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Elements of Symbolic Logic

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'c' must be analyzed for each of the $2^3 = 8$ possible combinations of truth-values of 'a', 'b', 'c'. We classify the formulas with respect to the number of propositional variables or to the kind of operations. The first group referring to only one propositional variable contains formulas presented in traditional logic as the laws of thought, such as the law of identity or the law of contradiction; we see that we are concerned here only with some special formulas out of a long list of other equally important laws of thought. The explicit formulation of these other formulas, few of which were known in traditional logic, is due to the work of the first logisticians such as de Morgan, Boole, Schröder, Peirce, Russell, and Whitehead.

We add some remarks concerning the equivalence relation. Though not all tautologies are equivalences, equivalences play an important role because their function in logic corresponds to the function of equations in mathematics. Most of our formulas are therefore equivalences. Because of formula 7a we obtain from every formula containing an equivalence another formula which states, instead, an implication; thus from 1b we get

$$a \vee a \supset a \qquad (5)$$

from 6c we get

$$(a \supset b) \supset (\bar{b} \supset \bar{a}) \qquad (6)$$

We do not include such formulas in our list because they can easily be obtained; we rather follow the practice of writing an equivalence sign instead of an implication wherever it is possible. In group 8 we collect those formulas for which an equivalence sign would have been false, calling them 'one-sided implications'. [Ex.]

TAUTOLOGIES IN THE CALCULUS OF PROPOSITIONS

Concerning one proposition:

1a.	$a \equiv a$	rule of identity
1b.	$a \vee a \equiv a$	rule of identity
1c.	$a.a \equiv a$	rule of identity
1d.	$\bar{\bar{a}} \equiv a$	rule of double negation
1e.	$a \vee \bar{a}$	tertium non datur
1f.	$\overline{a.\bar{a}}$	rule of contradiction
1g.	$a \supset \bar{a} \equiv \bar{a}$	reductio ad absurdum

Sum:

2a.	$a \vee b \equiv b \vee a$	commutativity of 'or'
2b.	$a \vee (b \vee c) \equiv (a \vee b) \vee c \equiv a \vee b \vee c$	associativity of 'or'

Product:

3a.	$a.b \equiv b.a$	commutativity of 'and'
3b.	$a.(b.c) \equiv (a.b).c \equiv a.b.c$	associativity of 'and'

Sum and product:

4a.	$a.(b \vee c) \equiv a.b \vee a.c$	1st distributive rule
4b.	$a \vee b.c \equiv (a \vee b).(a \vee c)$	2nd distributive rule
4c.	$(a \vee b).(c \vee d) \equiv a.c \vee b.c \vee a.d \vee b.d$	twofold distribution
4d.	$a.b \vee c.d \equiv (a \vee c).(b \vee c).(a \vee d).(b \vee d)$	
4e.	$a.(a \vee b) \equiv a \vee a.b \equiv a$	redundance of a term

Negation, product, sum:

5a.	$\overline{a.b} \equiv \bar{a} \vee \bar{b}$	breaking of negation line
5b.	$\overline{a \vee b} \equiv \bar{a}.\bar{b}$	
5c.	$a.(b \vee \bar{b}) \equiv a$	dropping of an always true factor
5d.	$a \vee b.\bar{b} \equiv a$	dropping of an always false term
5e.	$a \vee \bar{a}.b \equiv a \vee b$	redundance of a negation

Implication, negation, product, sum:

6a.	$a \supset b \equiv \bar{a} \vee b$	dissolution of implication
6b.	$a \supset b \equiv \overline{a.\bar{b}}$	
6c.	$a \supset b \equiv \bar{b} \supset \bar{a}$	contraposition
6d.	$a \supset (b \supset c) \equiv b \supset (a \supset c) \equiv a.b \supset c$	symmetry of premises
6e.	$(a \supset b).(a \supset c) \equiv a \supset b.c$	merging of implications
6f.	$(a \supset c).(b \supset c) \equiv a \vee b \supset c$	
6g.	$(a \supset b) \vee (a \supset c) \equiv a \supset b \vee c$	
6h.	$(a \supset c) \vee (b \supset c) \equiv a.b \supset c$	

Equivalence, implication, negation, product, sum:

7a.	$(a \equiv b) \equiv (a \supset b).(b \supset a)$	dissolution of equivalence
7b.	$(a \equiv b) \equiv a.b \vee \bar{a}.\bar{b}$	
7c.	$\overline{a \equiv b} \equiv (a \equiv \bar{b})$	negation of equivalence
7d.	$(a \equiv b) \equiv (\bar{a} \equiv \bar{b})$	negation of equivalent terms

One-sided implications:

8a.	$a \supset a \vee b$	addition of an arbitrary term
8b.	$a.b \supset a$	implication from both to any
8c.	$a \supset (b \supset a)$	arbitrary addition of an implication
8d.	$\bar{a} \supset (a \supset b)$	
8e.	$a.(a \supset b) \supset b$	inferential implication
8f.	$(a \supset b) \supset (a \supset b \vee c)$	addition of a term in the implicate
8g.	$(a \supset b) \supset (a.c \supset b)$	addition of a factor in the implicans
8h.	$(a \vee c \supset b) \supset (a \supset b)$	dropping of a term in the implicans
8i.	$(a \supset b.c) \supset (a \supset b)$	dropping of a factor in the implicate
8j.	$(a \supset b).(c \supset d) \supset (a.c \supset b.d)$	derivation of a merged implication
8k.	$(a \supset b).(c \supset d) \supset (a \vee c \supset b \vee d)$	
8l.	$(a \supset b).(b \supset c) \supset (a \supset c)$	transitivity of implication
8m.	$(a \equiv b).(b \equiv c) \supset (a \equiv c)$	transitivity of equivalence