition occurs in another only as the base of a truth-operation, and (2) that all propositions are ultimately related to the empirically given facts.

The case of "A thinks P" is also interesting for another reason. Wittgenstein thinks that the meanings of signs and the sense of propositions are wholly determined by the facts to which they refer. The relation of meaning will then be an internal connection between the proposition and the fact represented by it. This relation involves only the two terms, *symbol* and *referent of symbol*, and no third term such as a person (i.e. a psychological or metaphysical subject) for whom the symbol means something. Other

philosophers, who seem to see an ineradicable element of arbitrariness in symbolism, have insisted that meaning involves at least three terms, the person, the symbol, and the fact. The question is thus interesting and fundamental from three points of view. The example, "A thinks P", seems to violate the empiricism of the Logical Positivists; it seems to violate the so-called axiom of extensionality (that all propositions are truth-functions—extensional functions of the propositions occurring within them), and finally it seems to involve reference to a third and non-empirical element in the " meaning-relation ".

The elimination of this alleged exception proceeds along the following lines.[1] The proposition is a fact representing another fact. Hence it is possible to distinguish between the proposition, considered as a fact in its own right, and the proposition, as a meaning and thus as a vehicle of truth. Now, there is no subject in the metaphysical sense of a simple personal entity (according to Wittgenstein), so that the apparent relation of the proposition P to the subject A is really not a relation to A considered as a simple entity. Aside from this it is clear that "A says (thinks) P" is not a relation between a subject and a proposition in its symbolic use. For the proposition in its symbolic function does not occur in "A thinks P." "A thinks P" is really of the form "'P' says P" and this is not the relation between a subject and a proposition, but simply a co-ordination of the propositional sign with the fact for which it stands by means of the co-ordination of their objects. Thus "'aRb' represents the fact that a stands in a certain relation to b" means what is meant by "'a' is co-ordinated with a and 'b' is co-ordinated with b; consequently the order of 'a', 'b' is the same as the order of a,b". When the proposition occurs factually, we may speak of it. It is then not a symbol, but an objective. We may say that someone utters it, or we may say that it is written in black ink. One is a fact of

[1] Cf. *Principia Mathematica*, vol. i, 2nd ed., pp. 559–666. Wittgenstein, op. cit., *supra*, 5.54.2.

behaviour, the other of typography. When, however, the proposition occurs symbolically, nothing is said about it, but, by means of it, facts are represented. The case of " *A* thinks *P* " is not a case of the occurrence of *P* as a proposition, but simply as a fact. The allegedly exceptional character of " *A* thinks *P* " is thus eliminated.

Hence, " *A* thinks *P* " is no exception to the ultimately empirical content of propositions. It is likewise no exception to the doctrine that functions of propositions are always truth-functions. Finally the " meaning-relation " is a relation between propositions and their objectives involving no third term for whom the meaning is a meaning.

The proposition " *A* thinks *P* " is an assertion about the behaviour of a human being on the same level as " *P* is found in Chapter I of the Book of Genesis ". It is thus reducible to a set of elementary (and hence empirically grounded) propositions. *P*, in the significant use, does not occur in " *A* thinks *P* " any more than in " *P* is found . . ."

The proposition " *A* thinks *P* " is not an intensional function of *P* any more than " the sign *x* occurs in *P* [*x* occurs in *xRy*] ". Both are remarks about *P* considered as a fact. Therefore, all functions of *P* (in its symbolic occurrence) are extensional functions of *P*.

In the significant use, the meaning and truth of a proposition is wholly determined by the data which it represents. Hence the propositional meaning requires no supplementation by the postulation of a subject for whom *P* means something. Neither more nor less can be said in a proposition than is dictated by the facts which it represents. Hence, the " meaning-relation " obtains solely between proposition and fact, without introducing a person to complete the relation.

It will be observed that all this holds only on the basis of a doctrine which was found highly questionable. It can be maintained that the meaning and truth of propositions depend solely on the data only on the assumption that the data are simple combinations of logical atoms. This has

already been called in question. If the data are not simple, or assumed to be simple, then there will be other functions of propositions than the extensional ones, and the cases of " *A* thinks *P* " or " *X* occurs in *P* " cannot be eliminated as above.

Consider, for example, the following case. We wish to explain " *X* is yellow " in such a way that " *X* is yellow " is either wholly true or wholly false. The data are not assumed to be logically simple. It will then be necessary to state how " being yellow " is determined. This would involve the axioms by means of which " being yellow " was arbitrarily determined ; for example, " yellow " lies between the two colours to which the real numbers *a* and *b* are ascribed. This axiom would be a function of " *X* is yellow ", but not an extensional function since " *X* is yellow " might be false, whereas " ' *X* is yellow ' is determined by Axiom *N* " would be true. This shows that the conclusions given above only follow from the premise that the data are logically simple, and do not follow from the contrary hypothesis.

The third difficulty which confronts the doctrine that all propositions are, in principle, empirically grounded, is that there are apparently some propositions which assert the internal connection of meanings and hence do not simply assert the existence of empirical facts. Examples of such propositions would be : (1) " The rose is red " entails " the rose is coloured " ; (2) the meaning of " *P* " is contained in the meaning of " *P* and *Q* " ; (3) " *A* is a father " entails " *A* has a child " ; (4) *aRb* . *bRc* entails *aRc*.

Wittgenstein would undoubtedly admit that there are internal relations among meanings and among facts. He expressly denies, nevertheless, that these relations can be expressed by propositions. Any internal relation among facts or propositions is exhibited by the form of the facts or propositions. What is shown *in* the form of propositions cannot be expressed *by* propositions. Hence, no proposition expresses the existence of internal properties or relation.

An internal relation or property is defined as a relation or property which is not conceivable save in connection with the objects possessing it. The colour of an object must be spatial, the difference of brightness between two specific colours is necessarily what it is, etc. Propositions do not express this necessity, but reveal it in their forms. This doctrine will become clear in the next chapter. At present, it is sufficient to note that internal properties and relations are *shown* by propositional forms, but are not expressed by propositions as such.

The ways of avoiding exceptions to the empirical doctrine have been indicated. The consequences of this doctrine for philosophy and science remain to be developed. I shall outline the course of the development here, and develop it in detail in the succeeding chapters of this study.

V

If the meaning of propositions depends on the elementary propositions which are pictures of empirical facts, and if the truth of propositions finally depends upon the successful comparison of the elementary propositions with the facts, it follows that sentences which contain names or relational predicates of non-empirical entities will be simply nonsense. It will then be impossible significantly to assert anything non-empirical. Metaphysical doctrines postulating the existence of non-empirical entities will automatically be eliminated from significant discourse.

Non-elementary propositions are explicit truth-functions of the elementary propositions. Any given truth-function will be definite, and hence limited in its scope. It will therefore be impossible to assert general propositions having a possibly infinite set of instances, and thus the Cantorian idea of the real infinite becomes nonsense. This has important consequences for mathematics, for philosophy, and for science.

The mathematical infinite must be reinterpreted, so as to

avoid any assertions of infinite collections or magnitudes. Inasmuch as the proper infinite (the $\aleph_0$ of Cantor) is usually considered to be a necessary foundation for mathematical analysis (functions of a real variable, continuity, limits, and irrationals), this result involves a revision of mathematics. The infinite likewise disappears from philosophical doctrines.

The laws of natural science are usually considered to be general propositions whose scope is indefinite. For example, "all bodies fall with a constant vertical acceleration" is asserted for "all" bodies unqualifiedly. These laws will have to be reinterpreted if the scope of general propositions is limited to the elementary propositions from which they are constructed.

The doctrine that the laws of logic are tautological transformations of meaning makes it impossible to deduce anything unknown from the known. In this way deductive metaphysics, and indeed deduction in general, is eliminated.[1] Theoretical physics likewise requires much reinterpretation, since it is almost wholly concerned with deduction.

Finally, the so-called induction-problem is eliminated by the consideration that the general propositions, which inductive methods are supposed to establish, do not occur in significant discourse.

Thus a thorough revision of the usual conception of the sciences and philosophy together with logic and mathematics is entailed by the results of Wittgenstein's logical theory.

Science become a schematism by means of which singular propositions are constructed for the sake of empirical verification. In itself a scientific system cannot tell us very much about the world, for it is general and abstract, whereas the facts comprising the world are specific and empirical atoms. Science, then, is simply an organization of specific knowledge for purposes of recording and predicting specific events.

Prediction and verification are, therefore, not matters of inference. On this theory, inference *a posteriori* is never used, and can be given no justification. The theory of Probability

[1] i.e. deduction in the traditional sense of the term.

must likewise be changed so as to provide a place for a non-inferential use of the assessment of probability in the sciences.

Metaphysical thought is automatically rejected on the grounds that it consists of non-empirical assertions which are not capable of theoretical verification. If Wittgenstein's theory were true, it would be altogether impossible to make a non-empirical assertion. Wittgenstein and the other logical positivists must, therefore, be able to explain how metaphysical pseudo-propositions come to be expressed at all. For Wittgenstein, this occurs in one or more of the following ways :—

(1) The presentation of a propositional function, in which at least one constituent is really undetermined, in the guise of a completed proposition which contains no undetermined parts. For example : " There are at least three objects in the world." This has the form " $(\exists x, y, z)$ . . ." because the word " object " occurs as the argument place of a variable and not as a constant. The " proposition " is incomplete, and hence is nonsense as it stands. Indeterminacy of this kind is one source of metaphysical pseudo-assertion.

(2) The attempt to say what can only be shown. Attempts to define truth, falsehood, the logical constants, numbers, in general, formal concepts, all involve this fallacy.

Not only metaphysics but much of traditional logic and mathematics would have to be eliminated in so far as the attempt is made therein to express what is essentially inexpressible.

(3) The attempt to deduce facts. All deductive metaphysics consists of a combination of this fallacy with (1) and (2). A concept is defined in such a way that consequences about reality can be drawn from it which are not ostensibly contained in the concept. St. Thomas's first three proofs for the existence of God provide examples of this. They are all deductions from implicit definitions of non-empirical (and hence theoretically unverifiable) concepts.

Philosophy, properly so-called, is therefore not a system

of general propositions. It is simply the activity of making propositions clear. Philosophical work essentially consists in the discovery of the elementary propositions on which a given proposition depends. A perfect language would thus have no need of philosophy.

These, in general, are the radical consequences of Logical Positivism in respect of other philosophies and the disciplines of logic, mathematics, and natural science. The consequences for Logical Positivism as a philosophy itself remain to be considered.

The world is not a systemic totality. What we call the world is all the atomic facts which there are. Empirical reality is limited by the totality of true elementary propositions. This is a finite, because theoretically definite, totality. The world is not systemically organized, and therefore is not a closed system. The finitude of the world of atomic facts provides no essential boundary to the world. The world is thus a finite but unbounded collection of mutually independent empirical facts. The totality of the expressible is the totality of elementary propositions. These are pictures of empirical facts. Thought is therefore limited to picturing the empirical reality. The inevitable consequence of this is solipsism. As one of Wittgenstein's critics has expressed it : " Notre travail de pensée se limite perpetuellement à reproduire, à montre le donné ; nous y sommes enfermes ".[1] An individual can only give expression to the past, present, or expected facts of his own experience. This makes it impossible to construct significant assertions about the experience of any other individual.

This radical consequence involves a further alteration of the usual view of scientific knowledge. An objective science, which is valid irrespective of the presence or absence of a particular experience or experiencing subject, has no meaning in Wittgenstein's scheme. In the first place, science, for him, is an organization of experience. In the second place,

[1] Feys, R., " Le raisonnement en termes de faits dans la logistique russelliene," *Revue néo-scholastique*, Louvain, 1928, 2 série, 5, pp. 157 ff.

an experiencing subject is as meaningless as the objective substratum of experience, since both are non-empirical concepts. Thus we are presented with a solipsism without a subject.

In the succeeding chapter I shall attempt to develop all of the theses presented here and I shall criticize them, so far as is possible within the limits of analytical criticism. By this I mean that I do not propose to criticize Logical Positivism on the basis of any philosophy which it specifically rejects. If there are fatal difficulties with Logical Positivism, they will be discovered by a logical analysis of this system without reference to any philosophical theories.

Chapter II

LOGIC AND MATHEMATICS

In the first chapter I attempted to present that part of Wittgenstein's logical theory which especially concerned philosophical questions. In this chapter I shall try to develop certain theses which are specifically related to issues of logic and mathematics.

"Logic," writes Wittgenstein, "is the investigation of all regularity. And outside logic all is accident." [1] This might also be expressed by saying that logic is the investigation of all the essential, as opposed to accidental, regularity (in so far as men speak of regularity in natural processes). But, perhaps, whatever regularity there is in nature is formal in character and so belongs to the realm of the logical.[2] In any case, the task of logic is the study of the forms of propositions and the nature of the connections of propositions. The form of the elementary proposition is wholly determined by known facts or by facts which are anticipated on the basis of what is known. The forms of propositions describing entirely new kinds of facts cannot be known and such propositions could not even be constructed. Logic is, therefore, restricted in its application to the analysis of elementary propositions already known, together with the truth-functions of elementary propositions. This means that there is no logic of induction. In a larger sense, it has another significance, as I have already tried to show.

I have already discussed the nature of the elementary proposition. Generalization is the construction of special kinds of truth-functions of elementary propositions, in

[1] Op. cit., *supra*, 6.3. [2] Op. cit., *supra*, 6.3211.