THE CENTRAL ERROR IN THE TRACTATUS

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Robert Fogelin claimed there was an error in the logic of the Tractatus. I first cover his point here before going on to show that any error in this area derived from an even more fundamental one. Correcting that further error, moreover, does more than correct the logic of the Tractatus: it has repercussions for the metaphysics and theory of value found there, in line with later developments in Wittgenstein's philosophy. In what follows I use the Tractarian numbers to indicate the paragraphs spoken about.

1

The logic of the Tractatus was very influential, but there is a difficulty with it which was pointed out by Fogelin. In the Tractarian predicate logic, and specifically in its account of generality, Wittgenstein continues to employ his operator 'N', the form of Sheffer's Stroke he had used in connection with his propositional logic. He says, amongst other things (5.52) 'if ξ has as its values all the values of a function Fx for all values of x then $N(\xi) = \neg(\exists x)Fx$'. Fogelin focusses on a problem concerning multiple generality, which arises because 'N' tries to replace a quantifier without using an index to show which variable is being quantified over. Others (Geach and Soames, for instance) have tried to modify 'N' appropriately, to allow for this, but Fogelin has stuck to the text (c.f. Cheung 2000).

Fogelin points out that, in order to generate, say, '$\neg(\exists x)\neg(\exists y)Fxy$' it might seem one could say that if ξ has as its values all the values of the function

'$\neg(\exists y)Fxy$' for all values of x then $N(\xi) = \neg(\exists x)\neg(\exists y)Fxy$. But since '$\neg(\exists y)Fxy$' would itself have to be $N(\zeta)$ where ζ has as its values all the values of a function of two variables Fxy, this is either disallowed by the mention of functions of merely one variable in 5.52, or, as Fogelin shows (Fogelin 1976, 71) generates instead '$\neg(\exists x)(\exists y)Fxy$'.

So one would have to consider an infinite enumeration:

$N(\neg(\exists y)Fay, \neg(\exists y)Fby, \neg(\exists y)Fcy...)$.

But while enumerations are allowed in N(...) (5.501) Fogelin points out (Fogelin 1976, 72-3) that only a finite number of truth-operations were condoned (5.32). Other remarks in the Tractatus suggest that Wittgenstein was prepared to allow infinite totalities (see, e.g. Ramsey 1978, 158; Marion 1998, 34), but paragraph 5.32 is final against an infinite number of truth-operations. One might want to say that the operation 'N' is only applied once even in an infinite case: the application of 'N' to (p, q) produces '$\neg p.\neg q$', in which there are as many negations as there are arguments, but the whole transformation is just one application of 'N'. But negation is itself a truth-operation, and there is a further point: the kinds of enumeration envisaged in paragraph 5.501 were finite lists, and opposed there to the giving of a function Fx whose values are the propositions in question. It was Wittgenstein's view that there is no way of writing down an infinite list other than through such a variable form, or formal law. Ramsey disagreed, but Wittgenstein flatly rejected 'unordered generalities' (see, for instance, Fogelin 1983). Thus in the above case, we can see the way the enumeration is intended to develop by replacing 'a', 'b', 'c' with the variable 'x', even though that specific variable form of expression was not available, in its standard use, within the Tractatus, as Fogelin showed.

The impossibility of giving such a function in the above case would have been removed if the Tractatus language had included Skolem functions, for instance. With them in the language we might be able to write '¬(∃y)Fxy' as '¬Fxg(x)' for a certain function 'g(x)', and then apply paragraph 5.52 directly to a function of one variable, to get '¬(∃x)¬Fxg(x)', and thereby '¬(∃x)¬(∃y)Fxy', as required. Whatever the virtues of Geach's or Soames' emendations, however, there are quite fundamental reasons why the explicit language of the Tractatus could not accommodate Skolem functions, as we shall now start to see.

2

For there is another error in the logic of the Tractatus, which Fogelin missed, even though his discussion of the logic of the Tractatus was much fuller, and more critical than most. That further error concerns the logic of identity, which, as is well known, Wittgenstein handled - or tried to handle - using identity of sign (5.53). That requires using variables which are 'exclusive' rather than the standard 'inclusive' ones. Fogelin gives some examples (Fogelin 1976, 66-7). Thus 'There are at least two F's', written with inclusive variables

(∃x)(∃y)(Fx.Fy.x≠y),

becomes

(∃x)(∃y)(Fx.Fy)

with exclusive variables, and 'Somebody likes somebody' is

(∃x)(∃y)(Px.Py.Lxy) v (∃x)(Px.Lxx)

rather than

$(\exists x)(\exists y)(Px.Py.Lxy)$.

Fogelin goes on (Fogelin 1976, 67):

> It seems obvious - though a proof for this is needed - that Wittgenstein's method can shadow Russell's making numerical assignments to things already described under some other non-logical predicate. But Wittgenstein's procedures will not produce counterparts for what we might call pure occurrences of the identity sign, i.e. occurrences of the identity sign governing individuals not previously qualified by some non-logical predicate. It thus seems that the only occurrences of an identity sign that are not eliminable by Wittgenstein's procedure arise in expressions that Wittgenstein wishes to exclude from the language.

Fogelin, at the end here, had in mind paragraph 5.534, which says propositions like 'a=a' and '(∃x)(x=a)' cannot even be written down in a correct conceptual notation, and paragraph 4.1272, which says one cannot say 'There are objects', 'There are 100 objects', etc. Thus the example Fogelin considers (Fogelin 1976, 66):

$(\exists x)(y)(y=x)$,

we might read as 'there is just one object', but this is a pure identity proposition, without any non-logical predicate, and so it will not be expressible, Fogelin thinks, in Wittgenstein's preferred logical language.

But Fogelin seems to have been unaware of work by Hintikka in 1956, in which the needed proof he spoke about above was in fact given. Hintikka produced a general translation of all propositions available in standard predicate logic with identity (which there were expressed using inclusive variables, of course), so that only exclusive variables were involved. He wrote the exclusive translation of a formula 'L' as 'ϕ(L)', and summed up (Hintikka 1956, 235):

> We have already observed that if L is closed, ϕ(L) does not contain any identities; and it is well known that in the predicate calculus we have no need to consider formulae which are not closed. Hence the result we have just proved shows that everything expressible in terms of the inclusive quantifiers and identity may also be expressed by means of the...exclusive quantifiers <u>without using a special symbol for identity</u>.

In a footnote he immediately adds: 'Wittgenstein was, hence, right in saying that the identity sign is not an essential constituent of logical notation'.

Unfortunately for Fogelin, and in a different way for Hintikka, Hintikka's mode of re-expression does too much: for it also gives translations of Fogelin's pure identity propositions. This is implicit in what Hintikka says above, since pure identity propositions are just some of the propositions in standard predicate logic with identity. So it is surprising that Hintikka did not consider, or at least comment on, what Wittgenstein has to say in paragraphs 4.1272, and 5.534. But for Fogelin these remarks have more significance, and yet they cannot be supported in the way he supposes. To make the matter entirely clear I will give Hintikka's translation in Fogelin's chosen case above.

Hintikka's translation process, for a proposition K with two free (inclusive) variables x and y, follows the general plan of the translation for

'Somebody likes somebody' (Hintikka 1956, 231-2). He first finds two propositons K_0 and K_1 which contain no identity propositions, such that K is equivalent to

$K_0.(x \neq y) \vee K_1.(x=y)$.

Then the translation of (∃y)K becomes

$(\exists y)K_0 \vee K_1(x/y)$,

using exclusive variables. Fogelin's case is '(∃x)¬(∃y)¬(x=y)', and merely requires introducing arbitrary tautological (T), or contradictory (F) propositions for the appropriate K_0 and K_1. Thus '¬(x=y)' is equivalent to 'T.¬(x=y) v F.(x=y)', and so '(∃y)¬(x=y)' becomes '(∃y)T v F(x/y)'. Its negation '¬(∃y)¬(x=y)' becomes '(y)F.T(x/y)', and finally 'there is only one object' becomes

$(\exists x)((y)F.T(x/y))$.

The latter is not the formal contradiction it may seem, note, since it returns to the inclusive form via '(∃x)(y)(¬(x=y) ⊃ F)' (see Hintikka 1956, 230).

So, apparently, formal concepts are expressible: we can say 'There is just one object' in a language without the identity symbol.

3

What are we to make of this? Was Fogelin too hasty in simply endorsing Wittgenstein on 'There are 100 objects' etc? Was Wittgenstein completely wrong about formal concepts - which would be a really major error? Certainly Hintikka seems to be misleading if he intended in his footnote to defend the whole of the Tractatus in this area. But what defence of Fogelin's separation of pure identity propositions could there be, given Hintikka's quite general translation?

One aspect of this matter is that there are two sorts of claims possible regarding pure identity propositions: 'they cannot be said in a correct conceptual notation' could mean 'there is no exclusive representation of them', and it could mean 'they are nonsense propositions, since they have the non-factual status of tautologies and contradictions'. Hintikka showed that 'there is just one individual' can be re-expressed in the exclusive notation; but Fogelin was certainly more concerned elsewhere with the status of propositions, as, for instance, in his discussion of fully general propositions (Fogelin 1976, 60-4). Ramsey illustrates the two major aspects of this issue in the following passage (Ramsey 1978, 211):

> Next let us take 'There are at least two individuals' or '$(\exists x,y)(x\neq y)$'. This is the logical sum of the propositions $x\neq y$ which are tautologies if x and y have different values, contradictions if they have the same value. Hence it is the logical sum of a set of tautologies and contradictions; and therefore a tautology if any one of the set is a tautology, but otherwise a contradiction. That is, it is a tautology if x and y can take different values (i.e. if there are two individuals) but otherwise a contradiction.

Here Ramsey, because of his rather different concept of identity, is happy with 'there are two individuals' not only being expressed, but also being a condition which might be true or false, even though it itself is going to turn out to be either a tautology or a contradiction, and so not conditional at all. Wittgenstein was, on this basis, not entirely wrong about formal concepts: they are expressible, but their expression must result in vacuous tautologies or nonsensical contradictions.

We have not, however, seen the end to the significance of Hintikka's result, nor to the full assessment of paragraphs 4.1272, and 5.534. For we have not attended to the initial caveat in Hintikka's statement of his result: he said it applied only to closed formulas in the predicate calculus with identity. Hence ordinary identity statements of the form 'a=b' are not covered, and must be given a separate treatment. For one thing, they should not be lumped with other pure identity propositions as in paragraph 5.534. More significantly, despite Ramsey wanting to say they were tautologies or contradictions above, they are the only statements which come into Fogelin's special category: it is just plain identity statements which cannot be expressed in Wittgenstein's approved way, using exclusive individual terms, in a language lacking identity. The fact that they are not going to be expressed is just the content of paragraph 5.53, in which Wittgenstein announces his intention to use an exclusive language and abandon the identity symbol. The singularity of this decision, we shall now see, underpins a much broader set of opinions in the Tractatus, to do not just with the expression of generality flagged before, but also the independence of atomic propositions, the place of the synthetic a priori, and even the place of the will, and questions of value.

4

When Wittgenstein asserted 'Identity of object I express by identity of sign and not by using a sign of identity. Difference of objects I express by difference of sign.' he was putting himself in the place of God. We can only use the resulting exclusive language without identity when we <u>know</u> the identity relations between objects expressed inclusively. It is a privileged language in which the ignorance cancelled by '$x=y$' or '$x\neq y$', in their inclusive sense, cannot arise. The inclusive forms are used when we are not in a fully knowing state, and so when there is a question still to be asked. Wittgenstein had read Frege, and knew that if $x=y$ then to say that $x=y$ is to say that $x=x$, while if $x\neq y$ then to say that $x=y$ is to say that $x\neq x$. But the consequences '$x=x$' and '$x\neq x$' are not evident in the language expression '$x=y$', which is what we mere mortals have to deal with, in ignorance of the facts. The problem with '$x=y$', for Wittgenstein, was that it was certainly a necessary truth or falsity, and yet its truth or falsity was not evident in its face, and so it was not what was then called 'analytic a priori'. By removing identity statements from his logical system, Wittgenstein in the Tractatus was therefore on the way to eliminating one major portion of what was then understood as the 'synthetic a priori' from logic (c.f. Ayer 1958, ChIV). As Kripke later showed, however, (Kripke 1972), what others might now prefer to call the 'a posteriori necessity' of identity statements is still there, and should be acknowledged.

One has to be careful, when identifying identity propositions. Ramsey understood this very well (Ramsey 1978, 167-8):

> ...these are not real propositions at all; in 'a=b' either 'a', 'b' are names of the same thing, in which case the proposition says nothing, or of different things, in which case it is absurd. In neither case is it an assertion of fact; it

> only appears to be a real assertion by confusion with the case when 'a' or 'b' is not a name but a description. When 'a', 'b' are both names, the only significance which can be placed on 'a=b' is that we use 'a', 'b' as names of the same thing or, more generally, as equivalent symbols.

Ramsey here was clearly alluding to the analysis of descriptions provided by Russell: Russell had allowed 'identities' to be formed like 'a = ιxDx', but this is not a proper identity, being re-expressible '(y)(Dy ≡ y=a)'. After such cases are sieved out we are left with noms de plume, and aliases, for instance. But we can quite easily be ignorant of someone's alias, hence the usefulness of (inclusive) identity propositions, and the extreme oddity of the position Wittgenstein was putting himself in, by trying to avoid them.

Is such an identity statement an 'assertion of a fact'? If we are ignorant of anything it is merely the use of the names, since 'a=b' is empty of factual content about the world, as Ramsey says. This is also shown by its origin in pure acts of will, providing verbal alternatives for the same thing. But with the provision of this choice we immediately get the non-independence of certain elementary propositions, since we can express Leibniz' Law. Given a=b we can say

Pa ≡ Pb,

for any 'P', and even if 'Pa' then merely says the same as 'Pb' another way, still we have two elementary propositions which are equivalent. Upon Wittgenstein's return to philosophy in 1929, of course, he came to question his earlier dictum about the independence of elementary propositions. But instead of the much larger body of examples he considered, which convinced him elementary propositions need not be independent, he might just have considered how Leibniz' Law originates, through having a choice, as above.

Pairs of elementary propositions between which there is simply an entailment arise when other identities are settled upon, associated with the Skolem functions mentioned before, in connection with generality. We considered then how '¬(∃y)Fxy' might be replaced with '¬Fxg(x)', with 'g(x)' a Skolem function. In the simpler case of one variable, '(∃x)Fx' is equivalent to 'Fs' for a Skolem constant s, and so, since

Fa ⊃ (∃x)Fx,

we can obtain

Fa ⊃ Fs,

making these two elementary propositions also non-independent.

5

A formulation of Skolem functions and constants is provided by Hilbert's epsilon calculus. And that leads us to see how correcting the above error in the Tractatus is connected with later developments in Wittgenstein's philosophy. For '(∃x)Fx' is logically equivalent to 'FεxFx', where the referent of the epsilon term 'εxFx' is chosen from amongst the F's, if there are any, and from the universe at large if not (Leisenring 1969, 19). Then not only might '¬(∃y)Fxy' be replaced with '¬Fxg(x)', more particularly it may be replaced with '¬FxεyFxy'. In the constant case, if s = εxFx, then we get the implicational relation given above. An epsilon term, note, is a logically proper name, i.e. a 'complete' term for an individual, unlike the iota terms considered before, and so ‘εxFx = s’ is not further analysable, and comes into Ramsey’s non-descriptive class.

The epsilon term ‘εxFx’ Hilbert read ‘the first F’, because it refers to a paradigm object with the property in question, such items being guaranteed on account of the relation between the epsilon axiom

$(\exists x)Fx \supset F\varepsilon xFx$,

and the predicate calculus thesis

$(\exists y)((\exists x)Fx \supset Fy)$.

‘If anyone is just then Aristedes is just’ was the original example used by Hilbert when he introduced epsilon terms (Reid 1970, 270, see also Copi 1973, 110). Likewise ‘If anything is a material object then Moore’s hands are’ would be a good illustration of what is involved in applying the epsilon axiom with ‘Fx’ meaning ‘x is a material object’, and ‘εxFx’ being chosen to refer to Moore’s hands. Such paradigms as the standard metre, and colour samples, were notable features of Wittgenstein’s later understanding of concepts, as Fogelin has explained (Fogelin 1976, 112f): ‘If anything is a metre long, then the standard metre is a metre long’ insists on the centrality of a certain extra-linguistic object (related to Wittgenstein’s notion of a ‘simple’ by Fogelin) in connection with the measurement of length.

These kinds of cases, in fact, inspired ‘The Paradigm Case Argument’, which was widely applied in the period of Linguistic Philosophy as a general means of understanding all concepts. One could not doubt that a paradigm had the respective property without bringing into question the whole concept involved, so the temptation was to settle any question simply by affirming the paradigm. In its heyday The Paradigm Case Argument was even thought to answer not only most of the traditional problems in epistemology, but even those in axiology, to do with our power to make things happen (Black 1958), our freedom (Flew, 1966, Ch1), and our respect for moral value (Bambrough 1979, Ch2). Indeed Bambrough drew an explicit parallel between Moore's case in epistemology and his case in axiology, and, as above, ethical, social and political values are involved in 'If anyone is just then Aristedes is just', the original example used by Hilbert.

The generations of philosophers subsequent to the Linguistic Philosophy period have not understood things this way, due in part to observing an important flaw in The Paradigm Case Argument (c.f. Passmore 1961, 114f). For Aristedes still might be unjust - the only consequence being that that would make his nickname 'The Just' an ironic misnomer, and be the end of justice:

$$\neg F\varepsilon xFx \supset \neg(\exists x)Fx.$$

Likewise, while Moore's hands might be accepted as paradigm material objects, that properly only leaves us with the conditional truth above: Moore's hands are material objects <u>if anything is</u>. So the traditional epistemological problem was not solved, as Linguistic Philosophy had assumed, and more philosophy seemed to be needed, to defeat Skepticism in the area. But that move to more argument itself involves a misconception of the situation, from a Wittgensteinian perspective. Specifically it misrepresents the role of the will in the matter. For there is no proof of the existence of the external world which could improve on

Moore's procedures. There is a demonstration of it, in the way that Moore proceeded, but responding to that demonstration is a matter of human choice, a matter of 'how we act' (Shiner 1977/8). Likewise in Bambrough's case of giving an anaesthetic to a child who needs an operation: there is no proof that this is right, but appeal to this sort of case is reason's last resort, since if someone does not respond normally to children in pain they have lost touch with the concept of morality. If a person falls in line with Moore's and Bambrough's clear illustrations, then they start to acquire the standard concept of a material object, and the standards of morality. But humans are, logically, quite free not to fall in line, even if then, as the reversal of the conditional shows, nothing will be material, or moral for them, in the usual sense. They then 'act differently', and simply live a form of life at odds with the rest of us. It is a personal, existential choice, in other words, and there is no final argument against it.

Correcting the central error in the Tractatus therefore does more than correct the logic of the Tractatus: it has repercussions for the metaphysics and theory of value found there, in line with later developments in Wittgenstein's philosophy.

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