since George IV wished to know the truth of the one and did not wish to know the truth of the other. If 'the author of *Waverley*' stood for anything other than Scott, 'Scott is the author of *Waverley*' would be false, which it is not. Hence you have to conclude that 'the author of *Waverley*' does not, in isolation, really stand for anything at all; and that is the characteristic of incomplete symbols.

VII. THE THEORY OF TYPES AND SYMBOLISM: CLASSES

Before I begin to-day the main subject of my lecture, I should like to make a few remarks in explanation and amplification of what I have said about existence in my previous two lectures. This is chiefly on account of a letter I have received from a member of the class, raising many points which, I think, were present in other minds too.

The *first* point I wish to clear up is this: I did not mean to say that when one says a thing exists, one means the same as when one says it is possible. What I meant was, that the fundamental logical idea, the primitive idea, out of which both those are derived is the same. That is not quite the same thing as to say that the statement that a thing exists is the same as the statement that it is possible, which I do not hold. I used the word 'possible' in perhaps a somewhat strange sense, because I wanted some word for a fundamental logical idea for which no word exists in ordinary language, and therefore if one is to try to express in ordinary language the idea in question, one has to take some word and make it convey the sense that I was giving to the word 'possible', which is by no means the only sense that it has but is a sense that was convenient for my purpose. We say of a propositional function that it is possible, where there are cases in which it is true. That is not exactly the same thing as what one ordinarily means, for instance, when one says that it is possible it may rain to-morrow. But what I contend is, that the ordinary uses of the word 'possible' are derived from this notion by a process. E.g., normally when you say of a proposition that it is possible, you mean something like this: first of all it is implied that you do not know whether it is true or false; and I think it is implied, secondly, that it is one of a class of propositions, some of which are known to be true. When

I say, e.g., 'It is possible that it may rain to-morrow'—'It will rain to-morrow' is one of the class of propositions 'It rains at time *t*', where *t* is different times. We mean partly that we do not know whether it will rain or whether it will not, but also that we do know that that is the sort of proposition that is quite apt to be true, that it is a value of a propositional function of which we know some value to be true. Many of the ordinary uses of 'possible' come under that head, I think you will find. That is to say, that if you say of a proposition that it is possible, what you have is this: 'There is in this proposition some constituent, which, if you turn it into a variable, will give you a propositional function that is sometimes true.' You ought not therefore to say of a proposition simply that it is possible, but rather that it is possible in respect of such-and-such a constituent. That would be a more full expression.

When I say, for instance, that 'Lions exist', I do not mean the same as if I said that lions were possible; because when you say 'Lions exist', that means that the propositional function '*x* is a lion' is a possible one in the sense that there are lions, while when you say 'Lions are possible' that is a different sort of statement altogether, not meaning that a casual individual animal may be a lion, but rather that a *sort* of animal may be the *sort* that we call 'lions'. If you say 'Unicorns are possible', e.g., you would mean that you do not know any reason why there should not be unicorns, which is quite a different proposition from 'Unicorns exist'. As to what you would mean by saying that unicorns are possible, it would always come down to the same thing as 'It is possible it may rain to-morrow'. You would mean, the proposition 'There are unicorns' is one of a certain set of propositions some of which are known to be true, and that the description of the unicorn does not contain in it anything that *shows* there could not be such beasts.

When I say a propositional function is possible, meaning there are cases in which it is true, I am consciously using the word 'possible' in an unusual sense, because I want a single word for my fundamental idea, and cannot find any word in ordinary language that expresses what I mean.

Secondly, it is suggested that when one says a thing exists, it means that it is in time, or in time and space, at any rate in time. That is a very common suggestion, but I do not think that really there is much to be said for that use of the words; in the first place, because if that were all you meant, there would be no need for a separate word. In the second place, because after all in the sense, whatever that sense may be, in which the things are said to exist that one ordinarily regards as existing, one may very well wish to discuss the question whether there are things that exist without being in time. Orthodox metaphysics holds that whatever is really real is not in time, that to be in time is to be more or less unreal, and that what really exists is not in time at all. And orthodox theology holds that God is not in time. I see no reason why you should frame your definition of existence in such a way as to preclude that notion of existence. I am inclined to think that there are things that are not in time, and I should be sorry to use the word existence in that sense when you have already the phrase 'being in time' which quite sufficiently expresses what you mean.

Another objection to that definition is, that it does not in the least fit the sort of use of 'existence' which was underlying my discussion, which is the common one in mathematics. When you take existence-theorems, for instance, as when you say 'An even prime exists', you do not mean that the number two is in time but that you can find a number of which you can say 'This is even and prime'. One does ordinarily in mathematics speak of propositions of that sort as existence-theorems, i.e., you establish that there is an object of such-and-such a sort, that object being, of course, in mathematics a logical object, not a particular, not a thing like a lion or a unicorn, but an object like a function or a number, something which plainly does not have the property of being in time at all, and it is that sort of sense of existence-theorems that is relevant in discussing the meaning of existence as I was doing in the last two lectures. I do, of course, hold that that sense of existence can be carried on to cover the more ordinary uses of existence, and does in fact give the key to what is underlying those ordinary uses, as when one says that 'Homer existed' or 'Romulus did not exist', or whatever we may say of that kind.

I come now to a *third* suggestion about existence, which is also a not uncommon one, that of a given particular 'this' you can say 'This exists' in the sense that it is not a phantom or an image or a universal. Now I think that use of existence involves confusions which it is exceedingly important to get out of one's mind, really rather dangerous mistakes. In the first place, we must separate phantoms and images from universals; they are on a different level. Phantoms and images do undoubtedly exist in that sense, whatever it is, in which ordinary objects exist. I mean, if you shut your eyes and imagine some visual scene, the images that are before your mind while you are imagining are undoubtedly there. They are images, something is happening, and what is happening is that the images are before your mind, and these images are just as much part of the world as tables and chairs and anything else. They are perfectly decent objects, and you only call them unreal (if you call them so), or treat them as non-existent, because they do not have the usual sort of relations to other objects. If you shut your eyes and imagine a visual scene and you stretch out your hand to touch what is imaged, you won't get a tactile sensation, or even necessarily a tactile image. You will not get the usual correlation of sight and touch. If you imagine a heavy oak table, you can remove it without any muscular effort, which is not the case with oak tables that you actually see. The general correlations of your images are quite different from the correlations of what one chooses to call 'real' objects. But that is not to say images are unreal. It is only to say they are not part of physics. Of course, I know that this belief in the physical world has established a sort of reign of terror. You have got to treat with disrespect whatever does not fit into the physical world. But that is really very unfair to the things that do not fit in. They are just as much there as the things that do. The physical world is a sort of governing aristocracy, which has somehow managed to cause everything else to be treated with disrespect. That sort of attitude is unworthy of a philosopher. We should treat with exactly equal respect the things that do not fit in with the physical world, and images are among them.

'Phantoms', I suppose, are intended to differ from 'images' by being of the nature of hallucinations, things that are not merely imagined but that go with belief. They again are perfectly real; the only odd thing about them is their correlations. Macbeth sees a dagger. If he tried to touch it, he would not get any tactile sensation, but that does not imply that he was not *seeing* a dagger, it only implies that he was not *touching* it. It does not in any way imply that the visual sensation was not there. It only means to say that the sort of correlation between sight and touch that we are used to is the normal rule but not a universal one. In order to pretend that it is universal, we say that a thing is unreal when it does not fit in. You say 'Any man who is a man will do such-and-such a thing.' You then find a man who will not, and you say, he is not a man. That is just the same sort of thing as with these daggers that you cannot touch.

I have explained elsewhere the sense in which phantoms are unreal.* When you see a 'real' man, the immediate object that you see is one of a whole system of particulars, all of which belong together and make up collectively the various 'appearances' of the man to himself and others. On the other hand, when you see a phantom of a man, that is an isolated particular, not fitting into a system as does a particular which one calls an appearance of the 'real' man. The phantom is in itself just as much part of the world as the normal sense-datum, but it lacks the usual correlation and therefore gives rise to false inferences and becomes deceptive.

As to universals, when I say of a particular that it exists, I certainly do not mean the same thing as if I were to say that it is not a universal. The statement concerning any particular that it is not a universal is quite strictly nonsense—not false, but strictly and exactly nonsense. You never can place a particular in the sort of place where a universal ought to be, and vice versa. If I say '*a* is not *b*', or if I say '*a* is *b*', that implies that *a* and *b* are of the same logical type. When I say of a universal that it exists, I should be meaning it in a different sense from that in which one says that particulars exist. E.g., you might say 'Colours exist in the spectrum between blue and yellow.' That would be a perfectly respectable statement, the colours being taken as universals. You mean simply that the propositional function '*x* is a colour between blue and yellow' is one which is capable of truth. But the *x* which occurs there is not a particular, it is a universal. So that you arrive at the fact that the ultimate important notion involved in existence is the notion that I developed in the lecture before last, the notion of a propositional function being sometimes true, or being, in other words, possible. The distinction between what some people would call real existence, and existence in people's imagination or in my subjective activity, that distinction, as we have just seen, is entirely one of correlation. I mean that anything which appears to you,

* See *Our Knowledge of the External World*, Chap. III. Also Section XII of 'Sense-Data and Physics' in *Mysticism and Logic*.

you will be mistakenly inclined to say has some more glorified form of existence if it is associated with those other things I was talking of in the way that the appearance of Socrates to you would be associated with his appearance to other people. You would say he was only in your imagination if there were not those other correlated appearances that you would naturally expect. But that does not mean that the appearance to you is not exactly as much a part of the world as if there were other correlated appearances. It will be exactly as much a part of the real world, only it will fail to have the correlations that you expect. That applies to the question of sensation and imagination. Things imagined do not have the same sort of correlations as things sensated. If you care to see more about this question, I wrote a discussion in *The Monist* for January, 1915, and if any of you are interested, you will find the discussion there.

I come now to the proper subject of my lecture, but shall have to deal with it rather hastily. It was to explain the theory of types and the definition of classes. Now first of all, as I suppose most of you are aware, if you proceed carelessly with formal logic, you can very easily get into contradictions. Many of them have been known for a long time, some even since the time of the Greeks, but it is only fairly recently that it has been discovered that they bear upon mathematics, and that the ordinary mathematician is liable to fall into them when he approaches the realms of logic, unless he is very cautious. Unfortunately the mathematical ones are more difficult to expound, and the ones easy to expound strike one as mere puzzles or tricks.

You can start with the question whether or not there is a greatest cardinal number. Every class of things that you can choose to mention has some cardinal number. That follows very easily from the definition of cardinal numbers as classes of similar classes, and you would be inclined to suppose that the class of all things there are in the world would have about as many members as a class could be reasonably expected to have. The plain man would suppose you could not get a larger class than the class of all the things there are in the world. On the other hand, it is very easy to prove that if you take selections of some of the members of a class, making those selections in every conceivable way that you can, the number of different selections that you can make is greater than the original number of terms. That is easy to see with small numbers. Suppose you have a class with just three numbers, *a,b,c*. The first selection that you can make is the selection of no terms. The next of *a* alone, *b* alone, *c* alone. Then *bc*, *ca*, *ab*, *abc*, which makes in all 8 (i.e., 2^3) selections. Generally speaking, if you have *n* terms, you can make 2^n selections. It is very easy to prove that 2^n is always greater than *n*, whether *n* happens to be finite or not. So you find that the total number of things in the world is not so great as the number of classes that can be made up out of those things. I am asking you to take all these propositions for granted, because there is not time to go into the proofs, but they are all in Cantor's work. Therefore you will find that the total number of things in the world is by no means the greatest number. On the contrary, there is a heirarchy of numbers greater than that. That, on the face of it, seems to land you in a contradiction. You have, in fact, a perfectly precise arithmetical proof that there are *fewer* things in heaven or earth than are dreamt of in *our* philosophy. That shows how philosophy advances.

You are met with the necessity, therefore, of distinguishing between classes and particulars. You are met with the necessity of saying that a class consisting of two particulars is not itself in turn a fresh particular, and that has to be expanded in all sorts of ways; i.e., you will have to say that in the sense in which there are particulars, in that sense it is not true to say there are classes. The sense in which there are classes is a different one from the sense in which there are particulars, because if the senses of the two were exactly the same, a world in which there are three particulars and therefore eight classes, would be a world in which there are at least eleven things. As the Chinese philosopher pointed out long ago, a dun cow and a bay horse makes three things: separately they are each one, and taken together they are another, and therefore three.

I pass now to the contradiction about classes that are not members of themselves. You would say generally that you would not expect a class to be a member of itself. For instance, if you take the class of all the teaspoons in the world, that is not in itself a teaspoon. Or if you take all the human beings in the world, the whole class of them is not in turn a human being. Normally you would say you cannot expect a whole class of things to be itself a member of that class. But there are apparent exceptions. If you

take, e.g., all the things in the world that are not teaspoons and make up a class of them, that class obviously (you would say) will not be a teaspoon. And so generally with negative classes. And not only with negative classes, either, for if you think for a moment that classes are things in the same sense in which things are things, you will then have to say that the class consisting of all the things in the world is itself a thing in the world, and that therefore this class is a member of itself. Certainly you would have thought that it was clear that the class consisting of all the classes in the world is itself a class. That I think most people would feel inclined to suppose, and therefore you would get there a case of a class which is a member of itself. If there is any sense in asking whether a class is a member of itself or not, then certainly in all the cases of the ordinary classes of everyday life you find that a class is not a member of itself. Accordingly, that being so, you could go on to make up the class of all those classes that are not members of themselves, and you can ask yourself, when you have done that, is that class a member of itself or is it not?

Let us first suppose that it is a member of itself. In that case it is one of those classes that are not members of themselves, i.e., it is not a member of itself. Let us then suppose that it is not a member of itself. In that case it is not one of those classes that are not members of themselves, i.e., it is one of those classes that are members of themselves, i.e., it is a member of itself. Hence either hypothesis, that it is or that it is not a member of itself, leads to its contradiction. If it is a member of itself, it is not, and if it is not, it is.

That contradiction is extremely interesting. You can modify its form; some forms of modification are valid and some are not. I once had a form suggested to me which was not valid, namely the question whether the barber shaves himself or not. You can define the barber as 'one who shaves all those, and those only, who do not shave themselves'. The question is, does the barber shave himself? In this form the contradiction is not very difficult to solve. But in our previous form I think it is clear that you can only get around it by observing that the whole question whether a class is or is not a member of itself is nonsense, i.e., that no class either is or is not a member of itself, and that it is not even true to say that, because the whole form of words is just a noise without meaning. That has to do with the fact that classes, as I shall be coming on to show, are incomplete symbols in the same sense in which the descriptions are that I was talking of last time; you are talking nonsense when you ask yourself whether a class is or is not a member of itself, because in any full statement of what is meant by a proposition which seems to be about a class, you will find that the class is not mentioned at all and that there is nothing about a class in that statement. It is absolutely necessary, if a statement about a class is to be significant and not pure nonsense, that it should be capable of being translated into a form in which it does not mention the class at all. This sort of statement, 'Such-and-such a class is or is not a member of itself', will not be capable of that kind of translation. It is analogous to what I was saying about descriptions: the symbol for a class is an incomplete symbol; it does not really stand for part of the propositions in which symbolically it occurs, but in the right analysis of those propositions that symbol has been broken up and disappeared.

There is one other of these contradictions that I may as well mention, the most ancient, the saying of Epimenides that 'All Cretans are liars'. Epimenides was a man who slept for sixty years without stopping, and I believe that it was at the end of that nap that he made the remark that all Cretans were liars. It can be put more simply in the form: if a man makes the statement 'I am lying', is he lying or not? If he is, that is what he said he was doing, so he is speaking the truth and not lying. If, on the other hand, he is not lying, then plainly he is speaking the truth in saying that he is lying, and therefore he is lying, since he says truly that that is what he is doing. It is an ancient puzzle, and nobody treated that sort of thing as anything but a joke until it was found that it had to do with such important and practical problems as whether there is a greatest cardinal or ordinal number. Then at last these contradictions were treated seriously. The man who says 'I am lying' is really asserting 'There is a proposition which I am asserting and which is false'. That is presumably what you mean by lying. In order to get out the contradiction you have to take that whole assertion of his as one of the propositions to which his assertion applies; i.e., when he says 'There is a proposition which I am asserting and which is false', the word 'proposition' has to be interpreted as to include among propositions his statement to the effect that he is asserting a false proposition. Therefore you have to suppose that you have a certain totality, viz., that of propositions, but that that totality contains members which can only be defined in terms of itself. Because when you say 'There is a proposition which I am asserting and which is false', that is a statement whose meaning can only be got by reference to the totality of propositions. You are not saying which among all the propositions there are in the world it is that you are asserting and that is false. Therefore it presupposes that the totality of proposition is spread out before you and that some one, though you do not say which, is being asserted falsely. It is quite clear that you get into a vicious circle if you first suppose that this totality of propositions is spread out before you, so that you can without picking any definite one say 'Some one out of this totality is being asserted falsely', and that yet, when you have gone on to say 'Some one out of this totality is being asserted falsely', that assertion is itself one of the totality you were to pick out from. That is exactly the situation you have in the paradox of the liar. You are supposed to be given first of all a set of propositions, and you assert that some one of these is being asserted falsely, then that assertion itself turns out to be one of the set, so that it is obviously fallacious to suppose the set already there in its entirety. If you are going to say anything about 'all propositions', you will have to define propositions, first of all, in some such way as to exclude those that refer to all the propositions of the sort already defined. It follows that the word 'proposition', in the sense in which we ordinarily try to use it, is a meaningless one, and that we have got to divide propositions up into sets and can make statements about all propositions in a given set, but those propositions will not themselves be members of the set. For instance, I may say 'All atomic propositions are either true or false', but that itself will not be an atomic proposition. If you try to say 'All propositions are either true or false', without qualification, you are uttering nonsense, because if it were not nonsense it would have to be itself a proposition and one of those included in its own scope, and therefore the law of excluded middle as enunciated just now is a meaningless noise. You have to cut propositions up into different types, and you can start with atomic propositions or, if you like, you can start with those propositions that do not refer to sets of propositions at all. Then you will take next those that refer to sets of propositions of that sort that you had first. Those that refer to sets of propositions of the first type, you may call the second type, and so on.

If you apply that to the person who says 'I am lying', you will find that the contradiction has disappeared, because he will have to say what type of liar he is. If he says 'I am asserting a false proposition of the first type', as a matter of fact that statement, since it refers to the totality of propositions of the first type, is of the second type. Hence it is not true that he is asserting a false proposition of the first type, and he remains a liar. Similarly, if he said he was asserting a false proposition of the 30,000th type, that would be a statement of the 30,001st type, so he would still be a liar. And the counter-argument to prove that he was also not a liar has collapsed.

You can lay it down that a totality of any sort cannot be a member of itself. That applies to what we are saying about classes. For instance, the totality of classes in the world cannot be a class in the same sense in which they are. So we shall have to distinguish a hierarchy of classes. We will start with the classes that are composed entirely of particulars: that will be the first type of classes. Then we will go on to classes whose members are classes of the first type: that will be the second type. Then we will go on to classes whose members are classes of the second type: that will be the third type, and so on. Never is it possible for a class of one type either to be or not to be identical with a class of another type. That applies to the question I was discussing a moment ago, as to how many things there are in the world. Supposing there are three particulars in the world. There are then, as I was explaining, 8 classes of particulars. There will be 2^8 (i.e., 256) classes of classes of particulars, and 2^{256} classes of classes of classes of particulars, and so on. You do not get any contradiction arising out of that, and when you ask yourself the question: 'Is there, or is there not a greatest cardinal number?' the answer depends entirely upon whether you are confining yourself within some one type, or whether you are not. Within any given type there is a greatest cardinal number, namely, the number of objects of that type, but you will always be able to get a larger number by going up to the next type. Therefore, there is no number so great but what you can get a greater number in a sufficiently high type. There you

have the two sides of the argument: the one side when the type is given, the other side when the type is not given.

I have been talking, for brevity's sake, as if there really were all these different sorts of things. Of course, that is nonsense. There are particulars, but when one comes on to classes, and classes of classes, and classes of classes of classes, one is talking of logical fictions. When I say there are no such things, that again is not correct. It is not significant to say 'There are such things', in the same sense of the words 'there are' in which you can say 'There are particulars'. If I say 'There are particulars' and 'There are classes', the two phrases 'there are' will have to have different meanings in those two propositions, and if they have suitable different meanings, both propositions may be true. If, on the other hand, the words 'there are' are used in the same sense in both, then one at least of those statements must be nonsense, not false but nonsense. The question then arises, what is the sense in which one can say 'There are classes', or in other words, what do you mean by a statement in which a class appears to come in? First of all, what are the sort of things you would like to say about classes? They are just the same as the sort of things you want to say about propositional functions. You want to say of a propositional function that it is sometimes true. That is the same thing as saying of a class that it has members. You want to say that is it true for exactly 100 values of the variables. That is the same as saying of a class that it has a hundred members. All the things you want to say about classes are the same as the things you want to say about propositional functions excepting for accidental and irrelevant linguistic forms, with, however, a certain proviso which must now be explained.

Take, e.g., two propositional functions such as '*x* is a man', '*x* is a featherless biped'. Those two are formally equivalent, i.e., when one is true so is the other, and vice versa. Some of the things that you can say about a propositional function will not necessarily remain true if you substitute another formally equivalent propositional function in its place. For instance, the propositional function '*x* is a man' is one which has to do with the concept of humanity. That will not be true of '*x* is a featherless biped'. Or if you say, 'so-and-so asserts that such-and-such is a man' the propositional function '*x* is a man' comes in there, but '*x* is a featherless biped' does not. There are a certain number of things which you can say about a propositional function which would be not true if you substitute another formally equivalent propositional function. On the other hand, any statement about a propositional function which will remain true or remain false, as the case may be, when you substitute for it another formally equivalent propositional function, may be regarded as being about the class which is associated with the propositional function. I want you to take the words *may be regarded* strictly. I am using them instead of *is*, because *is* would be untrue. 'Extensional' statements about functions are those that remain true when you substitute any other formally equivalent function, and these are the ones that may be regarded as being about the class. If you have any statement about a function which is not extensional, you can always derive from it a somewhat similar statement which is extensional, viz., there is a function formally equivalent to the one in question about which the statement in question is true. This statement, which is manufactured out of the one you started with, will be extensional. It will always be equally true or equally false of any two formally equivalent functions, and this derived extensional statement may be regarded as being the corresponding statement about the associated class. So, when I say that 'The class of men has so-and-so many members', that is to say 'There are so-and-so many men in the world', that will be derived from the statement that 'x is human' is satisfied by so-and-so many values of x, and in order to get it into the extensional form, one will put it in this way, that 'There is a function formally equivalent to "x is human", which is true for so-and-so many values of x'. That I should define as what I mean by saying 'The class of men has so-and-so many members'. In that way you find that all the formal properties that you desire of classes, all their formal uses in mathematics, can be obtained without supposing for a moment that there are such things as classes, without supposing, that is to say, that a proposition in which symbolically a class occurs, does in fact contain a constituent corresponding to that symbol, and when rightly analysed that symbol will disappear, in the same sort of way as descriptions disappear when the propositions are rightly analysed in which they occur.

There are certain difficulties in the more usual view of classes,

in addition to those we have already mentioned, that are solved by our theory. One of these concerns the null-class, i.e., the class consisting of no members, which is difficult to deal with on a purely extensional basis. Another is concerned with unit-classes. With the ordinary view of classes you would say that a class that has only one member was the same as that one member. That will land you in terrible difficulties, because in that case that one member is a member of that class, namely, itself. Take, e.g., the class of 'Lecture audiences in Gordon Square'.* That is obviously a class of classes, and probably it is a class that has only one member, and that one member itself (so far) has more than one member. Therefore if you were to identify the class of lecture audiences in Gordon Square with the only lecture audience that there is in Gordon Square, you would have to say both that it has one member and that it has twenty members, and you will be landed in contradictions, because this audience has more than one member, but the class of audiences in Gordon Square has only one member. Generally speaking, if you have any collection of many objects forming a class, you can make a class of which that class is the only member, and the class of which that class is the only member will have only one member, though this only member will have many members. This is one reason why you must distinguish a unit-class from its only member. Another is that, if you do not, you will find that the class is a member of itself, which is objectionable, as we saw earlier in this lecture. I have omitted a subtlety connected with the fact that two formally equivalent functions may be of different types. For the way of treating this point, see *Principia Mathematica*, page 20, and Introduction, Chapter III.

I have not said quite all that I ought to have said on this subject. I meant to have gone a little more into the theory of types. The theory of types is really a theory of symbols, not of things. In a proper logical language it would be perfectly obvious. The trouble that there is arises from our inveterate habit of trying to name what cannot be named. If we had a proper logical language, we should not be tempted to do that. Strictly speaking, only particulars can be named. In that sense in which there are particulars,

* [These lectures were given 'in Dr. Williams's library in Gordon Square,' Russell informs me, on eight consecutive Tuesdays. Although University College London, stands nearby, this was probably the only lecture audience in Gordon Square proper.—R.C.M.]

you cannot say either truly or falsely that there is anything else. The word 'there is' is a word having 'systematic ambiguity', i.e., having a strictly infinite number of different meanings which it is important to distinguish.

Discussion

Question: Could you lump all those classes, and classes of classes, and so on, together?

Mr. Russell: All are fictions, but they are different fictions in each case. When you say 'There are classes of particulars', the statement 'there are' wants expanding and explaining away, and when you have put down what you really do mean, or ought to mean, you will find that it is something quite different from what you thought. That process of expanding and writing down fully what you mean, will be different if you go on to 'there are classes of classes of particulars'. There are infinite numbers of meanings to 'there are'. The first only is fundamental, so far as the hierarchy of classes is concerned.

Question: I was wondering whether it was rather analogous to spaces, where the first three dimensions are actual, and the higher ones are merely symbolic. I see there is a difference, there are higher dimensions, but you can lump those together.

Mr. Russell: There is only one fundamental one, which is the first one, the one about particulars, but when you have gone to classes, you have travelled already just as much away from what there is as if you have gone to classes of classes. There are no classes really in the physical world. The particulars are there, but not classes. If you say 'There is a universe' that meaning of 'there is' will be quite different from the meaning in which you say 'There is a particular', which means that 'the propositional function "x is a particular" is sometimes true'.

All those statements are about symbols. They are never about the things themselves, and they have to do with 'types.' This is really important and I ought not to have forgotten to say it, that the relation of the symbol to what it means is different in different types. I am not now talking about this hierarchy of classes and so on, but the relation of a predicate to what it means is different from the relation of a name to what it means. There is not one single concept of 'meaning' as one ordinarily thinks there is, so

that you can say in a uniform sense 'All symbols have meaning', but there are infinite numbers of different ways of meaning, i.e., different sorts of relation of the symbol to the symbolized, which are absolutely distinct. The relation, e.g., of a proposition to a fact, is quite different from the relation of a name to a particular, as you can see from the fact that there are two propositions always related to one given fact, and that is not so with names. That shows you that the relation that the proposition has to the fact is quite different from the relation of a name to a particular. You must not suppose that there is, over and above that, another way in which you could get at facts by naming them. You can always only get at the thing you are aiming at by the proper sort of symbol, which approaches it in the appropriate way. That is the real philosophical truth that is at the bottom of all this theory of types.

VIII. EXCURSUS INTO METAPHYSICS: WHAT THERE IS

I come now to the last lecture of this course, and I propose briefly to point to a few of the morals that are to be gathered from what has gone before, in the way of suggesting the bearing of the doctrines that I have been advocating upon various problems of metaphysics. I have dealt hitherto upon what one may call philosophical grammar, and I am afraid I have had to take you through a good many very dry and dusty regions in the course of that investigation, but I think the importance of philosophical grammar is very much greater than it is generally thought to be. I think that practically all traditional metaphysics is filled with mistakes due to bad grammar, and that almost all the traditional problems of metaphysics and traditional results—supposed results—of metaphysics are due to a failure to make the kind of distinctions in what we may call philosophical grammar with which we have been concerned in these previous lectures.

Take, as a very simple example, the philosophy of arithmetic. If you think that 1, 2, 3, and 4, and the rest of the numbers, are in any sense entities, if you think that there are objects, having those names, in the realm of being, you have at once a very considerable apparatus for your metaphysics to deal with, and you have offered to you a certain kind of analysis of arithmetical propositions. When you say, e.g., that 2 and 2 are 4, you suppose in