"super-empirical" proposition an empirical one which leads to the same actions. The second principle does nothing but formulate the consequence which this idea implies for a theory of meaning based on the relation of meaning to action. We may state it in the form: if two sentences will lead us under all possible conditions to the same actions, they have the same meaning. However, this formulation is possible only within the probability theory of meaning; for only if we introduce the predicate of weight can the relation of meaning and action be demonstrated. On the other hand, it becomes obvious from this formulation that the antimetaphysical function of the principle is kept. In our formulation also the principle denies any "super-empirical meaning" and states: *there is as much meaning in a proposition as can be utilized for action*. With this formulation, the close relation of the probability theory of meaning to pragmatism becomes still more obvious; we think, though, that our theory, by using the concepts of probability and weight, may furnish a better justification of the relation between meaning and action than pragmatism is able to give. This outcome of the probability theory of meaning—the connection of meaning and action—seems to me the best guaranty of its correspondence to empirical science and to the intention of language in actual life.

CHAPTER II

IMPRESSIONS AND THE EXTERNAL WORLD

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§ 9. The problem of absolute verifiability of observation propositions

The foregoing chapter was based on the assumption of the division of propositions into direct and indirect sentences. Direct sentences are sentences concerning immediately observable physical facts; such sentences—this was the presupposition—are absolutely verifiable, i.e., accessible to a determination of their truth-value within the frame of two-valued logic. Only for indirect sentences was the predicate of weight needed; such sentences are not controlled directly, but by means of their relation to direct sentences which confer on them a certain degree of probability.

This particular position of observation sentences, as direct sentences, is now to be examined. We must question their being accessible to direct verification. They deal with what is called a *physical fact;* our investigation, therefore, concerns the question whether we can verify a physical fact.

Before entering into detail, we must indicate that the word "fact" is used in a fluctuating sense. Sometimes physical laws are called facts because they are furnished by experience and not by deduction; but this is not what we shall here call a fact. Laws concern, on account of their claim to generality, an infinity of facts; we shall therefore distinguish them from facts, ascribing to this word a narrower sense.

To clarify our intention let us apply this distinction to some controversial examples. We know that the velocity of light is the upper limit for all velocities transmitting an effect; is this a fact or a law? According to our definition, generality characterizes law rather than fact, so this must be called a law. For the same reason we must call it a law that the Michelson interferometer shows the equality of the velocity of light in different directions because this result is stated for all apparatus of this kind. We obtain a fact if we proceed to consider the special experiment made by Michelson in 1883 with his special apparatus. To render the term more precise we may speak of a *single fact;* a single event, occurring at one definite spatiotemporal point, represents such a single fact.

We have now to apply our criticism to single facts and to ask whether single facts can be absolutely ascertained or whether propositions concerning single facts can be absolutely verified.

Let us consider the Michelson experiment. Every physicist knows that the statement concerning the equality of the velocity of light in different directions is not directly observed in the Michelson experiment but that it is inferred. Such a physical experiment is a rather complicated procedure. Directly observed are images in telescopes or on photographic plates, or indications of thermometers, galvanometers, etc. If we proceed from these experimental data to the statement concerning the velocity of light, this procedure is an inference, and an inference containing inductions. It contains, for instance, the presupposition that the temperature noted from time to time on the thermometer is valid also for the intervals of time between the moments of observation; that the laws of geometrical optics are valid for light passing through the telescope; that the lengths of the brass bars of the apparatus do not

change during the observation (compared with other bars in rest relative to them), etc. It is obvious, then, that the statement concerning the velocity of light is not absolutely certain, being dependent on the validity of inductions. So this statement, although concerning a single case, is not absolutely verifiable. We see that mere reference to a single case is not sufficient to insure absolute verifiability to a statement.

We arrive at a more favorable result if we proceed from the statement concerning the velocity of light to statements concerning the individual data of the instruments used. It seems to be absolutely certain that at least the thermometer registered, say, 15° C. It might be a bad instrument, and the temperature of the room might be different from that indicated; but that this individual thermometer reached at this particular moment the line corresponding to 15° C.—is not this single fact absolutely certain?

This question leads us from the rather abstract facts of physics to the concrete facts of daily life. A thermometer is a thing built of glass, and mercury, and wood; a thing comparable to tables, chairs, houses, trees, stones—in short, a thing belonging to the sphere of our daily environment. To ascertain the existence of such objects requires no theoretical conclusions; so it seems possible to obtain absolute truth in this case at least.

It is well known that this assumption has been attacked by almost all philosophers since Descartes; and I should say for good reasons. The correct way of substantiating this attack seems to me to be the following one.

A statement concerning a physical fact, even if it concerns a simple fact of daily life, never refers to a single fact alone but always includes some predictions. If we say, "There was a table in my room, before my eyes, at 7:15

P.M.," this contains the prediction: "If no table passes the doors from 7:15 to 7:20, and no fire or earthquake acts on my apartment, then there will be a table in my room at 7:20." Or simpler still: "If I put a book on the table, it will not drop." It is because such predictions are included in the statement that it is not absolutely true, for an absolute reliability of the predictions cannot be warranted.

It might be proposed that we can separate these predictions from the statement, and reduce it to a bare factual statement; that is, that we exclude consequences concerning the table after five minutes, or concerning books placed on the table, and restrict the statement to the table just as it is seen. Such a reduction is possible; if we perform it, however, the statement loses its definite character. Saying, "There is a table," normally means that I maintain that what is referred to is a material thing capable of resisting the pressure of other physical things; this is what is expressed in the implication concerning the book. If I renounce implications of such a kind, the object I saw might be a picture furnished by a concave mirror; indeed everybody knows that illusions occur in which the image produced by a concave mirror is taken for a material object. The difference between the material object and the illusion cannot be otherwise formulated; it is only the consequences—i.e., future observations—which distinguish these two categories. This is the essential point. It might be objected that the future observations could be replaced by past observations—that I might have put the book on the table a moment before, or touched the table with my hand a moment before. But if I infer from this that the table as I see it now, without a book on it and without my touching it, is a material table and not the image produced by a mirror, then I perform an induction running, "If I

were to touch it now, I would feel the resistance," or "If I were to put the book on the table now, it would not fall" —sentences which concern future observations and not past ones. It is true that past observations of the kind mentioned may suffice to substantiate my statement, but only because I base inductions on them; the statement concerning the table as a material object cannot be separated from predictions without losing its definite character; i.e., it would no longer indicate a definite physical object.

This, it seems to me, is the reasoning which proves indubitably that there is no statement concerning physical objects which is absolutely verifiable. Statements about simple physical objects are very sure but not absolutely sure. They are not sure because they are controllable; if we admit the possibility that later observation can control our statement about a present observation, we cannot exclude the case of a negative result of this control—that is, our statement cannot be maintained as certain. If in spite of that we take such statements as certain, we perform an idealization; we identify a high degree of probability with certainty. But, strictly speaking, this is not a case of truth but one of weight; even the observation sentences of daily life are not to be considered as direct sentences but as indirect sentences judged by the predicate of weight instead of the predicate of truth. The probability theory of meaning, therefore, is to be applied even to observation sentences of physics, or of daily life, if such sentences are to have meaning.

The attempt has been made to show that, although a physical statement never can be absolutely verified, it may at least be demonstrated in certain cases that the statement is false. If a book placed on a table does not stay lying there but falls down vertically, we might deem it sure that what is there observed is no material table. The principle

of absolute verification, so we might suppose, might be replaced by a principle of absolute falsification.[1] Such an idea, however, is not tenable. Any falsification also presupposes certain inductions based on observations of other things and may be assumed with probability only. In our example it may be the book which is the nonmaterial thing, or which has become so the moment after withdrawing my hand from it; the statement about the material table then would remain true. Our statements about physical things are interwoven in such a way that the rejection of one of the statements may always be replaced by the rejection of another. Our choice as to the rejection is entirely made by reflections determined by the rules of probability. There is, therefore, no absolute falsification, just as there is no absolute verification. There remains nothing but the probability theory of meaning if we wish to justify observation propositions in the sense in which they are actually used in science or in daily life.

§ 10. Impressions and the problem of existence

The result of the foregoing section cannot be taken as a proof that there are no verifiable sentences at all. The uncertainty as pointed out concerns only observation sentences referring to physical objects. Philosophers who share our interpretation of sentences of such a kind have maintained the idea that there are observation sentences of another kind which can be absolutely verified. These are sentences concerning impressions. We must now consider this concept and ask for its epistemological significance.

The way in which so-called impressions are introduced is given by a continuation of the reasoning with which we questioned the truth of an observation sentence.

[1] This attempt has been made by K. Popper, *Logik der Forschung* (Berlin, 1935); cf. also my criticism of this book in *Erkenntnis*, V (1935), 267.

It is true that a sentence stating the existence of a material table implies predictions and that a reduction of it to a bare report would destroy its physical reference. But what, then, would be the result of such a reduction? We arrive, it is said, at a fact of another type: we come to say that at least I see a table. This is true whether it is a material table or the optical image of such a table produced by a concave mirror; so this at last is an indubitable fact. Facts of such a kind are called "impressions."[2] Thus there are, it is maintained, absolutely verifiable statements; what they concern, however, is not physical facts but impressions.

We shall accept, for the present, this conception. We shall admit that there are immediately given facts of such a kind, which the word "impression" or "sensation" is to denote—facts which we describe in sentences capable of absolute verification. A criticism of this assumption may be postponed to the following chapter. In the same way as the first chapter was based on the presupposition of the absolute verifiability of observation propositions, so the present chapter will be based on the presupposition of the absolute verifiability of impression propositions. It is only the consequences of this presupposition, not its validity, in itself, which we want to study for the present. We shall study these consequences by making use of the results of the foregoing chapter, which showed the relevance of the concept of probability; in a similar way, we shall show that the probability character of the inferences which occur affects the consequences resulting from the introduction of impressions as a basis of knowledge.

Impressions are—this is the usual conception—phenomena occurring within my mind but produced by physical

[2] The words "presentation," "sensation," and "sense data" are used in the same sense.

things outside my mind. Thus the concept of impression leads to the distinction between my own mind and the external world. Impressions are events of my personal sphere, of my private world; it is a grave mistake, so the adherents of this conception argue, to think that what I observe are things of an independent existence—I observe only the impressions produced by such things, i.e., effects of external things on my private world.

We said that we shall admit impression sentences as being absolutely certain; we see, however, that this absolute certainty is restricted to events of a private world only. With the transition from my own subjective experience to the objective external world, uncertainty enters into my statements. But not only uncertainty as to special statements; there is superimposed a general uncertainty as to the world of external things at all. How do we know that there is such an external world outside our private world? It is the problem of the existence of external things which arises here.

As long as we regarded observation sentences as the basis of knowledge, the problem of existence did not occur. There is no difference as to existential character between observational facts and other facts indirectly inferred; it is only the introduction of the basis of one's own psychical experience which creates the existence problem. This problem, therefore, is due to a certain advance in philosophical inquiry; it originates from the attempt to reduce knowledge to an absolutely certain basis.

Indeed, for the naïve conception of the world, there is no problem of existence. The sphere of daily life is not disturbed by the question whether the things we observe around us are real, are existent; any doubt of such reality would be considered ridiculous, as an outcome of an unhealthy departure from the clear views of daily experience.

The man of common sense is convinced that he is right in asserting that the tables, the houses, the trees, and the people around him exist as he does. Not only is this believed for objects of personal experience but the communications of other people and of scientific men are also accepted as certain. That there are other continents besides the one on which we live; that other planets and stars exist incomparably bigger than our small island within the universe; that unseen physical entities such as electricity, atoms, X-rays exist—all this is considered as a matter of fact which it would be simply unreasonable to doubt. This world of concretely existent things is further enriched by other things which are called "abstract," but which are nonetheless conceived as existent also. There is the state, as a political body, never directly seen as a whole, but whose reality is imposed upon everyone by daily experience; there is the spirit of the nation whose existence we find emphasized every day in the leading articles of the newspapers; there is the soul, our own and that of other persons, the doubt of which might lead to disagreeable collisions with the church; there is the financial crisis, the reality of which needs no confirmation by the holy authorities. In short, there is a solid and compact world around us, filled up by less solid but not less real things; this world is given to us from the early days of childhood, and there is no question as to its existence.

The beginning of doubt concerning this matter-of-fact world marks, indeed, a departure from the sound pursuit of daily affairs. It is that departure which leads from mere submission to traditional conceptions toward an intellectual penetration into the formation of concepts and marks the very beginning of philosophical thinking. It is the issue of the attempt to understand what we think, to clarify the bearing and the legitimacy of human conceptions. It

is, therefore, an enterprise not less healthy than looking after everyday necessities; it is the sound desire to add to the struggle for existence an understanding of the struggle and of existence itself; and, if common sense attacks philosophy on account of its questioning fundamental concepts of life, this is only because the man of common sense does not realize that the desire for understanding may become as urgent as the desire for economic existence.

We preface this general remark to the following inquiry to meet the opinion of certain philosophers that an investigation of the question of the existence of external things is unreasonable and ridiculous. Such a position would be in itself an answer and would demand substantiation. It is true that the question of existence, as it is usually expressed, needs a correction; and it is precisely the task of the philosopher to clarify the question first before an answer can be given. But it is not legitimate to cut short the question by sophistical remarks. It has been argued by certain philosophers that a man who doubts the existence of external things ought to have his forehead knocked against a wall to convince him of the reality of the wall. I do not think this is philosophical reasoning. What the man saw might have better convinced him of external things than what he felt because what he saw was outside his body, whereas the pain he felt was inside; and it is just the question of whether there is something outside of himself which the man wanted to solve.

With this remark we are in the center of the problem of existence. Experience, even experience of daily life, compels us to distinguish between dreaming and being awake; there is a world of dreams as vivid as the waking-world—but nevertheless we know that we have to interpret this world as an internal world only, to which no external things correspond. Are we sure that the so-called "waking-

world" is better? That this world is of a greater regularity is no convincing argument; nor is it an argument that in this world we even happen to reflect about its reality. That may happen in the dream world also; there are indeed dreams in which we try to discover whether we are in a dream and decide that we are not—only to discover on waking that this decision was itself part of a dream. The question concerning the reality of our waking-world, therefore, cannot be rejected as unreasonable; it is as reasonable as the distinction between the waking-world and the world of dreams.

§ 11. The existence of abstracta

There is a second problem of existence distinct from that of impressions. This is the problem of abstracta. What of the existence of such things as the political state, the spirit of the nation, the soul, the character of a person? Do things of such a kind exist? If they exist, are they things alongside of such concrete things as houses or trees? Or are they things of another sphere of existence? But what, then, is this other sphere? Since the time of Greek philosophy this question has been constantly discussed; it formed the subject of the famous controversy between nominalism and realism; it split philosophers into parties as thoroughly as did the question of the reality of the external world.

In spite of all differences there is one common feature in the structure of the two problems of existence. One questions the existence of abstracta as distinct from concreta, the other questions the existence of concreta in relation to impressions. It is this relational character which is common to both problems. We shall therefore have to study the relations occurring here. As these relations are of a

simpler type in the problem of the existence of abstracta, we shall begin with this problem.

As to the problem of the existence of abstracta, it seems to me that the position of the realists was never a very good one. They insisted on the existence of abstract things, but they were always obliged to defend themselves by placing these things into a special sphere; the sphere of the "ideas" of Plato is the famous prototype of this kind of existence. There is, nevertheless, a strong natural feeling against such a procedure; the human mind needs a certain degree of perversion by sophistic training to be able to find some sense in such terms. The position of the nominalists, who maintained that only concrete things exist, looks much sounder, though I do not want to say that the ancient nominalists had already found the right form of solution.

The nominalistic idea is that abstracta are reducible to concreta, i.e., in terms of modern logic: that all propositions concerning abstracta can be translated into propositions concerning concreta only. To give an example: instead of saying, "The race of Negroes has its home in Africa," we can say, "All Negroes descend from forefathers who lived in Africa." In this way, the abstracta "race of Negroes" and "home" are eliminated and replaced by concreta, such as "descend" and "forefather"; the new terms which enter by this operation are logical concepts, such as "all." By the same method, such complex terms as the "political state" can be reduced to concreta. The logical method, in the general case, may be somewhat more complicated. It may turn out that to replace a statement containing an abstractum, more than one phrase containing concreta is needed. Thus the phrase, "The state is waging war," is to be translated into many propositions concerning soldiers, shooting, being wounded, and dying, men working in armament factories, others writing in

offices, etc. We speak in general of a *reduction by co-ordination of propositions;* to one abstract proposition we co-ordinate a group of concrete propositions in such a way that the meaning of the group is the same as the meaning of the abstract proposition.

The equivalence of meaning on both sides of the co-ordination is an outcome of the theory of meaning as developed in chapter i. There is an equivalence of truth-value on both sides; if the abstract proposition is true, the group of concrete propositions is true, and if the abstract proposition is not true, not all concrete propositions taken in conjunction are true. It may be objected that in some cases the abstract propositions may be true even if not all concrete propositions are true; this may be because the same abstract fact may be realized by different concrete facts. The abstract fact, for instance, that there is good weather may be realized by a clear sky and a calm atmosphere, or by a partially cloud-covered sky and some fresh wind, etc. This case finds its logical expression by the introduction of disjunctions which allow us to maintain the equivalence in an expanded form. Let a be the abstract proposition and $c_1, c_2, \ldots,$ the concrete propositions; then the equivalence is to be formulated[3]

$$a \equiv [c_1.c_2 \ldots . c_m] \vee [c_{m+1} \ldots . c_n] \vee \ldots . \vee [c_{r+1} \ldots . c_s] \quad (1)$$

In this way the exact logical construction of the abstracta is established. It follows from both the truth theory of meaning and the probability theory of meaning that both sides have the same meaning.

We see that the position of nominalism is connected with the verifiability theory of meaning; this, of course, is not a discovery of our time but the basic reason why both

[3] I use the signs of Russell: a period (.) for "and," $\vee$ for the inclusive "or," and $\equiv$ for logical equivalence.

theories have been developed in reciprocal relation. We have already mentioned that the nominalist Ockham was the father of our second principle of meaning. The nominalists were right in maintaining that the existence of abstracta is reducible to the existence of concreta.

What the ancient nominalists did not see was that it cannot be inferred from their theory of meaning that the abstracta do not exist. Whether or not we apply the category of existence to an abstractum is a matter of convention. We may say: "The race of Negroes exists." We know, then, that this means the same as, "Many Negroes exist, and they have certain biological qualities in common which distinguish them from other people." We may also say: "The race of Negroes does not exist." Then we have to add: "Many Negroes exist, and any proposition containing the term 'the race of Negroes' can be translated into propositions concerning those Negroes." We see, then, that the question whether or not abstracta exist, whether or not there is the term only or also a corresponding entity, is a pseudo-problem. The question is not a matter of truth-character but involves a decision—a decision concerning the use of the word "exist" in combination with terms of a higher logical order.

If we ask now which decisions are used in practice as far as the existence of abstracta is concerned, we meet the remarkable fact that there is no common rule, that the use of language decides sometimes for and sometimes against the existence of abstracta. To give some examples: the furniture belonging to a family is usually taken as existent; so is the company invited to a home, or a regiment of soldiers, or a court of justice. The decision is doubtful concerning such terms as "the state," or "human society," or "the third estate." In other cases there is a clear refusal to acknowledge existence: the height of a mountain

does not exist, nor does the mortality of children, nor does left-handedness. The question of the motives of these decisions must be analyzed psychologically. It seems that those abstracta are conceived as existent with which we have concern in practical life, and which are usually expressed by nouns. We sometimes have to do with left-handed people, but we seldom employ the term "left-handedness"; so this remains a term without an existent object. Reference to "the furniture," however, appears frequently, and furniture is therefore conceived as an existent thing. The decision may even depend on the profession of the speaker. For a merchant, supply and demand may be existent entities, whereas an electrician would conceive an electrical charge as existent. It is a remarkable psychological fact that this "feeling of existence" which accompanies certain terms is fluctuating and depends on the influence of the milieu. The pursuit of this question is of great psychological interest; for logic there is no problem at all.

The possibility of ascribing existence to abstracta, however, does not justify the position of realism. The abstractum is not a thing of another "sphere" but a thing existing in the ordinary world. The furniture exists in the same world as the tables and chairs which form its elements; like these, the furniture is a thing which has weight and can be paid for in money. The realist introduces this other sphere because he believes in a surplus meaning of the abstract term. This is, I think, due to a misunderstanding of a logical fact which seems to have bothered ancient logicians, but which can be interpreted by nominalism without any difficulty. It is the fact that the abstract thing and the things which form its concrete elements cannot be "added," cannot be put alongside of one another. We are not allowed to count, say, a table and three chairs

and a cupboard as six things, adding the furniture composed by these five things to them as a sixth thing. This is, however, a matter of the rules of language only; these rules contain prescriptions about the use of the terms "addition," "counting," "number," etc.—prescriptions which take account of the difference between the abstractum and its elements. To infer from this distinction the necessity of putting the abstracta into another "sphere" means mistaking a problem of language for a problem of being; a misunderstanding of the type which is responsible for the origin of the construction of so-called "ontology." The domain of the theory of abstracta has become a kind of maze composed of pseudo-problems.

Another pseudo-problem of this group is given by the problem of the spatial localization of certain abstracta. Does the state as a political body occupy a place in space? It may be answered that only the country belonging to the state, and not the state as a political institution, has a spatial extent. But this is a matter of convention only; it depends on the way in which we define spatial qualities. All qualities of the abstractum "state" are to be defined as relations between its concrete elements, so we may also define the spatial extent of the state as the space occupied by its inhabitants. The question whether a physical force is in space, or a melody, or the elasticity of a spring, is of the same type and is to be settled by a definition.

With these remarks the problem of the existence of abstracta finds its solution. This problem is a matter of decision and not a question of truth-character. Independently of the decision it may be stated that the existence of the abstracta is reducible to the existence of other things. This logical process may be called "reduction." The abstractum may be called a "complex"; the concreta on the right hand of formula (1) may be called the "internal elements"

of the complex. The inverse process may be called "composition." The elements compose the complex, the complex is reduced to its elements. Both relations may be united into the term "reducibility relation"; it is defined by the equivalence (1).

Let us add a remark which concerns a relation with which we must deal in this context: this is the relation of the whole to its parts. This relation is to be considered as a special case of the relation of reducibility as defined. The parts are internal elements of the whole, as a complex. There is, however, no strict definition as to the use of this term. We use it when the complex has a spatial extent, and the elements have also spatial extents which form parts, in the geometrical sense, of the geometrical extent of the complex, as in the case of a wall and its bricks, or an estate and its grounds and fields. In this case the concept of the whole and its parts is reduced to the concept of geometrical whole and its parts. This conception is not always maintained, and sometimes the use of terms fluctuates; shall we consider the trees as parts of the wood? The definition of the relation of the whole and its parts is not given strictly enough to settle this question unambiguously. An example of a nonspatial case of this relation is given by a fortune and its parts, which may consist in cash, shares, and estates. It seems that we speak of a whole and its parts in a situation in which we ascribe to the elements certain numerical or geometrical quantities, the arithmetical sum of which is ascribed to the complex. This is, however, not a sufficient condition for the term. If the complex has, in addition, many other qualities which do not fulfil this relation, we do not consider it as a whole composed of its elements. The political state is usually not considered as a whole built up by its inhabitants as parts, though the quantity "total population" is the sum of the

inhabitants; this is because the sum relation is not valid for many other qualities ascribed to the state.

Another example of the relation of reducibility is the case of the *Gestalt*. A melody is a *Gestalt* built up of tones; a drawing offers a *Gestalt* built up by pencil marks on the paper. This concept plays a great role in modern psychology, and for good reasons; but its logical nature as a special case of the relation of a complex to its internal elements has not always been pointed out by psychologists. They are right in saying that the *Gestalt* is not the "sum" of its elements, i.e., does not stand to these in the relation of the whole to its parts; but this does not imply that statements about the *Gestalt* have a surplus meaning over and above statements about its elements. On the contrary, the equivalence (1) holds here as well as in all other cases of the relation of reducibility. If this is disputed, the denial originates from an insufficient formulation of the statements about the elements, the relations between which must not be forgotten. The special conditions which a complex must fulfil to be called a *Gestalt* are, as yet, not so sharply demarcated that unambiguity is insured for all cases. This does not exclude, however, a useful application of the concept of *Gestalt* in many other cases.

The logical investigations which follow are independent of the special cases of the whole and its parts, or of the *Gestalt*. They concern the general case of the complex and its internal elements, expressed in the reducibility relation as formulated in (1).

§ 12. The positivistic construction of the world

We turn to the second problem of existence—the question of the existence of concreta. We begin our investigation with the consideration of the positivistic solution of the problem.

The positivistic conception of the existence problem may be summarized in one statement: The existence of concreta is to be reduced to the existence of impressions in the same way as the existence of abstracta is reduced to the existence of concreta.

This idea is an outcome of the positivistic conception of impressions as basic facts of knowledge (§ 10) in combination with the truth theory of meaning (§ 7). All observations are to be reduced, it is said, to impressions because it is only impressions that I can directly observe. Propositions concerning concrete physical things are, therefore, indirect sentences reducible to impression sentences as corresponding direct sentences; only the latter sentences can be directly verified. According to the principle of retrogression, this correspondence is an equivalence of meaning; therefore this correspondence is a reduction, in the sense defined in § 11.

Let us illustrate this by a simple example. The proposition, "There is a table," is inferred from certain impressions we have in looking at the table from different sides, by touching it, and the like. Now according to the principle of retrogression this inference is taken as an equivalence of meaning. Therefore the sentence, "The table exists," means the same as the sentence, "I have impressions of such and such kinds." It is the same relation as is valid for the reduction of abstracta; the table, therefore, is to be conceived as a complex, the elements of which are impressions.

This conception permits the positivists to interpret the existence of concreta in the same way as the existence of abstracta is interpreted. There is, they argue, no genuine problem of the existence of the external things; it is a pseudo-problem. We can say that external things exist; then this means the same as, "Impressions of such and

such kinds exist." We can say also that external things do not exist. Then we must admit that the term "external things" may nevertheless be used and expresses the same as propositions concerning impressions. To decide upon the first or the second mode of speech is a matter of convention only. To demand more, to ask whether the external things exist "beyond" the impressions, would be meaningless. This is the famous positivistic interpretation of the existence of the external world.

It is one of the advantages of this conception that there remains no doubt as to the "reality" of the external world. The existence of the world is as sure as the existence of my impressions; this is because the first contention means no more than the second. Any doubt of the reality of the external world is an outcome of a meaningless question which supposes an existence of the things "beyond" my impressions. It would be the same meaningless question as to ask whether the race of Negroes has an existence of its own beyond the existence of the individual Negroes. To deny the existence of an external world, consequently, is not rejected as false but as meaningless; the positivistic solution, therefore, pretends to establish the world of external things in absolute certainty.

In spite of that conclusion, the positivistic conception need not deny a difference between dreaming and being awake. If we state a difference between the two, this must be inferred from a difference in impressions; this difference involves perhaps the great regularity of the impressions of the waking-state in comparison with the irregularity of the impressions of the dream. The whole of my impressions, therefore, may be divided into two classes such that alternately groups of impressions belonging to one class or the other follow one another; let us call these classes the

"regular class" and the "irregular class." Applying the principle of retrogression, we find that the sentence, "I was dreaming," means, "My impressions belonged to the irregular class"; whereas the sentence, "I am awake," means, "My impressions belong to the regular class." The difference between dreaming and being awake is therefore saved by this theory; if anybody demands more, if he wants to maintain that the things he sees while being awake are "real" things whereas the things of the dream are "unreal" things, he says nothing because such a surplus contention would be meaningless. All that he wants to maintain by such words is sufficiently expressed by the already established difference between dreaming and being awake—because nothing else *can* be maintained.

These are the fundamental ideas of positivism as they are generally developed by their adherents. There is something very suggestive in these conceptions, something comparable to the convincing clarity of a religious conversion; and the ardor with which this interpretation of the existence problem has been emphasized by the preachers of positivism reminds one indeed of the fanaticism of a religious sect. I do not say this with the intention of discrediting positivism; on the contrary, it is just this strength of conviction which attracts our sympathies because of its manifest intensity and candor and its extreme desire to submit to the exigencies of intellectual cleanliness. But it is the danger of fanatic doctrines that they forget the necessary criticism of their basic conceptions; we must take care that admiration of the lucidity of the theory does not restrain us from a sober examination of its logical bases.

Our foregoing investigations of meaning lead us to an attack against one of the pillars of the positivistic doc-

trine. It is the principle of retrogression which we must question here. We found in § 7 that the relation between direct and indirect sentences is only a probability connection, not an equivalence. Thus the main idea of the positivistic reduction is not tenable. In the relation between abstracta and concreta, the co-ordination of propositions is an equivalence; only on account of this fact is the existence of abstracta reducible to the existence of concreta. If it turns out now that for the relation between concreta and impressions the co-ordination is of another character, the analogy does not hold; we are not then justified in saying that the existence of concreta is reducible to the existence of impressions. This means that the sentence, "The table exists," does not have the same meaning as the sentence, "I have impressions of this and this kind." The instinctive aversion we feel against submitting to the religious conversion turns out to have a sound logical basis. The positivistic interpretation of existence is not valid; there is a surplus meaning in the statement about the existence of external things. The positivist turns out to be a victim of the schematization which replaces a high probability by truth and takes the connections between propositions as relations ruled by the predicates of truth and falsehood. This schematization is allowable only for certain purposes; if it is made the basis for judging a question of principle, such as the question of the interpretation of existence, it leads to a profound discrepancy between epistemological construction and actual knowledge.

It will now be our task to develop another solution of the problem of existence—a solution in accordance with the probability character of the relations between propositions. To exhibit this solution we must first enter into a more detailed analysis of the nature of probability connections.

§ 13. Reduction and projection

We found that the transition from external things to impressions cannot be interpreted as a reduction; it is of another type of logical structure. To understand the nature of this structure, let us begin with the consideration of two examples.

The relation of reduction may be illustrated by the relation between a wall and the bricks of which it is built. Every proposition concerning the wall may be replaced by a proposition about the bricks. To say that the wall has a height of three meters reads in translation that there are bricks stuck together by mortar and piled upon one another to the height of three meters. The wall is a complex of the bricks; the bricks are the internal elements of the wall. The wall is not the "sum" of the bricks; this means that, if the bricks are separated from one another and scattered over the ground, the wall no longer exists, whereas the individual bricks may be unchanged. The wall is dependent upon a certain configuration of the bricks. This is included into our concept of "complex"; since all propositions concerning the complex are equivalent to propositions about the elements, the qualities of the complex will change if the relations between the elements change. The existence of the complex is dependent on certain relations between the elements, such that the complex may cease to exist whereas the elements still exist.

The inverse relation does not hold. If the elements cease to exist, the complex can no longer exist either. If the bricks are destroyed, the wall is also destroyed. This is what we mean by reducibility of existence: the existence of the complex is dependent on the existence of the elements in such a way that the nonexistence of the elements implies the nonexistence of the complex. This may be transformed into the statement that the existence of the complex

implies the existence of the elements. The latter statement is only another formulation of the former. It is, however, to be distinguished from the converse relation according to which the nonexistence of the complex would imply the nonexistence of the elements, or the existence of the elements would imply the existence of the complex; as we saw, this inverse relation does not hold. There is, consequently, an asymmetry between the complex and its internal elements; it is just this asymmetry by which these two terms are distinguished, and which is meant by saying, "The existence of the complex is reduced to the existence of its internal elements." We do not make the inverse statement; the elements have, so to speak, a more solid existence.

It might be objected that a clever architect might be able to exchange the bricks, one after the other, for other bricks, in such a careful way that the existence of the wall remains undisturbed; the original bricks might even be ground to powder so that these elements no longer exist whereas the complex persists. This objection, however, is to be overcome by a more correct use of words. The wall made up of the exchanged bricks is a complex of other elements; if we speak nevertheless of the same wall, this complex "wall" is to be defined in such a way that it is constituted by one system, or another, of elements. That is, the complex is to be constituted by a disjunction of elements; or the propositions concerning the complex are equivalent to a disjunction of propositions about elements, as we stated formerly in the general formula (1) in § 11. Most of the complexes of usual language are of this complicated type. A melody may be played in different keys; the melody is defined by means of a disjunction of propositions. Our existence theorem, then, is to be formulated as follows: the existence of the complex implies the existence of *one* of the systems of elements but not the existence of a determinate one of the systems; and the nonexistence of *all* the systems of elements implies the nonexistence of the complex. We shall call such a complex a *disjunctive complex*.

We may give a more determinate form to the relation of the elements to the complex. We saw that the existence of the elements is not a sufficient condition for the existence of the complex. But it becomes a sufficient condition if some additional relations between the elements are fulfilled. If the bricks are arranged in such and such a way, the wall exists. Let us call these additional relations the *constitutive relations* between the elements. Then we may say for the simple as well as for the disjunctive complex: The complex exists if one of the corresponding systems of elements exists and fulfils the constitutive relations. This formulation expresses what we call the dependence of the complex on its elements. The elements may "produce" the complex; whether or not they produce it depends only on their internal relations. We must add, of course, that for this purpose the elements must be completely given; only in this case do we need no introduction of further elements to produce the complex. That is, only in this case can the constitutive relations be formulated with reference to these elements alone. Let us call such a set of elements a *complete set of elements*. The tones which the musician plays on the piano form such a complete set, that is, a set sufficient for the existence of the melody; it is not necessary to play other keys also. The constitutive conditions are formed here by the relations which constitute the temporal order of the tones, the length of the time intervals between them, and the like.

After this analysis of the concept of reduction we turn now to the consideration of another logical structure which

is also characterized by a co-ordination of propositions, but which shows different qualities.

We imagine a number of birds flying within a certain domain of space. The sun rays falling down from above project a shadow-figure of every bird on the soil, which characterizes the horizontal position of the bird. To characterize the vertical position also, let us imagine a second system of light rays running horizontally and projecting the birds on a vertical plane which may be represented by a screen of the kind employed in the cinemas. We have, then, a pair of shadows corresponding to every bird; which of the shadows belongs to the same bird may be indicated by the outlines of the shadows. This correspondence allows us to determine the spatial position of every bird from the position of the corresponding pair of shadows and to determine the spatiotemporal movement of the birds from the spatiotemporal changes in the pairs of shadows. We can express this in the form of a co-ordination of propositions: every proposition concerning the movement of the birds is co-ordinated with a proposition about the changes of the pairs of shadows.

By this method the spatiotemporal position of the birds is projected into a system of marks which can be taken as a representative of the original birds. Analogous methods would allow us to construct marks for other qualities of the birds; for this we would have to employ other effects coming from the birds. The singing of the birds might be recorded, and the curved lines on the record would be the marks of the singing. Everything which can be observed from outside must be communicated to us by a physical process and can, therefore, be transformed into a physical thing outside the birds; this physical thing is our mark for the quality in question. We obtain in this way a system of marks which contains representatives for any quality of

the birds observable from below, and which enables us to construe a co-ordination of propositions: every proposition concerning the birds is co-ordinated with a proposition, or a set of propositions, concerning the marks.

We contrive, in this way, to obtain a co-ordination analogous to the case of reduction illustrated in the example of the wall and the bricks. There are, however, some specific differences between the two cases; let us enumerate those qualities in which the second case differs from the first.

First, there is no equivalence of the co-ordinated propositions. This is because there is only a probability connection between the birds and the marks; if we see the marks only, we may infer with a certain probability that they are produced by birds, and if we see the birds only, we may infer with a certain probability that they will produce the marks. This lack of certainty is due to the fact that natural processes can never be foreseen with certainty. Whether or not the shadow-figures will be produced depends on numerous physical factors other than the presence of the birds alone, e.g., on the conditions on the screen. Conversely, whether or not there are birds as causes of observed shadow-figures cannot be inferred with certainty because there might be other physical processes having the same effect on the screen. Consequently there is no strict relation between the truth-values of the co-ordinated propositions. The proposition about the birds may be true, and that about the marks may be false; conversely, the proposition about the birds may be false, and that about the marks may be true.

Second, there is no reduction of existence. The birds have an existence independent of the existence of the marks. Using a mode of speech similar to our description of the existence qualities valid for reduction, we may say: neither does the existence of the birds imply the existence

of the marks, nor does the existence of the marks imply the existence of the birds. The same is valid for nonexistence. This may be taken as a definition of what we mean by saying that the existence of the birds is not reducible to the existence of the marks. The shadow-figures may vanish while the birds still exist because other conditions may interfere; and the birds may be destroyed without the shadow-figures disappearing—because these may be produced by other physical causes.

In the example concerning the wall and the bricks we called the transition in question a *reduction;* in opposition to this we shall call the transition from the birds to the marks a *projection.* To express the parallelism we shall speak in both cases of a complex and its elements; to show the difference, however, we distinguish between a reducible complex and a projective complex, and call the elements of the former *internal elements,* the elements of the latter *external elements.* The birds are thus to be called a projective complex constructed by means of the marks as external elements. The most conspicuous feature of the projection is that it does not furnish a reduction of existence; this is because the relations between the projective complex and its elements are probability connections only. The probability character of these relations may be used to formulate the definition of the projection: A projection is a co-ordination of propositions, by means of a probability connection, in such a way that one term, or one set of terms, called the "complex," occurs only on one side of the co-ordination, and another term, or set of terms, called the "external elements," occurs only on the other side of the co-ordination. As the relation of probability connection is symmetrical (cf. § 7), there is no absolute difference between the elements and the complex of a projection; the terms may be interchanged. Thus the shadow-figures may

be called a projective complex of the birds as external elements. Which side is denoted as the side of the elements depends on psychological conditions; usually we choose that side which is more easily accessible to observation.

To see the difference between both kinds of transition, let us consider a transition in which the birds are a reducible complex: this is the case when we consider as elements the cells of which the birds are constructed, or the atoms. These would be internal elements. An attempt might be made to conceive the projective complex as a disjunctive complex, by considering a disjunction of sets of elements which contains the internal elements as one set. But it is easily seen that the relations stated above for disjunctive complexes are not fulfilled. The existence of the complex implies, then, the existence of a determinate set of elements, i.e., of the set of internal elements; and it is not possible to add, to a set of external elements, constitutive conditions in such a way that the existence of the complex is implied. The projection is of a type logically different from a reduction.

Let us now apply the concepts which we have developed to the problem of the relation between impressions and external things. We pointed out that there is no equivalence between propositions concerning external things and propositions concerning impressions; there is only a probability connection. This relation is thus a projection and not a reduction; the existence of the external things is not reducible to the existence of impressions; the external things have an independent existence. It is the same kind of independence as between the birds and their shadows. Thus the naïve conception of independence of existence, as illustrated by this example, may be applied to the problem of external things and impressions as well; the idea that external things will persist after our death,

when our impressions have vanished, may be conceived as valid in the same sense as the idea that the birds may persist when, on account of a cessation of the radiation, their shadows disappear. If we should consider, however, statements concerning external things as equivalent to statements about impressions, this would be interpreting the relation between external things and impressions as a reduction; so the existence of external things would be reduced to the existence of impressions. The external things, according to this theory, would vanish with the ceasing of our impressions—an idea which nobody seriously wants to maintain.

This interpretation of the existence problem will be attacked by positivism. We shall be answered that positivism does not maintain for external things and impressions a relation comparable to the relation between the wall and the bricks. Positivists agree with us in desiring to conceive the relation between external things and impressions as analogous to the relation between the birds and their shadows, i.e., as a projection. What they do not admit is that this relation of projection requires a probability connection. It is justifiable, they say, to talk of a projection also in a case when there are equivalence relations. What is to be altered for this purpose is only the form of the co-ordination of propositions. In the example of the wall the co-ordination is performed in such a way that the nonexistence of the bricks implies the nonexistence of the wall. There may be, however, another form of a co-ordination for which, in spite of the equivalence, the nonexistence of the elements does not imply the nonexistence of the complex. This can be attained if the existence of the complex at a certain time t_1 is defined by certain conditions valid for the elements at another time t_2. To give an example: We said that the melody is a reducible complex of the

tones by which it is formed; we would substantiate this by saying that the melody vanishes when the tones disappear. We can, however, define the existence of the melody in such a way that the melody persists during the time intervals between the tones. We define: "The melody exists throughout the time-stretch running from the first tone to the last tone" means "There are tones at different individual times." Although the elements, the tones, do not exist in the time intervals between two tones, the melody does, and thus the existence relations for a projective complex are valid for the melody. This is even the usual way of conceiving the melody; for if we asked anybody whether the melody existed during all the time, from the beginning to the end of the music, he would surely answer in the affirmative.

To this objection we answer in the following way. It is true that such a definition of the complex may be given; but we are not obliged to do so—in the case of an equivalence we may always introduce another co-ordination for which the existence of the complex vanishes with the existence of the elements. The melody may be defined in such a way that it exists only at the moments when there are tones and vanishes in the intervals between the tones; such a kind of definition is equivalent to the one given above. Thus we arrive at an element of arbitrariness, just as has been already pointed out (§ 11) in the case of abstracta: the question whether or not the complex exists independently of its elements becomes a matter of convention. It is this arbitrariness which we do not accept for the problem of the existence of concreta. We maintain that a conception for which external things vanish with our impressions is not equivalent to the conception of an independent existence. Only in the case of probability connections is there no such equivalence; it is, therefore, only

the conception of the projection as a probability connection between complex and elements which furnishes the admissible interpretation of the existence of the external world.

The preceding reflections nevertheless necessitate a slight correction of our interpretation of the reducibility of existence. We shall call the existence of the complex reducible to the existence of the elements when it is at least possible to introduce an equivalent system of propositions, in which the existence of the complex ceases with the existence of the elements. This definition of the term "reducible," however, does not require a change in our definition of reduction as a co-ordination for which all statements concerning the complex are equivalent to statements concerning the elements. The latter definition implies the possibility of defining the existence of the complex in such a way that the complex vanishes with its elements.

There remain some objections which we must now consider. They concern the question whether it is true that the probability connection can protect us from such consequences as pointed out for the equivalence connection, i.e., from the reducibility of the existence of external things to the existence of impressions. These objections will be considered in the following sections.

§ 14. A cubical world as a model of inferences to unobservable things

The objection which we consider first starts with questioning the analogy between the example of the birds and our situation in the recognition of external things. We said that the birds have an existence independent of their shadows on the screen; but to substantiate this we made use of the fact that there are other and direct observations of

the birds which do not need any consideration of the shadows. We see the birds directly in their places within space; it is therefore easy to distinguish them from the shadows as different physical entities. In the case of our knowledge of the external world, however, we have nothing but impressions as a basis of the observation; is it logically possible to infer from here the separate existence of something which has an existence of its own, in the sense defined above, i.e., an existence which is not reducible to the existence of impressions?

This objection can be more precisely formulated in the following way. It is true that we use a probability inference when we infer from a given set of impressions to the existence of a physical thing. But is this more than an inference to new impressions? It seems impossible that by probability inferences the domain of impressions can ever be left; probability inferences, it may be supposed, will always remain within the domain from which they start. Thus statements about external things, in spite of the occurrence of probability inferences, will be equivalent to statements about impressions; not to statements about the observed set of impressions from which the probability inference starts but to statements about a certain wider set of impressions.

To discuss this objection it will be advisable to stay with the example of the birds at first and to carry through the discussion on this subject, since it is less exposed to misinterpretations. To obtain the same logical structure as in the problem of the inference from impressions to external things, we shall, however, alter this example in such a way that nothing but the shadows of the birds are visible. Thus we have comparable conditions in both problems.

We imagine a world in which the whole of mankind is imprisoned in a huge cube, the walls of which are made of

sheets of white cloth, translucent as the screen of a cinema but not permeable by direct light rays. Outside this cube there live birds, the shadows of which are projected on the ceiling of the cube by the sun rays; on account of the translucent character of this screen, the shadow-figures of the birds can be seen by the men within the cube. The birds themselves cannot be seen, and their singing cannot be heard. To introduce the second set of shadow-figures on the vertical plane, we imagine a system of mirrors outside the cube which a friendly ghost has constructed in such a way that a second system of light rays running horizontally projects shadow-figures of the birds on one of the vertical walls of the cube (Fig. 2). As a genuine ghost this invisible friend of mankind does not betray anything of his construction, or of the world outside the cube, to the people within; he leaves them entirely to their own observations and waits to see whether they will discover the birds outside. He even constructs a system of repulsive forces so that any near approach toward the walls of the cube is impossible for men; any penetration through the walls, therefore, is excluded, and men are dependent on the observation of the shadows for all statements they make about the "external" world, the world outside the cube.

Will these men discover that there are things outside their cube different from the shadow-figures?

At first, I think, they will not. They observe black figures running on the screens quite irregularly, disappearing at the edges and reappearing. They will develop a cosmogony in which the world has the shape of a cube; outside the cube is nothing, but on the walls of the cube are dark spots running about.

After some time, however, I think there will come a Copernicus. He will direct telescopes to the walls and discover that the dark spots have the shape of animals;

and, what is more important still, that there are corresponding pairs of black dots, consisting of one dot on the

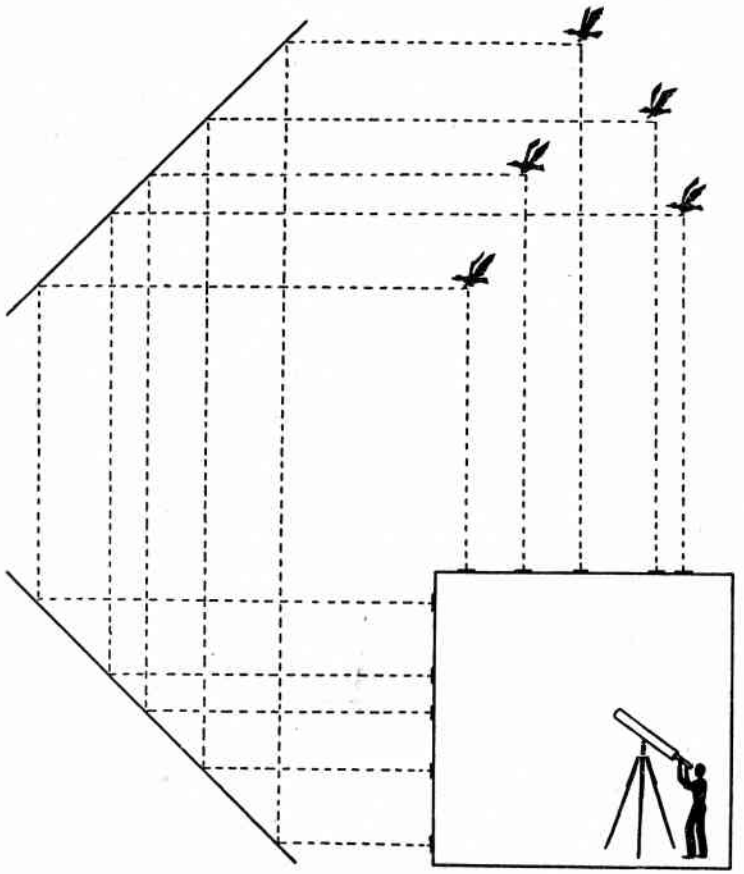

FIG. 2.—A cubical world where only the shadows of external things are visible.

ceiling and one dot on the side wall, which show a very similar shape. If a_1, a dot on the ceiling, is small and shows a short neck, there is a corresponding dot a_2 on the side

wall which is also small and shows a short neck; if b_1 on the ceiling shows long legs (like a stork), then b_2 on the side wall shows on most occasions long legs also. It cannot be maintained that there is always a corresponding dot on the other screen but this is generally the case. If a new dot appears, whether or not there may be a corresponding dot already on the other screen, the new dot always starts from the edge of the screen but never appears immediately within the interior of the screen. There is no correspondence between the locomotions of the dots of one pair; but there is a correspondence as to internal motions. If the shade a_1 wags its tail, then the shade a_2 also wags its tail at the same moment. Sometimes there are fights among the shades; then, if a_1 is in a fight with b_1, a_2 is always simultaneously in a fight with b_2. It happens sometimes that one of the shades has its tail plucked out during a fight; then the corresponding shade on the other surface of the cube has its tail plucked out simultaneously. This is what is observed by means of the telescope.

Copernicus, after these discoveries, will surprise mankind by the exposition of a very suggestive theory. He will maintain that the strange correspondence between the two shades of one pair cannot be a matter of chance but that these two shades are nothing but effects caused by one individual thing situated outside the cube within free space. He calls these things "birds" and says that these are animals flying outside the cube, different from the shadow-figures, having an existence of their own, and that the black spots are nothing but shadows. I am, indeed, inclined to assert that such a Copernicus would arise among the people of the cube; the discovery of our real Copernicus, it seems to me, presupposed much more perspicacity and imagination.

The people, I think, would become convinced by this theory; the question is, however, whether certain philosophers would be convinced. The positivists would attack Copernicus and argue in the following way:

What you maintain, they would say, is not false but biased. You say that there are things independent in their existence of the black dots; but you could say, on the same grounds, that these things are identical with the black dots. There is a correspondence between each of your "birds" and a pair of black dots; all that is said about your birds is inferred from the black dots and is therefore equivalent to statements about the dots. You believe in a surplus meaning of your hypothesis of the birds, compared with a description of the movement of the dots; but this is an illusion—both modes of speech have the same meaning. We admit your great discoveries concerning the relations between the dots, showing that there are corresponding dots on each of the two shade-covered surfaces of our cubical world. But your interpretation of this correspondence as an outcome of an individual identity of things outside the cubical world does not add a new content to your discoveries. This is only your way of speaking—other people prefer to speak of pairs of dots on the screens.

This means, in our terms, that the distinction between the projective complex and the reducible complex would be meaningless. Copernicus conceives the birds as a projective complex; the positivists answer him that he might conceive them, with equal reason, as a reducible complex with respect to the same elements, the black dots. The argument would be continued as follows:

We admit that this equivalence holds for our world only. If a man were once able to penetrate through the walls of the cube, he could distinguish between your hypothesis of the birds and the corresponding statement about the pairs of dots; if he were to see the birds above him, your hy-

pothesis would be confirmed; if not, it would be refuted. But then there would be verifiable facts which distinguish your hypothesis from the pure description of the movement of the dots. For our world, however, there is a law of nature excluding any penetration of the walls of the cube; so, for our world, your hypothesis has the same meaning as the pure description of the dots.

In our terms, this argument would assert that the hypothesis of Copernicus has a surplus meaning only if we accept logical meaning, but that for physical meaning it has no surplus meaning when compared with the statement about the dots. It is this question which we now have to examine.

The positivistic interpretation is based on the presupposition of absolute verifiability. From within the cube, there is no possibility of obtaining a clear "yes" or "no" for the hypothesis of Copernicus; from an observation post outside the cube, such a clear distinction would be obtained. If we insist that only a clear "yes" or "no" is to be taken as an answer, the positivistic conclusion holds; this, I think, is the reason why the positivistic conception is so suggestive. It is, indeed, conclusive if we accept nothing but truth and falsehood as predicates of propositions; but it is no longer so if we introduce intermediate values—if we introduce the predicate of weight.

With regard to the predicate of weight the two conceptions are not equivalent. Judged from the facts observed the hypothesis of Copernicus appears highly probable. It seems highly improbable that the strange coincidences observed for one pair of dots are an effect of pure chance. It is, of course, not impossible that, when one shade has its shade-tail plucked off, at the same moment the same thing happens to another shade on another plane; it is not even impossible that the same coincidence

is sometimes repeated. But it is improbable; and any physicist who sees this will not believe in a matter of chance but will look for a causal connection. Reflections like this would incline the physicists to believe in the hypothesis of Copernicus and to refuse the equivalence theory.

This means that the physicist insists on the surplus meaning of his interpretation not because it has logical meaning but because it has physical probability meaning. It is only physical truth meaning for which the positivistic interpretation is valid; but, if we admit physical probability meaning, there is a surplus meaning for the hypothesis of the birds (for the conception of the birds as a projective complex of the shades) because it obtains a weight different from that of the hypothesis of the pairs of dots, i.e., from the interpretation of the birds as a reducible complex of the shades. It is the different conception of the second principle of meaning which furnishes this distinction. The positivistic conception demands that two statements have the same meaning if they are equally determined as true or false by all possible facts; the probability conception demands the same meaning only if the statements obtain the same weight by all possible facts. It is to be admitted that the observable facts do not furnish a difference as to absolute truth or absolute falsehood of the two theories in question; but the weight conferred on them by the facts observable within the cube is different. Whereas the positivistic definition of meaning must therefore consider the two theories in question as having the same meaning, the probability definition of meaning furnishes a different meaning for both theories—although the domain of observable facts is the same, and although only the postulate of physical possibility is employed in the definition of meaning. The physicist, therefore, is not dependent on the

acceptance of the dubitable concept of logical meaning and employs physical meaning as well as the positivist, but only in the probability form and not in the truth form.

The positivist, to defend his position, will answer in the following way: Your hypothesis, he will say to the physicists, obtains a different weight compared to my hypothesis only because it furnishes different consequences within the domain of our observable facts. Your theory, for instance, leads to the consequence that the coincidences between the shades of one pair will continue, will always be repeated; the conception that the coincidences are due to chance, however, leads to the contrary prophecy, to the consequence that the coincidences will not be repeated. To remove this difference we shall change our conception in such a way that it furnishes the same observable consequences as your hypothesis within the domain of observable facts, and that it differs only in the consequences for unobservable facts, for facts outside the cube. That is, we shall maintain our conception in such a way that the birds remain a reducible complex of the shades, but that all consequences for facts within the cube are the same as in the case of the birds being a projective complex of the shades.

This idea, if it were tenable, would prove that a difference between a reducible and a projective complex cannot be maintained, provided we keep to physical meaning.

Carrying through this idea the positivist would have to interpret the correspondence between the dots of one pair as a form of causal connection. He would have to say that there is a kind of coupling between the elements of one pair. If an element a_1 of one pair is approaching an element b_1 of another pair in a certain way called "fight," recognizable by a kind of excited dance of the shades and mutual bites with their beaks, there is—the positivist has

to say—a causal effect transferred from a_1 to its corresponding dot a_2 on the other screen, and from b_1 to its corresponding dot b_2, in such a way that a_2 and b_2 enter into the same relations called "fight." With this hypothesis the positivist no longer would interpret the coincidences as chance but as an outcome of a causal law; and his conception, therefore, would furnish, as a consequence, the continuation of the coincidences for all the future. Thus his theory is altered in such a way that it does not differ from the physicist's conception as far as prophecies for future observable events are concerned.

The physicist, however, would not accept this improved theory. He is too clever a man to object to the positivist that such a causal connection is impossible; but he will say that it is very improbable. It is not because he wants to combine with the term "causal connection" some metaphysical feelings such as "influence from one thing to another" or "transsubstantiation of the cause into the effect." Our physicist is quite a modern man and needs no such anthropomorphisms. He simply states that, wherever he observed simultaneous changes in dark spots like these, there was a third body different from the spots; the changes happened, then, in the third body and were projected by light rays to the dark spots which he used to call shadow-figures. Freed from all associated representations his inference has this form: Whenever there were corresponding shadow-figures like the spots on the screen, there was in addition, a third body with independent existence; it is therefore highly probable that there is also such a third body in the case in question. It is this probability inference which furnishes a different weight for the projective complex and the reducible complex.

What is very remarkable here is that the two theories obtain, from the facts observed within the cube, different

weights, although both theories furnish for future facts within the cube the same weights.[4] The probability conception of meaning, therefore, allows us to distinguish between theories which furnish, for all observable consequences of a certain domain, the same weight, even if nothing but facts of this domain are at our disposal for the probability inferences.

It may be said that this is possible only if the theories in question differ at least in logical meaning. This is not false; as we have already pointed out, two theories which have the same logical meaning cannot obtain different probability meaning. But the concept of probability meaning has the smaller extension; not all propositions having different logical meaning have also different probability meaning. We cannot say, therefore, that we accept the theory of the physicist as meaningful because it has logical meaning. We accept it because it has physical probability meaning.

We might attempt another substantiation of the necessity of accepting logical meaning. It might be said that, although not every difference of logical meaning renders a difference of probability meaning, those cases in which the difference occurs can be carried through only on account of the difference in logical meaning. To speak more clearly: if we could not at least imagine a difference in logical meaning, it would not be possible to calculate a different weight for the two theories. But this, I think, would be a grave mistake. The concept of logical meaning is valid

[4] Remark for the mathematician: There is a relation between the "forward probabilities" from the theory to the facts and the "backward probabilities" from the facts to the theory; this relation is expressed by the rule of Bayes. But in this rule there occurs still a third set of probabilities usually called misleadingly "a priori probabilities," or, better, "initial probabilities." It is these initial probabilities which are involved in the reflections of the physicist about causal connections. Thus the "backward probabilities" may be different, although the "forward probabilities" are equal, on account of different "initial probabilities."

only within the sphere of idealization in which physical propositions are taken as absolutely verifiable; if we take into consideration that truth signifies, strictly speaking, nothing but a high weight, we find inversely that truth meaning is to be reduced to probability meaning. We see this if we consider once more our example of the birds. The objection here would read thus: You are entitled to infer, with probability, that there are birds outside the cube only because you can at least imagine that you penetrate through the ceiling and see the birds; this penetration, although excluded by a law of nature, is logically possible, and therefore the object of your probability inference has meaning. The fault of this reasoning becomes obvious if we now introduce the case of a penetration of the ceiling. If a man were able to pierce a hole through the ceiling, and to see the birds—would this be an absolute verification of the theory of the cube-Copernicus? We have shown that there are no statements capable of absolute verification. The man could construct an interpretation for which the birds were not material bodies but only optical images produced by light rays coming from the shadows, deflected in such a way that the rays coming from the dots of one pair met at a certain point in space and ran from there into the observer's eyes. Relative to what one sees this cannot be called false but only very improbable. So what is obtained by a "direct observation" is an increase of weight for the theory of the birds but not a verification. The objection in question, therefore, would finally maintain that a theory can be meaningfully inferred with probability only if it is at least logically possible to construct facts which confer a higher degree of probability on the theory. I do not think this conception will be seriously maintained.

Statements made in terms of the later verification of a theory which is for the time being only rather probable

on the basis of observed facts have the advantage of being an intuitive representation of the theory—but they are not the sole form in which the meaning of the theory is to be expressed. To say, "The statement that the birds are a projective complex of the shadow figures means that, if we should penetrate through the ceiling, we should see the birds," is only a short and intuitive way of expressing what is meant—nothing more. In this way we pick out one of the consequences of the theory which if observed would make the theory highly probable; but by no means do we obtain by this method the full meaning of the theory. What we get is an intuitive representation of the theory. We say, for instance: " 'Next year there will be a European war' means 'There will be airplanes above London, and shooting, and wounded men in the hospitals.' " Or we say: "A visit to New York means seeing skyscrapers and streets full of cars and men rushing for business." In this way we take certain representations for the whole; but it must not be forgotten that many other features are dropped by this method. The method is the more dangerous in case the chosen representations are not physically accessible but only accessible to our imagination. This is the case when it is physically impossible to obtain high degrees of weight for a theory. It may be advantageous, for certain purposes, to visualize the statement by imagining just the inaccessible results which would furnish the higher weight; but it must not be forgotten that we then obtain a representation only. Thus it may be permissible to visualize the concept "atom" by imagining the impressions of an observer whose size is of submicroscopic dimensions. But to insist in such cases that only the facts conferring a high weight on the theory are to be taken as its meaning is an outcome of the schematized conception of the two-valued logic. Actually, such a division of facts

does not correspond to the practice of science. Considering observations of the physically inaccessible domain, we do not obtain facts which verify statements concerning things situated there but only facts which confer a higher weight to such statements. But then there is only a difference of degree with respect to statements based on facts observed within the accessible domain. The probability theory of meaning, therefore, does right to admit statements as different in meaning if these statements obtain different weights from observed facts—without regard to the question whether or not there will be, later on, a better determination of the weight.

It is, however, not false to employ the concept of logical meaning in the sense of a meaning defined by the logical possibility of obtaining a high weight. We may say that physical probability meaning is a domain between physical truth meaning and logical meaning; it allows us to make inferences which infringe upon the domain of logical meaning, although it is based on the physical possibility of ascribing a weight. The probability theory of meaning therefore allows us to maintain propositions as meaningful which concern facts outside the domain of the immediately verifiable facts; it allows us to pass beyond the domain of the given facts. This *overreaching* character of probability inferences is the basic method of the knowledge of nature.

An example taken from physics may illustrate the significance of the probability theory of meaning. Einstein's theory of relativity forms the famous domain for examples of the application of the verifiability theory of meaning; but, if we consider this theory more exactly, we find that it is physical probability meaning, and not physical truth meaning, which is here applied. Let us consider Einstein's theory of simultaneity. We send at the moment t_1, from the spatial point A, a light signal to the spatial point B, arriving there at the moment t'_2; here the signal is reflected and returns to A at the moment t_3. t_2 may be a moment at A, between t_1 and t_3, but arbitrarily chosen in this interval. Then, according to Einstein, the statement s, "t_2 is absolutely simultaneous with

t'_2," has no meaning. This is usually substantiated by saying that this statement is not verifiable, i.e., has no physical truth meaning. This is, however, not correct; Einstein maintains more—he maintains that the statement s cannot be provided with a weight, and so has no physical probability meaning. Just because probability meaning is a "more tolerant" concept than physical truth meaning, the denial of probability meaning is a stronger postulate than the denial of physical truth meaning.

To show this, let us first note that the statement s has logical meaning. This reads: "If there were no upper limit to the velocity of signals, a signal of infinite velocity[5] leaving B at t'_2 would reach A at t_2." This, of course, would be true only for one determinate t_2 between t_1 and t_3, so that this time-point is distinguished as absolutely simultaneous to t'_2. For any other t_2, the statement would be false; but then it has meaning as well. We are allowed, therefore, to say that the statement s has logical meaning for every t_2. If Einstein rejects the statement s, he decides in favor of physical meaning. But he demands more than physical truth meaning; he demands that all other facts of nature are of such a kind that they do not furnish, for a determinate t_2, a higher probability of being a specific time-point than for other values of t_2.

Such a distinction might be given by the transportation of watches. Einstein demands that two watches equally regulated during a common stay at A, and moved in different ways and with different velocities toward B, will show at B, after their arrival, a difference in their readings. We can imagine a world in which this is not the case, but in which the indications of two watches are in correspondence after the different transportations from A to B. In this world transported watches would define a simultaneity which we call *transport time*,[6] and we would say: If there were no upper limit to the velocity of signals, the infinite velocity would determine with a great probability, as simultaneous to t'_2, *that* time-point t_2 which corresponds to the transport time. In this world absolute simultaneity would have a physical probability meaning, though no physical truth meaning. Einstein refuses to believe in the existence of experiments, like the described transportation of watches, which would distinguish a certain t_2 as probably being the time-point of the arrival of infinitely quick signals. Thus Einstein refuses physical probability meaning to absolute simultaneity, which is, as we see, a stronger postulate than the refusal of physical truth meaning.

[5] The concept of infinite velocity may here be eliminated and replaced by a more complicated statement about the limit of the times of arrival belonging to signals of finite velocity, which defines a fictive "first signal" (cf. the author's *Axiomatik der relativistischen Raum-Zeit-Lehre* [Braunschweig, 1924], p. 24).

[6] *Ibid.*, p. 76.

Our conception of the example of the cubical world, which accepts the statement about the birds outside the screens as meaningful and different from statements about the dots on the screen, is therefore not in contradiction to the principles of modern physics. The cubical world as described would correspond not to Einstein's world but to a world in which a transport time would be definable. The principles of the theory of relativity have been wrongly interpreted as supports for the concept of physical truth meaning; what they actually support is the concept of physical probability meaning.

§ 15. Projection as the relation between physical things and impressions

We proceed now to the application of our concepts of reduction and projection to the problem of the existence of the external world.

By analogy with the example of the cubical world our contention reads: Impressions are only effects produced within our body by physical things, in the same sense as the shadows are effects of the birds. Thus impressions are only external elements relative to the physical things; these things are projected to our impressions but not reduced to our impressions. The "external world" therefore has an existence of its own, independent of our impressions.

This is the so-called realistic conception of the world. Let us see what positivism answers. The answer is known to us from the example of the cubical world. It reads:

"What you maintain is not false but biased. You say there are things independent in their existence of your impressions; but you could say, on the same grounds, that these things are a reducible complex of your impressions. There is a correspondence between your impressions and your external things; all that is said about your external things is inferred from impressions and is therefore equivalent to statements about impressions. You believe in a surplus meaning of your hypothesis of the external world;

but this is an illusion—both modes of speech have the same meaning."

We need not repeat our discussion of this objection. We summarize only: It is not true that our statements concerning external things are equivalent to statements about impressions, although they are inferred from them. It is not true that the statement, "The external world is a reducible complex of impressions," has the same meaning as the statement, "The external world is a projective complex of impressions." This might be said, perhaps, if we accept physical truth meaning; but then there are no physical statements at all because there are no absolutely verifiable statements about the physical world. If we want to obtain meaningful statements, we must introduce physical probability meaning; and then the assumed equivalence between the reducible complex and the projective complex does not hold. There is a surplus meaning in saying that there is an external world independent of our impressions.

The reason, it seems, why positivists maintain this equivalence is to be found in their idea that it is not possible to infer from a certain domain of things to another domain. It is the neglect of the overreaching character of the probability inference which leads positivists to their equivalence theory. They believe that we are obliged to interpret probability inferences by the principle of retrogression, and so they do not see that the probability inference passes beyond the given observations. This error about the logical nature of the probability inference is the root of the positivistic doctrine of existence.

To clarify this error, let us consider the application of the principle of retrogression to probability inferences. Thus we come back to a form of the positivistic argument stated in the beginning of § 14. Let i be the conjunction

of statements about the impressions (forming the class I) from which the probability inference starts and e the statement about external things (forming the class E) which is inferred from i with probability. It is true, then, that i is not equivalent to e. But what is maintained is that there is a more comprehensive conjunction i' of statements about impressions (class I'), including predictions about future impressions, which is equivalent to e.

Let us ask whether there is such a conjunction i'. The first thing we can say is that if there is such a class the corresponding class I' cannot be finite, as the observable consequences of a physical statement do not form a closed class.[7] But we can say more. Even statements about an infinite class of impressions are not equivalent to the physical statement. This becomes obvious if we consider impressions as physical effects caused in our body by the external object and apply a general theorem concerning causes and effects.

If we have a cause and collect from all its effects a certain class which may be infinite, but which does not contain the cause itself, the cause and the class of effects stand in the relation of projection; a statement about the cause is not equivalent to any set of statements about the class of effects. They are in a probability connection only. The statement, "The sun is a ball of glowing gases of high temperature," is not equivalent to any set of statements about physical facts outside the sun, even if the set is infinite and even if it comprehends all points of a surface surrounding the sun; we get by these observations a set of elements from which we may with probability infer the sun's exist-

[7] We have to take account of the fact that an infinite class of impressions may be described by a finite class of propositions. If we say, e.g., "If there is a gravitational field at all points within a certain space, the impression of heaviness is obtainable"; this is one proposition, but it concerns an infinity of impressions. The denial of this sentence would also require an infinity of observations.

ence and qualities, but which is by no means of equivalent meaning. Only if we were to include the sun itself in the set of observed facts, would there be an equivalence; but in this case all other facts might be dropped, and nothing would remain but a trivial tautology.

There is no difference if the effects produced consist of impressions. We cannot say, therefore, that there is a conjunction of statements i' to which e is equivalent. This would be permissible only if I' were to include the physical object, i.e., if we include the case that our body might become identical with the physical object. This is not logically impossible; but the positivist will scarcely be ready to accept this idea as the only correct interpretation of his thesis that there are statements about a class of impressions which are equivalent to the physical statement. This would mean that a statement about the sun is equivalent to a statement about impressions because it is not logically impossible that one day the sun may be a part of my body, and the movement of its glowing gases signifies, within myself, an observational process. We may leave this interpretation to the novelist, I think, and keep to our probability theory of meaning which needs no such equivalences.

We have to say, therefore, that the physical statement e is not equivalent to statements i' about a class I' of physically attainable impressions. We cannot determine a class I' of impressions such that, if i' is true, e is also necessarily true. This is what I call the overreaching character of probability inferences in application to the problem of impressions and the external world. The nonequivalence between e and any conjunction of statements i is what is meant by saying, "The external things have an existence of their own independent of my impressions."

To show the failure of the positivistic equivalence theory, let us consider an example. We take the proposition, "External things will continue to exist when I am dead." Common sense is convinced that this proposition, if it is true, may be considered as a proof that the existence of external things is not reducible to the existence of impressions; external things are, on the contrary, to be conceived as a projective complex of impressions. The positivist maintains that both interpretations are equivalent; so he has to say that the proposition, "External things will cease to exist when I am dead," has the same meaning as the former. Let us give to both propositions a more precise formulation. The first, which may be called e_1, is to read: "Until and after my death, external things will persist as is usually expected." The second proposition e_2 may be: "Until my death external things will persist as is usually expected; but, after my death, external things will vanish." If the positivist maintains that these two propositions e_1 and e_2 are equivalent, the reason lies in the fact that both hypotheses have the same observable consequences, or, strictly speaking, that they furnish the same weight for all possible predictions which I can make for the stretch of life lying before me. But we saw, nevertheless, that such hypotheses may obtain different weights from the observable facts. This is obviously the case here. Seeing that many people who are similar to me expire without producing such fatal consequences to the physical world, I infer with high probability that the same will be the case when I die. This is a correct reasoning comparable to a great number of similar inferences occurring in physics and never questioned there because they do not concern my own person. Thus the probability theory of meaning furnishes a different meaning to both sentences and accords with common sense.

Introducing the concept of logical meaning, we could

also say that the proposition e_1 is meaningful and different from e_2 because it is logically possible that I awake, after my death, and verify the existence of the physical world. This interpretation is permissible in the sense stated above, as an intuitive representation of what is meant. But, if we were to accept this interpretation as the only justification for statements about the world after our death, we would be led into great difficulties. As we have pointed out (§§ 6, 8, 14), logical meaning is too wide a concept; it is not compatible with the conceptions of modern physics. Thus a man who accepts a statement about the world after his death as meaningful only because it has logical meaning would be obliged to accept absolute simultaneity as well. On the other hand, a relativist who insists on the postulate of absolute verifiability would be obliged to consider statements about the world after his death as meaningless. It is only probability meaning which leads us out of this dilemma, justifying jointly the statement about the world after my death, and the rejection of absolute space-time conceptions.

It is not always an easy matter to discuss this question with positivists. They usually become offended when they are told that they do not believe in the existence of the physical world after death. They emphasize that this is a misunderstanding of their theories and demonstrate their conviction of the persistence of the external world after their death by taking out life insurance policies in favor of their families. They do not acknowledge our reasoning but insist that for them also there is a difference between the statements, "The external world persists after my death" and "The external world does not persist after my death." The difference is, they say, that the first statement includes certain statements concerning the death of other people without the world's being annihilated, whereas the

second statement would contain statements about the world's vanishing simultaneously with the death of other people. This, however, is not the problem in question. The two statements we previously formulated are not the same as the two statements compared by the positivist. The second statement, in our formulation, reads otherwise. We formulated it in such a way that the difference of the two statements begins only with my death, saying that until my death all should be the same as usual. These statements cannot be distinguished within the positivistic theory of meaning, i.e., by means of the concept of physical truth meaning. I do not doubt the seriousness of the positivists as far as life insurance policies are concerned; what I want to maintain is that they cannot justify this carefulness because their theory furnishes no means of distinguishing between the statements e_1 and e_2 formulated by us.

§ 16. An egocentric language

We showed in the preceding section that propositions about external things are not equivalent to propositions about impressions. To give a new illustration to this conclusion, let us now consider an objection which attacks our result from another point of view. This objection starts from reflections which we introduced at the end of § 13. We showed there that in the case of a reduction the relation between the complex and its elements may be defined in different ways. Only for one kind of co-ordination of propositions does the existence of the complex vanish with the existence of the elements; for another kind of co-ordination, this consequence may be avoided. We maintained that the possibility of a co-ordination which has this consequence will suffice for us to call this case a reduction, and the complex a reducible complex. It may be objected, how-

ever, that perhaps the situation in the case of probability connections is not otherwise; that in this case it is also possible to introduce a co-ordination of propositions for which the existence of the complex vanishes with the existence of the elements. If this is true, it will show that there is no genuine difference between projection and reduction, but that this is a difference of language only. The objection in question, therefore, is proved as valid if we succeed in constructing a language for which the existence of the projective complex is dependent on the existence of the elements.

We shall find a way to construct such a language by starting from the very contention which we intend to actualize in our new language. We shall try to exclude the independent existence of external things by establishing this idea in the form of a principle which we make the basis of our language. To facilitate our task, let us consider an example. Let us imagine a man who is convinced that all things cease to exist as soon as he ceases to look at them; how could he defend his conviction against the objections made to him by common sense and by scientific thinking? He could defend himself if he had sufficient imagination to invent complicated logical constructions which connect the different impressions perceived by him in certain time intervals. He could interpret the reappearance of the things at the moment when he looks at them by saying that his looking produces the things. Thus he has to introduce a new kind of causality; but, if he is careful and consistent, he can carry through his conception. There are experiences which show that there is a certain "development" in a physical state. We put a kettle of cold water on the fire, come back after five minutes, and see the water boiling. The man in question would have to say that his looking at the kettle produces the things in the same advanced state which the things would have acquired by their

intrinsic development if he had observed them and had not interrupted their existence. His new causality thus obtains strange qualities but not impossible ones. He will find even stranger qualities when he takes into consideration observed effects produced by the things at a moment when he does not look at the things. He looks at a tree and observes it as existent; then he turns and no longer sees the tree but its shadow. His conception, then, compels him to say that there is an aftereffect of the tree—the shadow—which persists for a long time when the tree itself has already vanished. This would mean a change in the laws of optics; but it could be consistently carried through.

Would this conception ever lead to contradictions with observable facts? Obviously not, because all experiences are interpreted by the same principle. The laws of optics as obtained by this man from experience would differ from our laws of optics. They would be divided into two classes by means of the clauses "if I observe certain things" and "if I do not observe certain things." The laws of the first class are equal to our laws; the laws of the second class, however, speak of strange aftereffects and things appearing fitfully in different states of development. This furnishes a rather complicated description of the world, but it does not lead to any contradiction of experience. If there is a seeming contradiction, this is only because the distinction of the two classes of phenomena has not been consistently carried through; it can, therefore, be eliminated by a change of interpretation.

We may raise the question whether the hypothesis of this man, though at least compatible with the facts, does not obtain a rather low degree of weight, i.e., can be demonstrated as being very improbable. It turns out that even in this respect there is no difficulty for him. There is one kind of experience which might be considered as a diffi-

culty: the man sees that, when other persons turn their eyes away from things, these things still persist. If he admits the similarity between himself and other persons, this would render a high probability that the things will also persist when *he* does not look at them. But this is only valid under the presupposition of the similarity mentioned; so our hypothetical man may turn his inference in the opposite direction and maintain: I have an exceptional position in the world because the things vanish only when I do not look at them, whereas they persist when other persons do not look at them. When this conception is introduced, the probability inference from other people's nondisturbance of the existence of things to his nondisturbance of them is not valid. The methods of probability, therefore, do not furnish a result which throws into question the hypothesis of our example.

We may be astonished at such a result. We have so far maintained that the existence of things which are not observed may be inferred with high probability, even in the case when a direct observation of the things is excluded by certain physical laws, as in the case of the birds and the cubical world. We find now that we can introduce another conception for which the things do not exist at all when they are not observed and that this conception may obtain a high degree of probability as well. Is not this a contradiction?

The seeming contradiction is dissolved when we enter into a more detailed analysis of the second conception. We shall find, then, that our plan of constructing another language has been actualized in our example—that the man who conceived the things as vanishing when he does not observe them speaks another language than we do and that the apparent contradiction is due to a different meaning of words. This is to be understood in the following way.

Any description of the world presupposes certain postulates[8] concerning the rules of the language used in the description. The description of unobserved facts depends on certain assumptions concerning causality and therefore depends on postulates about causality. The postulate normally in use for this purpose requires us to construct homogeneous causal laws, as far as it is possible. The last clause is necessary because it is not always possible to construct homogeneous causal laws; thus it is not possible to construct for things seen in a dream the same laws as for things seen during waking. (Things seen in a dream are not seen once more in the next dream, etc.) But experience shows that for the things seen in the waking-state it is possible to describe the state of things during the interval between two observations in such a way that the principle of homogeneity of causality is satisfied.[9] This is done when we consider the things as existent during these intervals, whereas considering the things as nonexistent implies changes of causal laws, as we found in our example. The postulate of the homogeneity of causality, therefore, decides in favor of the conception of the existence of nonobserved things.

The man who conceived nonobserved things as nonexistent, however, decides in favor of another postulate. He

[8] Whether or not these postulates are conventions must be specially examined (cf. the remarks about equivalent and nonequivalent languages in § 17).

[9] There is, strictly speaking, a difference between homogeneity of causal processes and homogeneity of causal laws. The first postulate demands that the causal processes in physical things are not disturbed by our observation; the latter postulate demands only that, if there is a disturbance, this is to be according to causal laws for other phenomena. The first postulate cannot always be maintained; we know that scientific instruments of a more sensitive type are disturbed by the observer (by slight mechanical impacts, by the change of temperature caused by the observer, etc.). Quantum mechanics has even shown that there is a principle of disturbance by observation which cannot be reduced below a certain minimum. The second postulate, the equality of causal laws for the disturbance by the observer and for other physical phenomena, has turned out to be always maintainable in modern physics.

renounces a postulate concerning causality; his alternative postulate is the principle that things do not exist when they are not observed. Thus this assumption is for him no empirical matter but a decision and, therefore, beyond question. His scientific language, however, is altered by this procedure, and we must now point out in what respect.

The first and very obvious change is that his word "existence" does not correspond to our word "existence" but to our word "existence observed by me," or, simply, "being observed by me." Let us call the language of the man the *egocentric language;* then we may establish the following correspondence:

Egocentric Language	Usual Language
1. Things do not exist when I do not observe them.	1. Things are not observed by me when I do not observe them.
2. Things are produced at any time when I turn my eyes in a certain direction.	2. Things are observed by me at any time when I turn my eyes in a certain direction.

Both propositions are not about things directly but about observations of things. The first is a tautology, as is obvious in the expression within our usual language; this is because this proposition is nothing but the formulation of the postulate accepted by the man in question. The second is not always true, as it may happen (expressed in usual language) that the thing has been removed, or disappeared, while I was turning away; this character of not being always true is valid for both languages.

Let us now try to express a sentence which concerns not our observation of the thing but the independent existence of the thing. Take the sentence, "The thing exists during a certain time interval Δt." We pronounce such a sentence if we observe the thing at least at certain moments within the interval Δt, or if we discover that the observation is prevented by other things which do not exclude, however, that certain effects of the thing are observed by us. A

stone which we saw may be covered at a second observation by a person, whereas the shadow of the stone is still to be seen. We express this idea in the following way in both languages:

Egocentric Language	Usual Language
3. If I turn my eyes during the time interval Δt in a certain direction, the thing is produced, or I can construct, applying my causal laws to the things which I observe, a cause which prevents the thing's existence. This cause must be of such a kind that it does not prevent the existence of certain other things which would be, if the thing were to exist, according to my causal laws the effect of the thing.	3*a*. If I turn my eyes during the time interval Δt in a certain direction, the thing is observed, or I can construct, applying my causal laws to the facts which I observe, a cause which prevents the thing's observation. This cause must be of such a kind that it does not prevent the observation of certain other effects which my causal laws ascribe to the thing.

This sentence is of a better truth-value than sentence 2 because it takes into account the possibility of a disturbance of the observation. But even sentence 3 is not always true; it may happen that the thing (in usual language) really vanishes, as a cloud may vanish by being evaporated. Thus sentence 3 can be pronounced only with a certain probability, although with a higher probability than sentence 2.

The question remains: Is proposition 3 equivalent to the usual proposition, "The thing exists during a certain time interval Δt"? This means: Do we call the latter proposition true, when proposition 3 is true, and inversely? It is obvious that this is not the case. We can only say that, if proposition 3 is verified, there is a high probability for the thing's existence; and conversely, if the thing exists, there is a high probability for proposition 3. The first statement is an expression of our general idea that observations never can furnish an absolute verification of a sentence about

physical things. The second statement takes into account the case of an exception to the known rules of causality; it might happen that the laws of optics are suddenly superseded, and the thing, though being in its place, is not seen. Thus we have to say, proposition 3 is equivalent only to the following proposition:

Egocentric Language	Usual Language
3. As before.	3*b*. It is very probable that the thing exists during the interval Δt.

We find that proposition 3 is equivalent not to a proposition concerning the existence of a thing but to a sentence ascribing a probability to the existence of a thing.[10] We come to a similar result if we examine other examples. We find that normal propositions about the existence of things cannot be expressed in the egocentric language; this language can only express sentences about a probability for the existence of things.

This remarkable feature of the egocentric language is to be interpreted in the following way. The egocentric language confers existence only upon observed things, or, what amounts to the same thing, upon impressions.[11] Impressions are the basis of a probability inference directed to other things. A statement about impressions is therefore not equivalent to a sentence about physical things; it can only be equivalent to a statement conferring a probability upon a sentence about other things. The egocentric language dealing with impressions only cannot be equivalent to a language concerning the physical world.

[10] Strictly speaking, this is not an equivalence but a unilateral implication from the egocentric language to a probability statement about the realistic language (cf. our remark at the end of § 17).

[11] I do not mean by this that observed things and impressions are identical. But there is a one-one correspondence between them, and therefore the egocentric language can be formulated either for observed things or for impressions.

It can only be equivalent to a language conferring probability upon statements about the physical world.

Our investigation, therefore, confirms our thesis that the relation between impressions and the physical world is a projection and not a reduction. Impressions remain external elements of the world and cannot be considered as internal elements. The positivistic idea that this distinction is a matter of definition only, and that a projection may be changed into a reduction without any change of meaning, is not tenable. The egocentric language which would take the form of conceiving the physical world as a reducible complex of impressions cannot furnish propositions equivalent to propositions concerning the existence of physical things but only equivalent to sentences concerning a probability for the existence of physical things. The egocentric language is not equivalent to the physical language but only to a part of it; this is the part concerning the basis of probability inferences. It is precisely the part concerning physical things, given by the result of the probability inferences, which finds no equivalent in the egocentric language.

These results show that the positivistic conception of the problem of existence is no longer tenable. The positivistic conception that the question concerning the existence of the external world is a pseudo-problem is based on the idea that the physical language is equivalent to an egocentric language. For only in case of such an equivalence are we entitled to contest the uncertainty character of the logical process leading from impressions to external things; if this is nothing but an equivalence transformation of language, there remains no uncertainty as to the existence of external things. We see, however, that this is erroneous; there is no logical equivalence between statements about impressions and statements about external things—the latter are ob-

tained by inductive inferences based on the former. There remains, therefore, always an uncertainty in this inference. As in the case of any statement concerning a special physical thing, so the general statement that there are physical things at all, that there is an external world, can be maintained with probability only. The degree of probability of the general statement is higher than that of any special statement; this is due to the fact that the general statement may be conceived as a disjunction of special statements, a case for which the rules of probability furnish a higher numerical value. But there is no reason to maintain that the general statement is certain.

That there is a remaining uncertainty may be made clear by the following consideration. We know that during a dream we have the feeling of the reality of the world observed, and we know that after awaking we are obliged to correct our conception—that we must acknowledge it was only a private world in which we lived. Can we exclude the case that a similar discovery will happen tomorrow with respect to the world of today? Can we ever ascertain, with no doubt remaining, that we are not asleep? Or are we sure that there will never be a third world, of a stronger reality still than the second, which stands to the second in the same relation as this to the first, the dream world? In denying such possibilities, we can never pass beyond a certain high degree of probability.

To these latter reflections it might be argued that our actual inferences to the external world start from a restricted class of impressions, limited by the impressions of today; and it might be argued that it is only this limitation which furnishes the uncertainty. If we could foresee all future impressions, we should know whether we should "awaken" some day; the statement of the existence of the external world would then be equivalent to the statement

that there are no such impressions of "awakening" within the whole class. This argument is not valid, however, because even the knowledge of the whole class of impressions of a man's life does not furnish a basis from which we could infer with certainty the existence of external things. I do not admit that we can ever describe a class of impressions about which we may say that, if all impressions of my life are of this class, there is, with certainty, an external world. On the contrary: to any class of impressions described, even if it contains an infinity of impressions, we may imagine additional elements such that the enlarged class will lead to the conclusion that the world of the original class was a sort of dream world only. *All definable classes of impressions are of a type leading only to probability statements about an external world.* This is what we formulated as the nonequivalence of the realistic and the egocentric language; and this is what gives the reason for the uncertainty of our knowledge about the existence of an external world.

§ 17. Positivism and realism as a problem of language

With the reflections of the preceding section our inquiry about the difference of the positivistic and the realistic conception of the world has taken another turn; this difference has been formulated as the difference of two languages. This form of consideration, which has been applied particularly by Carnap, seems to be a means appropriate to the problem in question, and we shall make use of it for an illustration of our results.

The conception of the difference in question as a difference of language corresponds also to our idea that the question of meaning is a matter of decision and not of truth-character. If, in the preceding sections, we defended the idea that the positivistic conception of the world is not tenable, it was

because we wanted to maintain that the positivistic interpretation of existence propositions does not correspond to our common language, or to that kind of meaning which we have to attach to our words if our actions are to be considered as justifiable in terms of our knowledge about external things. Our statement, therefore, belonged to the descriptive task of epistemology (§ 1), maintaining a deviation of the positivistic interpretation from the realistic language of knowledge as a given sociological phenomenon. If we proceed now to regard the differences of the positivistic and the realistic languages, we pass from the descriptive task to the critical task of epistemology; with this turn we consider meaning as a matter of free decision, and ask for the consequences to which each form of decision leads, and thus for the advantages and disadvantages which may be used to determine our choice if we ourselves want to make a decision.

In spite of our reference to a free decision, we should not like to say that the decision in question is arbitrary. Although such a characterization cannot be called false, it is a very misleading mode of expression. If we speak of the arbitrariness of language, we intend to express the fact that different languages may express the same ideas in spite of all differences of external form; and that, consequently, the choice of the language does not influence the content of speech. This conception has its origin in certain characteristics of common languages; it does not matter whether a scientist expresses his ideas in English, or French, or German, and thus the irrelevance of the choice of the language has become the very prototype of arbitrary decision. This conception presupposes, however, the equivalence of the languages in question. Only in the case of equivalent languages are their differences matters of convention. There are, however, other cases in which the languages are not

equivalent; our consideration of the egocentric language led us to an example of this kind. In such a case the decision for or against one of the languages signifies what we called a volitional bifurcation. If we speak in such a case of an arbitrary decision, the word "arbitrary," therefore, is misleading; it suggests the idea that the decision in question is not relevant, does not influence our results. This, however, would be entirely erroneous.

If the languages in question are not equivalent, if the decision between them forms a case of a volitional bifurcation, this decision is of the greatest relevance: it will lead to consequences concerning the knowledge obtainable. The man who speaks the egocentric language cannot express certain ideas which the man with the realistic language may formulate; the decision for the egocentric language, therefore, entails the renunciation of certain ideas, and may, consequently, become highly relevant. We do not thereby say that the egocentric language is "false"; such a criticism would be a misunderstanding of the character of a volitional decision. It is rather the method of entailed decisions which we have to apply here; we can show that the decision for the egocentric language leads to a scientific system of a restricted character which does not correspond to the system constructed by the realistic language in its full extension.

Let us extend similar considerations to the general case of two languages. Using a symbolism corresponding to that of § 15, we will assume a domain I of elements as the basis of our language; let us assume, further, that statements i concerning these elements are absolutely or practically verifiable. With the latter term we include cases in which the statements i possess a high weight. There may be, in addition, a domain E of elements outside the domain I of elements; the elements of the domain E are in such re-

lation to those of I that some verified statement i confers a determinate probability to a statement e about the domain E. This relation is not simply a one-one correspondence; to every i, there belongs a class of statements e, each of which is co-ordinated to i with a different probability (and conversely). Let us assume now two statements e_1 and e_2 with the following characters:

α) A determinate verified statement i confers on e_1 and e_2 different probabilities which are, however, not so high that they may be considered as practical truth

β) e_1 and e_2 differ with respect to predictions of facts happening outside the domain I

γ) e_1 and e_2 do not differ with respect to predictions of facts happening within the domain I

We will now introduce two languages; the narrower language may be defined by truth meaning in combination with the principle of retrogression, the wider language by probability meaning. The wider language will call the statements e_1 and e_2 different. The narrower language will also accept them as meaningful because they involve predictions for the domain I; this language, however, cannot acknowledge any difference between e_1 and e_2 because the predictions involved for I are the same, and all difference is based on a calculation of probabilities which are too low to serve as practical verification. Thus the narrower language calls the statements e_1 and e_2 equivalent. For this language, there is as much meaning in a statement as can be (absolutely or practically) verified within I; this language, therefore, may replace both the statements e_1 and e_2 by the statement i, if i is conceived as involving the same predictions for I, and call i equivalent to e_1 and e_2.

Of such a kind is the language of the positivist concerning the cubical world. In realistic language, e_1 and e_2 are two different hypotheses about the birds and i is the co-ordinated description of the pairs of shades. The restriction to the domain I, as basis, is due in this case to the physical conditions; a statement e is therefore excluded from absolute or practical verification. Whether a statement e, however, has a meaning different from i is not determined by the physical conditions but depends on the choice of the language. If we decide for physical truth meaning, we obtain the narrower language and are to call i and e equivalent; if we decide for probability meaning, we obtain the wider language and are to call i and e different.

We cannot forbid anyone to choose the definition of meaning he prefers. If he makes his decision, however, the previous considerations form a logical signpost for him. He may be right in saying that, as long as a hole in the walls is excluded, he cannot distinguish between the statements i and e; this is true if he decides for physical truth meaning, i.e., for the narrower language. What would be entirely false, however, would be an utterance from his side that *we* cannot differentiate between i and e as long as there is no hole in the walls. We can; this is because we may choose the wider language, based on probability meaning.

These considerations demonstrate a restricting quality of truth meaning. If the domain of basic elements is restricted, truth meaning leads to a restricted language, for which statements concerning elements outside the basis are meaningless unless they are conceived as equivalent to statements concerning elements of the basis. Probability meaning, on the contrary, is free from such restrictions; it may pass beyond the basis of the language.

Let us apply these results to the language of impressions. If the basic domain of the language is restricted to the impressions of one man, attainable by him during the stretch of his life, statements about things happening

before or after his lifetime are meaningless except in so far as they are interpreted as being equivalent to statements concerning impressions of his lifetime. The two statements e_1 and e_2 used in § 16 concerning events after one's death are of this type; they possess the qualities α–γ and cannot be distinguished within this language. This is the decisive difficulty of positivism. The strictly positivistic language—thus we may call this language—contradicts normal language so obviously that it has scarcely been seriously maintained; moreover, its insufficiency is revealed as soon as we try to use it for the rational reconstruction of the thought-processes underlying actions concerning events after our death, such as expressed in the example of the life insurance policies (§ 16).[12] We have said that the choice of a language depends on our free decision but that we are bound to the decisions entailed by our choice: we find here that the decision for the strictly positivistic language would entail the renunciation of any reasonable justification of a great many human actions. The pragmatic idea that the definition of meaning is to be chosen in adaptation to the system of human actions, that it is to be determined by the postulate of utilizability, decides, therefore, against the strictly positivistic language.

To avoid these difficulties, positivists have attempted some generalizations of their language by an enlargement of the basis. Instead of the impressions of one man, they have considered the impressions of living beings in general as the basis. Such an expansion, however, contradicts the epistemological intentions of positivism which were to construct the world on the basis of one's own psychical experience; if this domain is once passed, there is no reason

[12] We may add that similar examples might be constructed for events situated before our lifetime, with the difference, however, that in this case the problem of action is not so directly concerned.

to stop just with the impressions of other people and to exclude other things. I should say that speaking of the independent existence of a table or of a stone seems much more permissible than speaking of the impressions of other people. Moreover, the expansion described does not suffice to solve all difficulties. There remain similar difficulties for events situated before the origin or after the expiration of mankind.[13]

Another expansion would be the introduction of a mixed basis. The basis determined by the impressions of our lifetime is defined by the postulate of physical possibility, i.e., by restricting meaning to impressions, the occurrence of which is physically possible. We might enlarge this basis by deviating from this postulate to a certain extent, admitting logically possible impressions situated at any time or any place in the world. In thus extending the domain of possible impressions throughout time and space positivists usually refuse to countenance expansions brought about by physical changes of the human body. It cannot be called logically impossible that the human body should become as small as an atom, or as large as the planetary system; the usual positivistic objections against the direct meaning of sentences about the elementary particles of matter refer therefore to physical possibility and not to logical possibility. But, if the case of logical possibility is once admitted for the spatiotemporal extension of the linguistic basis, it might be admitted as well for other extensions. It is true that we cannot forbid anybody to exclude the latter expansion and yet admit the former; we cannot see, however, much cogency in the construction of such a mixed basis. The arbitrary character of its limits becomes evident in some of its consequences: sentences

[13] This has been emphasized, with good reason, by C. I. Lewis, "Experience and Meaning," *Philosophical Review*, XLIII (1934), 125.

about events after our death are admitted as meaningful; sentences about the atom are prohibited, or reduced to sentences about macroscopic bodies. In spite of such scarcely justifiable qualities, this seems to be the basis which implicitly underlies most of the positivistic theories.[14]

It might be proposed to admit logical possibility to its full extent: to introduce a basis encompassing all kinds of logically possible impressions, including those which would occur with changes in the human body. This, we might suppose, would be the widest possible basis; with it we would presuppose nothing but the logical necessity of an impression basis—for that there is such a basis of impressions, that knowledge is conferred upon us through the medium of impressions, seems to be logically necessary. Or can we imagine that we may on some occasion get out of our private world?

This question, I think, is not to be answered in the negative—at least if the term "my own experience" is to have a meaning different from the purely logical term "basis of inferences." That there is such a private world is not a logical necessity but a matter of fact only, caused by the physiological organization of the human body. That I have to speak of *my* impressions, that I am separated from the impressions of other people, is by no means logically necessary. It is a matter of fact in the same sense as the people of the cubical world are bound to the interior of their cubical world. I could imagine other worlds in which impressions are not always bound together to the bundle "I"—worlds in which perhaps sometimes the ego splits into two egos which afterward unite again (cf. § 28). I can by no means maintain, with certainty, that all future experience will be of the same kind as present experience, will consist of colored figures and loud tones and resisting tactile sensations. This world may change in a way which we cannot imagine. Thus the statement, "Knowledge is bound to impressions as basic facts," is not absolutely certain.

[14] The refusal to admit physical changes of the human body finds its expression in Mach's struggle against atomism (cf. § 25).

It follows that the basis of all logically possible "impressions" is not the widest basis possible and would involve some restrictions; it seems, we must add, that a widest basis cannot be properly defined at all. To say, "All inferences about external things must start from elements of such and such kind" will never be permissible because we cannot define this "kind" in such a way that human beings are necessarily restricted to elements of the type described in order to have a basis of knowledge. Thus truth meaning will always lead to a restricted language, given any basis whatever.

The way to keep free from restrictions is pointed out by probability meaning: probability meaning, applied to any basis whatever, leads to an unrestricted language. This, it seems to me, is a decisive argument for preferring probability meaning. We may begin with a rather small domain of basic elements and construct upon it statements concerning elements of another domain without being obliged to borrow their meaning from statements about the basic domain. Thus probability meaning leads to the realistic language of actual science; we start from the rather small domain of our own observations and construct the whole world upon it. The positivistic postulate that the meaning of statements about this wider world is to be interpreted in terms of statements about the basic domain turns out to be not an obvious principle but the product of too narrow a conception of scientific language. This ambitious postulate is to be logically qualified as a proposal for a certain

restricted language; there is, however, no reason for us to accept a proposal which involves the renunciation of a great deal of human knowledge. Our situation with regard to external things is not essentially different from that of the inhabitants of the cubical world with respect to the birds outside: imagine the surface surrounding that world to contract until it surrounds only our own body, until it finally, with some geometrical deformations, becomes identical with the surface of our body—we arrive, then, at the actual conditions for the construction of human knowledge, all our information about the world being bound to the traces which causal processes project from external things to the surface of our body. We may therefore apply the analysis of the cubical-world model to the case of the relation between impressions and external things. What was shown for the cubical world is that only physical truth meaning binds us to the domain I of given facts; if we accept physical probability meaning, we may pass beyond the domain I even if all observable facts are restricted to it. The same is valid for the relation of impressions to external things. Only if we confine ourselves to physical truth meaning are our sentences bound to impressions alone. If we accept physical probability meaning, we are not bound to this domain; our statements may pass beyond it and refer to external things. This is what the logical signpost states; we do not forbid anyone to decide for the definition of meaning he likes—but if he decides for truth meaning, such as do the positivists, we do not admit that he substantiates his decision by saying that a statement about external things, as distinct from statements about impressions, cannot be conceived as meaningful. The equivalence is valid only for his definition of meaning; there is another definition of meaning, however, based on

the probability concept, which may differentiate between statements about external things and statements about impressions, even though it is not physically possible to extend the domain of observable facts beyond the domain of impressions.

A critical survey of the problem of impressions and external things therefore leads us to a confirmation of our refusal to accept the positivistic doctrine. The theory of the equivalence of statements about impressions and statements about external things originates from a too narrow conception of meaning; we are not restricted to this conception—and actual language has never been limited to such a narrow precept.

It may be proposed to formulate the relation of the positivistic to the realistic language in the following way. Since impressions furnish only probabilities for external events, a statement equivalent to a statement about impressions would be a statement concerning a probability of external events. If we introduce the name of statements of the second level for statements of the latter type, we may say that the impression language is equivalent to the second-level language of science. This would be a far-reaching change in the intent of positivism, since with this idea the existence of an independent realistic language not equivalent to the impression language is admitted. We might, indeed, agree with such a conception; we must add, however, that it can be carried through only in the sense of an approximation. There is, first, the difficulty that statements about impressions only imply probability statements about things but are not equivalent to such statements; the construction of the whole equivalent class of impressions would lead to difficulties similar to those described for the original positivistic conception. Second, the second-level language is, strictly speaking, not a two-valued language but once more a probability language, only of a higher level (cf. our criticism of impression statements in the following chapter and our remark on weights of higher levels in § 43). The interpretation indicated is, however, likely to be the best interpretation of positivism we can have: in the first approximation positivism is considered as equivalent to the language of science; in the second approximation positivism is considered as equivalent to the second-level language of science. The second approximation is much more exact than the first.

§ 18. The functional conception of meaning

If we now summarize the results of the present chapter, we find that it is the neglect of the probability character of the relations between impressions and external things which constitutes the fault of the postivistic construction of the world. The true-false conception of knowledge is valid only in the sense of an approximation; it must be applied, therefore, under careful control, and the consequences to which it leads must be interpreted in full consciousness of the merely approximative character of the presuppositions. Positivism, therefore, if it is to be considered as a permissible conception of the world, must be conceived as an approximation; only in this sense may it become of scientific value.

In this sense it is indeed frequently used and with success. If a new scientific theory is started, we imagine a set of impressions which if observed would make the theory highly probable; we say, then, that we understand the theory. If its truth is in question, we imagine another set of impressions which if observed would make the theory highly improbable; we say, then, that we understand how a refutation of the theory would run. The positivistic method thus provides us with a good intuitive representation of the theory; but it does no more.

In this process of making a theory intuitively clear it may also be permissible to supersede the postulate of physical possibility and to introduce imagined impressions which are logically possible only. If this expansion is not always consistently carried through, if some logical possibilities are admitted and others rejected, we shall not oppose such a mixed basis; it may even be advisable to refrain from drawing too narrow limits. We read Gulliver's voyage to the Lilliputians and picture with pleasure impressions we should have in this miniature country, al-

though it is not physically possible to go there. Reading Einstein's theories, we imagine a man who sets his watch right by the arrival of light rays with a super-astronomic precision; although this is not physically possible, it may be a good representation of Einstein's definition of simultaneity. We fancy rotating atoms and jumping electrons as though we could see them with a microscope, and that may be a good help for understanding Bohr's theories. The physicists have shown that we must be very careful in such constructions, that some of the tacitly assumed conditions of our macroscopic world are no longer valid for sub-microscopic dimensions; but in picturing a world which is constructed half by the postulates of physical laws, half by suppositions extending beyond physics, we may understand some essential features of the world which had previously escaped our notice and advance toward an intuitive understanding of theories which would otherwise remain in the mists of abstraction.

We must not forget, however, that the set of impressions fancied is not equivalent to the intension of the theory in question. Assuming this is just the illegitimate consequence to which the neglect of the probability character of knowledge leads. It means disregarding the fact that every describable set of impressions, if observed, furnishes probability only for physical statements. It means overstraining the bearing of approximative concepts and deducing from them consequences for which the limits of the approximation do not hold. It means restricting one's self to an intuitive representation—the occurrence of some determinate impressions—instead of exhausting the meaning of the whole sentence. It is not, as positivists pretend, the only admissible conception of meaning but an oversimplified theory of meaning.

The origin of this theory of meaning, it seems to me, is

to be found in the idea that the meaning of a sentence is something which may be pointed out, which may be seen and known. This "something" is constructed by positivism in the set of impressions belonging to the sentence. What we obtain in this way, however, are only images, associated representations. It is a psychological conception of meaning which positivism maintains—based, however, on some metaphysical remainders taken over from traditional philosophy—from a substantial conception of meaning. It is this deep-rooted misconception from which the positivistic theory of meaning originates.

The meaning of a proposition is not "something"—there is no question at all of the form, "*What* is the meaning?" A proposition has meaning—that is, a proposition has certain qualities; but it does not have a co-ordinated something which *is* the meaning. We had better say: a proposition *is significant*—the substantival term, "has meaning," is always to be understood in the sense of the adjectival term "is significant." This corresponds to our usage of words in the two principles of the theory of meaning which define not the use of the term "meaning" but that of the term "has meaning." The first denotes under what conditions a proposition has meaning, the second denotes under what conditions two propositions have the same meaning; this is all we need—we need not know what the meaning is.

To understand a proposition is the desire of every good-intentioned scholar, and it appears perhaps a heartless radicalism if we maintain that there is no understanding in the sense of "knowing the intension." What we call understanding, however, is nothing but producing associated images, representing some effects connected with the sentence, forming an intuitive representation. We do not intend to forbid this, certainly. We are convinced that this is a very good and fertile way of working in science, that

intuitive images may make thinking distinct and creative, that it is perhaps just these associations to which is due the intense joy combined with all productive and reproductive scientific thinking. What we object to, however, is the identification of the associated images with the meaning of the propositions, and the substitution of an intuitive representation for the full and complete intension. In other words, we refuse to deduce the meaning of meaning from psychological processes.

Thinking works in a tunnel; we do not see intensions, contents. Propositions are tools with which we operate; all we can demand is to be able to manipulate these tools. The darkness of the tunnel may be lighted by the searchlights of intuitive images fitfully appearing and wandering. Let us not confound blurred images with the full class of operations for which the tools are good.

Reference to impressions is permissible in the sense of an intuitive representation—if we accept this, however, we may accept other representations as well. The realistic conception of the world possesses images of this kind as well as the positivistic conception; and I do not see any reason why these conceptions should not be permissible in the same sense as positivistic images. Positivists have attacked realism in pretending that it is meaningless to imagine external things which we do not observe, and then have insisted that the only permissible interpretation of propositions about external things is to realize the impressions we should have when the things were observed. This, it seems to me, is the attack of one metaphysician against another; it cannot be the task of scientific philosophy to decide for one side in this struggle. An unprejudiced analysis of scientific propositions shows that the positions of positivism and realism are both rooted in the psychological sphere and that the concept of meaning should be freed

from all such psychological components if it is to correspond to the practice of thinking.

Meaning is a function of propositions; it is that function which is expressed in their usefulness as instruments for our actions upon the world. Meaning is not a substantial something attached to a proposition, like "ideas" or "impressions," but a quality; the physical things called "symbols" have a certain function as to operations on all other things—this function is called meaning. It is this functional conception of meaning only which opens the field for the introduction of the concept of probability into the theory of meaning. Probability meaning, as we defined it, must be considered within the framework of this functional theory. It seems to me that only this combination with the probability theory can provide the functional theory of meaning with the tools necessary for a satisfactory theory of scientific propositions, a theory adapted to the actual procedure of science. This is what is shown by the analysis of the relations between impressions and the external world.

CHAPTER III

AN INQUIRY CONCERNING IMPRESSIONS