THE UNIVERSITY OF CHICAGO PRESS, CHICAGO 37
Cambridge University Press, London, W.C. 1, England
W. J. Gage & Co., Limited, Toronto 2B, Canada

Experience and Prediction

An Analysis of the Foundations and the Structure of Knowledge

BY

HANS REICHENBACH, PH.D.

Professor of Philosophy in the University of California at Los Angeles
Formerly Professor in the Universities of Berlin and Istanbul

THE UNIVERSITY OF CHICAGO PRESS
CHICAGO · ILLINOIS

Published February 1938. Second Impression 1947. Composed and printed by THE UNIVERSITY OF CHICAGO PRESS, *Chicago, Illinois, U.S.A.*

PREFACE

The ideas of this book have grown from the soil of a philosophic movement which, though confined to small groups, is spread over the whole world. American pragmatists and behaviorists, English logistic epistemologists, Austrian positivists, German representatives of the analysis of science, and Polish logisticians are the main groups to which is due the origin of that philosophic movement which we now call "logistic empiricism." The movement is no longer restricted to its first centers, and its representatives are to be found today in many other countries as well —in France, Italy, Spain, Turkey, Finland, Denmark, and elsewhere. Though there is no philosophic system which unites these groups, there is a common property of ideas, principles, criticisms, and working methods—all characterized by their common descent from a strict disavowal of the metaphor language of metaphysics and from a submission to the postulates of intellectual discipline. It is the intention of uniting both the empiricist conception of modern science and the formalistic conception of logic, such as expressed in logistic, which marks the working program of this philosophic movement.

Since this book is written with the same intentions, it may be asked how such a new attempt at a foundation of logistic empiricism can be justified. Many things indeed will be found in this book which have been said before by others, such as the physicalistic conception of language and the importance attributed to linguistic analysis, the connection of meaning and verifiability, and the behavioristic conception of psychology. This fact may in part be justi-

fied by the intention of giving a report of those results which may be considered today as a secured possession of the philosophic movement described; however, this is not the sole intention. If the present book enters once more into the discussion of these fundamental problems, it is because former investigations did not sufficiently take into account one concept which penetrates into all the logical relations constructed in these domains: that is, the concept of probability. It is the intention of this book to show the fundamental place which is occupied in the system of knowledge by this concept and to point out the consequences involved in a consideration of the probability character of knowledge.

The idea that knowledge is an approximative system which will never become "true" has been acknowledged by almost all writers of the empiricist group; but never have the logical consequences of this idea been sufficiently realized. The approximative character of science has been considered as a necessary evil, unavoidable for all practical knowledge, but not to be counted among the essential features of knowledge; the probability element in science was taken as a provisional feature, appearing in scientific investigation as long as it is on the path of discovery but disappearing in knowledge as a definitive system. Thus a fictive definitive system of knowledge was made the basis of epistemological inquiry, with the result that the schematized character of this basis was soon forgotten, and the fictive construction was identified with the actual system. It is one of the elementary laws of approximative procedure that the consequences drawn from a schematized conception do not hold outside the limits of the approximation; that in particular no consequences may be drawn from features belonging to the nature of the schematization only and not to the co-ordinated object. Mathematicians know that for many a purpose the number π may be sufficiently approximated by the value 22/7; to infer from this, however, that π is a rational number is by no means permissible. Many of the inferences of traditional epistemology and of positivism as well, I must confess, do not appear much better to me. It is particularly the domain of the verifiability conception of meaning and of questions connected with it, such as the problem of the existence of external things, which has been overrun with paralogisms of this type.

The conviction that the key to an understanding of scientific method is contained within the probability problem grew stronger and stronger with me in the face of such basic mistakes. This is the reason why, for a long time, I renounced a comprehensive report of my epistemological views, although my special investigations into different problems of epistemology demanded a construction of foundations different from those constructed by some of my philosophical friends. I concentrated my inquiry on the problem of probability which demanded at the same time a mathematical and a logical analysis. It is only after having traced out a logistic theory of probability, including a solution of the problem of induction, that I turn now to an application of these ideas to questions of a more general epistemological character. As my theory of probability has been published for some years, it was not necessary to present it with all mathematical details once more in the present book; the fifth chapter, however, gives an abbreviated report of this theory—a report which seemed necessary as the probability book has been published in German only.

It is this combination of the results of my investigations on probability with the ideas of an empiricist and logistic conception of knowledge which I here present as my con-

tribution to the discussion of logistic empiricism. The growth of this movement seems to me sufficiently advanced to enter upon a level of higher approximation; and what I propose is that the form of this new phase should be a probabilistic empiricism. If the continuation suggested comes to contradict some ideas so far considered as established, particularly by positivist writers, the reader will bear in mind that this criticism is not offered with the intention of diminishing the historical merits of these philosophers. On the contrary, I am glad to have an occasion for expressing my indebtedness to many a writer whose opinions I cannot wholly share. I think, however, that the clarification of the foundations of our common conceptions is the most urgent task within our philosophic movement and that we should not recoil from frankly admitting the insufficiencies of former results—even if they still find defenders within our ranks.

The ideas of this book have been discussed in lectures and seminars at the University of Istanbul. I welcome the opportunity to express my warmest thanks to friends and students here in Istanbul for their active interest which formed a valuable stimulus in the clarification of my ideas, especially to my assistant, Miss Neyire Adil-Arda, without whose constant support I should have found it very much harder to formulate my views. For help in linguistic matters and reading of proofs I am grateful to Miss Sheila Anderson, of the English High School at Istanbul; to Professor Charles W. Morris, Mr. Lawrence K. Townsend, Jr., and Mr. Rudolph C. Waldschmidt, of the University of Chicago; to Mr. Max Black, of the Institute of Education of the University of London; and to Miss Eleanor Bisbee, of Robert College at Istanbul.

HANS REICHENBACH

UNIVERSITY OF ISTANBUL
TURKEY
July 1937

TABLE OF CONTENTS

CHAPTER I

MEANING

CHAPTER I

MEANING

§ 1. The three tasks of epistemology

Every theory of knowledge must start from knowledge as a given sociological fact. The system of knowledge as it has been built up by generations of thinkers, the methods of acquiring knowledge used in former times or used in our day, the aims of knowledge as they are expressed by the procedure of scientific inquiry, the language in which knowledge is expressed—all are given to us in the same way as any other sociological fact, such as social customs or religious habits or political institutions. The basis available for the philosopher does not differ from the basis of the sociologist or psychologist; this follows from the fact that, if knowledge were not incorporated in books and speeches and human actions, we never would know it. Knowledge, therefore, is a very concrete thing; and the examination into its properties means studying the features of a sociological phenomenon.

We shall call the first task of epistemology its *descriptive task*—the task of giving a description of knowledge as it really is. It follows, then, that epistemology in this respect forms a part of sociology. But it is only a special group of questions concerning the sociological phenomenon "knowledge" which constitutes the domain of epistemology. There are such questions as "What is the meaning of the concepts used in knowledge?" "What are the presuppositions contained in the method of science?" "How do we know whether a sentence is true, and do we know that at all?" and many others; and although, indeed,

these questions concern the sociological phenomenon "science," they are of a very special type as compared with the form of questions occurring in general sociology.

What makes this difference? It is usually said that this is a difference of internal and external relations between those human utterances the whole of which is called "knowledge." Internal relations are such as belong to the content of knowledge, which must be realized if we want to understand knowledge, whereas external relations combine knowledge with utterances of another kind which do not concern the content of knowledge. Epistemology, then, is interested in internal relations only, whereas sociology, though it may partly consider internal relations, always blends them with external relations in which this science is also interested. A sociologist, for instance, might report that astronomers construct huge observatories containing telescopes in order to watch the stars, and in such a way the internal relation between telescopes and stars enters into a sociological description. The report on contemporary astronomy begun in the preceding sentence might be continued by the statement that astronomers are frequently musical men, or that they belong in general to the bourgeois class of society; if these relations do not interest epistemology, it is because they do not enter into the content of science—they are what we call external relations.

Although this distinction does not furnish a sharp line of demarcation, we may use it for a first indication of the design of our investigations. We may then say the descriptive task of epistemology concerns the internal structure of knowledge and not the external features which appear to an observer who takes no notice of its content.

We must add now a second distinction which concerns psychology. The internal structure of knowledge is the system of connections as it is followed in thinking. From

such a definition we might be tempted to infer that epistemology is the giving of a description of thinking processes; but that would be entirely erroneous. There is a great difference between the system of logical interconnections of thought and the actual way in which thinking processes are performed. The psychological operations of thinking are rather vague and fluctuating processes; they almost never keep to the ways prescribed by logic and may even skip whole groups of operations which would be needed for a complete exposition of the subject in question. That is valid for thinking in daily life, as well as for the mental procedure of a man of science, who is confronted by the task of finding logical interconnections between divergent ideas about newly observed facts; the scientific genius has never felt bound to the narrow steps and prescribed courses of logical reasoning. It would be, therefore, a vain attempt to construct a theory of knowledge which is at the same time logically complete and in strict correspondence with the psychological processes of thought.

The only way to escape this difficulty is to distinguish carefully the task of epistemology from that of psychology. Epistemology does not regard the processes of thinking in their actual occurrence; this task is entirely left to psychology. What epistemology intends is to construct thinking processes in a way in which they ought to occur if they are to be ranged in a consistent system; or to construct justifiable sets of operations which can be intercalated between the starting-point and the issue of thought-processes, replacing the real intermediate links. Epistemology thus considers a logical substitute rather than real processes. For this logical substitute the term *rational reconstruction* has been introduced;[1] it seems an appropriate phrase to indi-

[1] The term *rationale Nachkonstruktion* was used by Carnap in *Der logische Aufbau der Welt* (Berlin and Leipzig, 1928).

cate the task of epistemology in its specific difference from the task of psychology. Many false objections and misunderstandings of modern epistemology have their source in not separating these two tasks; it will, therefore, never be a permissible objection to an epistemological construction that actual thinking does not conform to it.

In spite of its being performed on a fictive construction, we must retain the notion of the descriptive task of epistemology. The construction to be given is not arbitrary; it is bound to actual thinking by the postulate of correspondence. It is even, in a certain sense, a better way of thinking than actual thinking. In being set before the rational reconstruction, we have the feeling that only now do we understand what we think; and we admit that the rational reconstruction expresses what we mean, properly speaking. It is a remarkable psychological fact that there is such an advance toward understanding one's own thoughts, the very fact which formed the basis of the mäeutic of Socrates and which has remained since that time the basis of philosophical method; its adequate scientific expression is the principle of rational reconstruction.

If a more convenient determination of this concept of rational reconstruction is wanted, we might say that it corresponds to the form in which thinking processes are communicated to other persons instead of the form in which they are subjectively performed. The way, for instance, in which a mathematician publishes a new demonstration, or a physicist his logical reasoning in the foundation of a new theory, would almost correspond to our concept of rational reconstruction; and the well-known difference between the thinker's way of finding this theorem and his way of presenting it before a public may illustrate the difference in question. I shall introduce the terms *context of*

discovery and *context of justification* to mark this distinction. Then we have to say that epistemology is only occupied in constructing the context of justification. But even the way of presenting scientific theories is only an approximation to what we mean by the context of justification. Even in the written form scientific expositions do not always correspond to the exigencies of logic or suppress the traces of subjective motivation from which they started. If the presentation of the theory is subjected to an exact epistemological scrutiny, the verdict becomes still more unfavorable. For scientific language, being destined like the language of daily life for practical purposes, contains so many abbreviations and silently tolerated inexactitudes that a logician will never be fully content with the form of scientific publications. Our comparison, however, may at least indicate the way in which we want to have thinking replaced by justifiable operations; and it may also show that the rational reconstruction of knowledge belongs to the descriptive task of epistemology. It is bound to factual knowledge in the same way that the exposition of a theory is bound to the actual thoughts of its author.

In addition to its descriptive task, epistemology is concerned with another purpose which may be called its *critical task*. The system of knowledge is criticized; it is judged in respect of its validity and its reliability. This task is already partially performed in the rational reconstruction, for the fictive set of operations occurring here is chosen from the point of view of justifiability; we replace actual thinking by such operations as are justifiable, that is, as can be demonstrated as valid. But the tendency to remain in correspondence with actual thinking must be separated from the tendency to obtain valid thinking; and so we have to distinguish between the descriptive and the critical task. Both collaborate in the rational reconstruc-

tion. It may even happen that the description of knowledge leads to the result that certain chains of thoughts, or operations, cannot be justified; in other words, that even the rational reconstruction contains unjustifiable chains, or that it is not possible to intercalate a justifiable chain between the starting-point and the issue of actual thinking. This case shows that the descriptive task and the critical task are different; although description, as it is here meant, is not a copy of actual thinking but the construction of an equivalent, it is bound by the postulate of correspondence and may expose knowledge to criticism.

The critical task is what is frequently called *analysis of science;* and as the term "logic" expresses nothing else, at least if we take it in a sense corresponding to its use, we may speak here of the logic of science. The well-known problems of logic belong to this domain; the theory of the syllogism was built up to justify deductive thinking by reducing it to certain justifiable schemes of operation, and the modern theory of the tautological character of logical formulas is to be interpreted as a justification of deductive thinking as conceived in a more general form. The question of the synthetic a priori, which has played so important a role in the history of philosophy, also falls into this frame; and so does the problem of inductive reasoning, which has given rise to more than one "inquiry concerning human understanding." Analysis of science comprehends all the basic problems of traditional epistemology; it is, therefore, in the foreground of consideration when we speak of epistemology.

The inquiries of our book will belong, for the most part, to the same domain. Before entering upon them, however, we may mention a result of rather general character which has been furnished by previous investigations of this kind —a result concerning a distinction without which the

process of scientific knowledge cannot be understood. Scientific method is not, in every step of its procedure, directed by the principle of validity; there are other steps which have the character of volitional decisions. It is this distinction which we must emphasize at the very beginning of epistemological investigations. That the idea of truth, or validity, has a directive influence in scientific thinking is obvious and has at all times been noticed by epistemologists. That there are certain elements of knowledge, however, which are not governed by the idea of truth, but which are due to volitional resolutions, and though highly influencing the makeup of the whole system of knowledge, do not touch its truth-character, is less known to philosophical investigators. The presentation of the volitional decisions contained in the system of knowledge constitutes, therefore, an integral part of the critical task of epistemology. To give an example of volitional decisions, we may point to the so-called *conventions*, e.g., the convention concerning the unit of length, the decimal system, etc. But not all conventions are so obvious, and it is sometimes a rather difficult problem to find out the points which mark conventions. The progress of epistemology has been frequently furthered by the discovery of the conventional character of certain elements taken, until that time, as having a truth-character; Helmholtz' discovery of the arbitrariness of the definition of spatial congruence, Einstein's discovery of the relativity of simultaneity, signify the recognition that what was deemed a statement is to be replaced by a decision. To find out all the points at which decisions are involved is one of the most important tasks of epistemology.

The conventions form a special class of decisions; they represent a choice between *equivalent* conceptions. The different systems of weights and measures constitute a

good example of such an equivalence; they illustrate the fact that the decision in favor of a certain convention does not influence the content of knowledge. The examples chosen from the theory of space and time previously mentioned are likewise to be ranked among conventions. There are decisions of another character which do not lead to equivalent conceptions but to divergent systems; they may be called *volitional bifurcations*. Whereas a convention may be compared to a choice between different ways leading to the same place, the volitional bifurcation resembles a bifurcation of ways which will never meet again. There are some volitional bifurcations of an important character which stand at the very entrance of science: these are decisions concerning the aim of science. What is the purpose of scientific inquiry? This is, logically speaking, a question not of truth-character but of volitional decision, and the decision determined by the answer to this question belongs to the bifurcation type. If anyone tells us that he studies science for his pleasure and to fill his hours of leisure, we cannot raise the objection that this reasoning is "a false statement"—it is no statement at all but a decision, and everybody has the right to do what he wants. We may object that such a determination is opposed to the normal use of words and that what he calls the aim of science is generally called the aim of play—this would be a true statement. This statement belongs to the descriptive part of epistemology; we can show that in books and discourses the word "science" is always connected with "discovering truth," sometimes also with "foreseeing the future." But, logically speaking, this is a matter of volitional decision. It is obvious that this decision is not a convention because the two conceptions obtained by different postulates concerning the aims of science are not equivalent; it is a bifurcation. Or take a question as to the meaning of a certain

concept—say, causality, or truth, or meaning itself. Logically speaking this is a question of a decision concerning the limitation of a concept, although, of course, the practice of science has already decided about this limitation in a rather precise way. In such a case, it must be carefully examined whether the decision in question is a convention or a bifurcation. The limitation of a concept may be of a conventional character, i.e., different limitations may lead to equivalent systems.

The character of being true or false belongs to statements only, not to decisions. We can, however, co-ordinate with a decision certain statements concerning it; and, above all, there are two types of statements which must be considered. The first one is a statement of the type we have already mentioned; it states which decision science uses in practice. It belongs to descriptive epistemology and is, therefore, of a sociological character. We may say that it states an *object fact*, i.e., a fact belonging to the sphere of the objects of knowledge,[2] a sociological fact being of this type. It is, of course, the same type of fact with which natural science deals. The second statement concerns the fact that, logically speaking, there is a decision and not a statement; this kind of fact may be called a *logical fact*. There is no contradiction in speaking here of a fact concerning a decision; although a decision is not a fact, its character of being a decision is a fact and may be expressed in a statement. That becomes obvious by the cognitional character of such a statement; the statement may be right or wrong, and in some cases the wrong statement has been maintained for centuries, whereas the right statement was discovered only recently. The given examples of Helm-

[2] The term "objective fact" taken in the original sense of the word "objective" would express the same point; but we avoid it, as the word "objective" suggests an opposition to "subjective," an opposition which we do not intend.

holtz' and Einstein's theories of space and time may illustrate this. But the kind of fact maintained here does not belong to the sphere of the objects of science, and so we call it a logical fact. It will be one of our tasks to analyze these logical facts and to determine their logical status; but for the present we shall use the term "logical fact" without further explanation.

The difference between statements and decisions marks a point at which the distinction between the descriptive task and the critical task of epistemology proves of utmost importance. Logical analysis shows us that within the system of science there are certain points regarding which no question as to truth can be raised, but where a decision is to be made; descriptive epistemology tells us what decision is actually in use. Many misunderstandings and false pretensions of epistemology have their origin here. We know the claims of Kantianism, and Neo-Kantianism, to maintain Euclidean geometry as the only possible basis of physics; modern epistemology showed that the problem as it is formulated in Kantianism is falsely constructed, as it involves a decision which Kant did not see. We know the controversies about the "meaning of meaning"; their passionate character is due to the conviction that there is an absolute meaning of meaning which we must discover, whereas the question can only be put with respect to the concept of meaning corresponding to the use of science, or presupposed in certain connections. But we do not want to anticipate the discussion of this problem, and our later treatment of it will contain a more detailed explanation of our distinction between statements and decisions.

The concept of decision leads to a third task with which we must charge epistemology. There are many places where the decisions of science cannot be determined precisely, the words or methods used being too vague; and

there are others in which two or even more different decisions are in use, intermingling and interfering within the same context and confusing logical investigations. The concept of meaning may serve as an example; simpler examples occur in the theory of measurement. The concrete task of scientific investigation may put aside the exigencies of logical analysis; the man of science does not always regard the demands of the philosopher. It happens, therefore, that the decisions presupposed by positive science are not clarified. In such a case, it will be the task of epistemology to suggest a proposal concerning a decision; and we shall speak, therefore, of the *advisory task* of epistemology as its third task. This function of epistemology may turn out to be of great practical value; but it must be kept clearly in mind that what is to be given here is a proposal and not a determination of a truth-character. We may point out the advantages of our proposed decision, and we may use it in our own expositions of related subjects; but never can we demand agreement to our proposal in the sense that we can demand it for statements which we have proven to be true.

There is, however, a question regarding facts which is to be considered in connection with the proposal of a decision. The system of knowledge is interconnected in such a way that some decisions are bound together; one decision, then, involves another, and, though we are free in choosing the first one, we are no longer free with respect to those following. We shall call the group of decisions involved by one decision its *entailed decisions*. To give a simple example: the decision for the English system of measures leads to the impossibility of adding measure numbers according to the technical rules of the decimal system; so the renunciation of these rules would be an entailed decision. Or a more complicated example: the decision expressed in

the acceptance of Euclidean geometry in physics may lead to the occurrence of strange forces, "universal forces," which distort all bodies to the same extent, and may lead to even greater inconveniences concerning the continuous character of causality.[3] The discovery of interconnections of this kind is an important task of epistemology, the relations between different decisions being frequently hidden by the complexity of the subject; it is only by adding the group of entailed decisions that a proposal respecting a new decision becomes complete.

The discovery of entailed decisions belongs to the critical task of epistemology, the relation between decisions being of the kind called a logical fact. We may therefore reduce the advisory task of epistemology to its critical task by using the following systematic procedure: we renounce making a proposal but instead construe a list of all possible decisions, each one accompanied by its entailed decisions. So we leave the choice to our reader after showing him all factual connections to which he is bound. It is a kind of logical signpost which we erect; for each path we give its direction together with all connected directions and leave the decision as to his route to the wanderer in the forest of knowledge. And perhaps the wanderer will be more thankful for such a signpost than he would be for suggestive advice directing him into a certain path. Within the frame of the modern philosophy of science there is a movement bearing the name of *conventionalism;* it tries to show that most of the epistemological questions contain no questions of truth-character but are to be settled by arbitrary decisions. This tendency, and above all, in its founder Poincaré, had historical merits, as it led philosophy to stress the volitional elements of the system of knowledge

[3] Cf. the author's *Philosophie der Raum-Zeit-Lehre* (Berlin: De Gruyter, 1928), § 12.

which had been previously neglected. In its further development, however, the tendency has largely trespassed beyond its proper boundaries by highly exaggerating the part occupied by decisions in knowledge. The relations between different decisions were overlooked, and the task of reducing arbitrariness to a minimum by showing the logical interconnections between the arbitrary decisions was forgotten. The concept of entailed decisions, therefore, may be regarded as a dam erected against extreme conventionalism; it allows us to separate the arbitrary part of the system of knowledge from its substantial content, to distinguish the subjective and the objective part of science. The relations between decisions do not depend on our choice but are prescribed by the rules of logic, or by the laws of nature.

It even turns out that the exposition of entailed decisions settles many quarrels about the choice of decisions. Certain basic decisions enjoy an almost universal assent; and, if we succeed in showing that one of the contended decisions is entailed by such a basic decision, the acceptance of the first decision will be secured. Basic decisions of such a kind are, for instance, the principle that things of the same kind shall receive the same names, or the principle that science is to furnish methods for foreseeing the future as well as possible (a demand which will be accepted even if science is also charged with other tasks). I will not say that these basic decisions must be assumed and retained in every development of science; what I want to say is only that these decisions are actually maintained by most people and that many quarrels about decisions are caused only by not seeing the implication which leads from the basic decisions to the decision in question.

The objective part of knowledge, however, may be freed from volitional elements by the method of reduction trans-

forming the advisory task of epistemology into the critical task. We may state the connection in the form of an implication: If you choose this decision, then you are obliged to agree to this statement, or to this other decision. This implication, taken as a whole, is free from volitional elements; it is the form in which the objective part of knowledge finds its expression.

§ 2. Language

It may be questioned if every process of thinking needs language. It is true that most conscious thinking is bound to the language form, although perhaps in a more or less loose way: the laws of style are suspended, and incomplete groups of words are frequently used instead of whole propositions. But there are other types of thought of a more intuitive character which possibly do not contain psychological elements which can be regarded as constituting a language. This is a question which psychologists have not yet brought to a definite solution.

What cannot be questioned, however, is that this is the concern of psychology only and not of epistemology. We pointed out that it is not thinking in its actuality which constitutes the subject matter of epistemology but that it is the rational reconstruction of knowledge which is considered by epistemology. And rationally reconstructed knowledge can only be given in the language form—that needs no further explanation, since it may be taken as a part of the definition of what we call rational reconstruction. So we are entitled to limit ourselves to symbolized thinking, i.e., to thinking formulated in language, when we begin with the analysis of knowledge. If anyone should raise the objection that we leave out by this procedure certain parts of thinking which do not appear in the language form, the objection would betray a misunderstanding of

the task of epistemology; for thinking processes enter into knowledge, in our sense of the term, only in so far as they can be replaced by chains of linguistic expressions.

Language, therefore, is the natural form of knowledge. A theory of knowledge must consequently begin with a theory of language. Knowledge is given by symbols—so symbols must be the first object of epistemological inquiry.

What are symbols? It cannot be sufficiently emphasized that symbols are, first of all, physical bodies, like all other physical things. The symbols used in a book consist of areas of ink, whereas the symbols of spoken language consist of sound waves which are as physically real as the areas of ink. The same is true for symbols used in a so-called "symbolic" way, such as flags or crucifixes or certain kinds of salutation by a movement of the hand; they all are physical bodies or processes. So a symbol in its general character does not differ from other physical things.

But, in addition to their physical characteristics, symbols have a property which is generally called their *meaning*. What is this meaning?

This question has occupied philosophers of every historical period and stands in the foreground of contemporary philosophical discussion, so we cannot be expected to give a definite answer at the very beginning of our study. We must start with a provisional answer which may lead our investigation in the right direction. Let us formulate our first answer as follows: *Meaning is a function which symbols acquire by being put into a certain correspondence with facts.*

If "Paul" is the name of a certain man, this symbol will always occur in sentences concerning actions of, or the status of, Paul; or if "north" means a certain relation of a line to the North Pole of the earth, the symbol "north" will occur in connection with the symbols "London" and

"Edinburgh," as for example, in the sentence, "Edinburgh is north of London," because the objects London and Edinburgh are in the relation to the North Pole corresponding to the word "north." So the carbon patch "north" before your eyes has a meaning because it occurs in relation to other carbon patches in such a way that there is a correspondence to physical objects such as towns and the North Pole. Meaning is just this function of the carbon patch acquired by this connection.

One thing has to be considered in order that we may understand this situation. Whether a symbol has the function of meaning does not merely depend on the symbol and the facts in question; it also depends on the use of certain rules called the rules of language. That the order of the town names in the sentence previously cited must be the given one, and not the converse one, is stipulated by a rule of the language, without which the meaning of the word "north" would be incomplete. So it may be said that only the rules of language confer meaning on a symbol. At one time there were found certain stones covered with wedge-formed grooves; it was a long time before men discovered that these grooves have a meaning and were in ancient times the writing of a cultured people, the "cuneiform-writing" of the Assyrians. This discovery comprehends two facts: first, that it is possible to add a system of rules to the grooves on the stones in such a way that they enter into relations with facts of the kind occurring in human history; second, that these rules were used by the Assyrians and that the grooves were produced by them. This second discovery is of great importance to history, but to logic the first discovery is the important one. To confer the name of symbols upon certain physical entities it is sufficient that rules can be added to them in such a way that correspondence to facts arises; it is not necessary that the

symbols be created and used by man. It sometimes happens that large stones decay, through atmospheric action, in such a way that they assume the form of certain words; these words have a meaning, although they were not made by men. But the case is still special in so far as these symbols correspond to the rules of ordinary language. It might also happen that forms, obtained by natural processes, would convey European history to us if a certain new system of rules were added—although that does not seem to be very probable. There would still be the question of whether we could find these rules. But very frequently we invent new systems of rules for certain special purposes for which special symbols are needed. The signposts and lights in use for the regulation of motor traffic form a system of symbols different from ordinary language in symbols and rules. The system of rules is not a closed class; it is continuously being enlarged according to the requirements of life. We must therefore distinguish between known or unknown symbolic characters, between actual and virtual symbols. The first are the only important ones, since only actual symbols are employed, and therefore the word "symbol" is used in the sense of "actual symbol" or "symbol in use." It is obvious that a symbol acquires this character not by inner qualities but by the rules of language and that any physical thing may acquire the function of a symbol if it fulfils certain given rules of language, or if suitable rules are established.

§ 3. The three predicates of propositions

After this characterization of language in its general aspect, we must now proceed to a view of the internal structure of language.

The first salient feature we observe here is that symbols follow one another in a linear arrangement, given by the

one-dimensional character of speech as a process in time. But this series of symbols—and this is the second conspicuous feature—is not of uniform flow; it is divided into certain groups, each forming a unity, called propositions. Language has thus an atomistic character. Like the atoms of physics, the atoms of language contain subdivisions: propositions consist of words, and words of letters. The proposition is the most important unity and really performs the function of the atom: as any piece of matter must consist of a whole number of atoms, so any speech must consist of a whole number of propositions; "half-propositions" do not occur. We may add that the minimum length of a speech is one proposition.

We express this fact by saying that meaning is a function of a proposition as a whole. Indeed, if we speak of the meaning of a word, this is possible only because the word occurs within propositions; meaning is transferred to the word by the proposition. We see this by the fact that groups of isolated words have no meaning; to utter the words "tree house intentionally and" means nothing. Only because these words habitually occur in meaningful sentences, do we attach to them that property which we call their meaning; but it would be more correct to call that property "capacity for occurring within meaningful sentences." We shall abbreviate this term to "symbolic character" and reserve the term "meaning" for propositions as a whole. Instead of the term "symbolic character" we shall also use the term "sense"; according to this terminology, words have *sense*, and propositions have *meaning*. We shall also say that meaning is a predicate of propositions.

The origin of this unique propositional form arises from a second predicate which also belongs to propositions only and not to words. This is the character of being true or

false. We call this predicate the *truth-value* of the proposition. A word is neither true nor false; these concepts are not applicable to a word. It is only an apparent exception if occasionally the use of words contradicts this rule. When children learn to talk, it may happen that they point to a table, utter the word "table," and receive the confirmation "yes." But in this case the word "table" is only an abbreviation for the sentence, "This is a table," and what is confirmed by "yes" is this sentence. (The word "yes" in itself is a sentence, meaning, "The sentence stated before is true.") Analogous cases occur in a conversation with a foreigner whose knowledge of a language is rather incomplete. But, strictly speaking, a conversation consists of sentences.

The atomic sentences which form the elements of speech may be combined in different ways. The operations of combination are enumerated by logic; they are expressed by such words as "and," "or," "implies," etc. By these operations some atomic propositions may be closely connected; in this case, we may speak of molecular propositions.[4]

> Macbeth shall never vánquish'd be until
> Great Birnam wood to high Dunsinane hill
> Shall come against him.

The apparition states here, to inform Macbeth, a molecular proposition. It is one of the rules of language that in such a case the speaker wants to maintain only the truth of the whole molecular proposition, leaving open the question of the truth of the clauses; so Macbeth is right in inferring that the atomic proposition concerning the strange removal of the wood is not maintained by the apparition and that the implication asserted will not affect him. It is a

[4] The words "sentence" and "statement" are also in use. But this distinction being of little importance and rather vague, we shall make no distinction between "propositions" and "sentences" and "statements."

bad habit of all oracles that they make use in this way of the liberalism of logic, which allows the expression of propositions without their assertion, only to deceive a man in respect to a future fact which their superhuman eyes already see.

There are various ways in which language expresses this intention to leave the question of truth open. As for implication, this renunciation is usually expressed by the use of the particle "if," or "in case," whereas the particle "when" expresses the same implication with the additional condition that the premise will be fulfilled at a certain time. "If Peter comes, I shall give him the book" differs from "When Peter comes, I shall give him the book" in this respect; only in the second case is the first clause asserted separately, so that we may infer here that Peter will come. What is left open by "when" is only the time of the realization. The particle "until" used by the apparition is not quite clear, and, if Macbeth had been a logician, he might have asked the crowned child if she could repeat her molecular proposition by saying "when" instead of "until," after putting the first clause into the positive form. Another way of showing that the proposition is not maintained as true is by the use of the interrogative form. To put a question means to utter a sentence without stating its truth or its falsehood, but with the wish to hear the opinion of another man about it. Grammatically the interrogative form is expressed by the inversion of subject and predicate; some languages have a special particle for this purpose which they add to the unchanged proposition, like the Latin *ne* or the Turkish *mi*. On the other hand, a molecular sentence, running from a full stop to the next full stop, is maintained as true.

There is a third predicate of propositions which must be mentioned in this context. Only a small proportion of the

propositions occurring in speech are of such a type that we know their truth-value; for most propositions the truth-value has not yet been determined at the moment when they are uttered. It is the difference between verified and unverified sentences of which we must now speak. To the class of unverified sentences belong, in the first place, all propositions concerning future events. These are not only propositions concerning matters of importance which cannot be thoroughly analyzed, like questions regarding our personal position in life, or questions concerning political events; the greater part of such propositions concern rather insignificant events, like tomorrow's weather, or the departure of a tram, or the butcher's sending the meat for dinner. Though all these propositions are not yet verified, they do not appear in speech without any determination of their truth-value; we utter them with the expression of a certain opinion concerning their truth. Some of them are rather certain, like those concerning the sun's rising tomorrow, or the departure of trains; others are less certain, e.g., if they concern the weather, or the coming of a tradesman who has been summoned; others are very uncertain, like propositions promising you a well-paid position if you follow the instructions of a certain advertisement. Such propositions possess for us a determinate *weight* which takes the place of the unknown truth-value; but while the truth-value is a property capable of only two values, the positive and the negative one, the weight is a quantity in continuous scale running from the utmost uncertainty through intermediate degrees of reliability to the highest certainty. The exact measure of the degree of reliability, or weight, is probability; but in daily life we use instead appraisals which are classified in different steps, not sharply demarcated. Words such as "unlikely," "likely," "probable," "sure," and "certain" mark these steps.

Weight, therefore, is the third predicate of propositions. It is in a certain contrast to the second predicate, truth-value, in so far as only one of these two predicates is used. If we know the truth or falsehood of a proposition, we need not apply the concepts of probability; but, if we do not know this, a weight is demanded. The determination of the weight is a substitute for verification, but an indispensable one, since we cannot renounce forming an opinion about unverified sentences. This determination is based, of course, on formerly verified sentences; but the concept of weight applies to unverified sentences. Thus in the system of propositional weights we construct a bridge from the known to the unknown. It will be one of our tasks to analyze the structure of this bridge, to look for the bridging principle which enables us to determine the degree of propositional weight and to ask for its justification. For the moment, however, we shall be content to point out that there is a weight ascribed to unverified sentences, in science as well as in daily life. To develop the theory of weight, which shall turn out to be identical with the theory of probability, is one of the aims of our inquiry. The theory of propositions as two-valued entities was constructed by philosophers in ancient times and has been called logic, while the theory of probability has been developed by mathematicians only in the last few centuries. We shall see, however, that this theory may be developed in a form analogous to logic, that a theory of propositions as entities with a degree of probability may be put by the side of the theory of propositions as two-valued entities, and that this probability logic may be considered as a generalization of ordinary logic. Although this is to be developed only in the fifth chapter of our book, we may be allowed to anticipate the result and to identify weight and probability.

An appraisal of weight is needed particularly when we

want to make use of propositions as a basis for actions. Every action presupposes a certain knowledge of future events and is therefore based on the weight of propositions which have not yet been verified. Actions—unless they be nothing but a play of muscles—are processes intentionally started by men in the pursuit of certain purposes. Of course the purpose is a matter of volitional decision and not of truth or falsity; but whether the inaugurated processes are adapted to attain the purpose is a matter of truth or falsity. This aptness of means must be known before their verification and hence can be based only on the weight of a sentence. If we want to climb up a snow-covered mountain, that of course is our personal decision; and, if anybody does not like it, he may decide against the climb. But that our feet will sink down in the snow when we step on it; that, on the contrary, planks of two meters length will carry our feet; and that we shall slide down the slopes with them almost as quickly and lightly as a bird in the air—this is to be stated in a proposition which, fortunately, possesses a high weight, if our legs are sufficiently trained. Without knowing this it would be rather imprudent to attempt a realization of our desires to get up the snowy slopes. The same situation holds for any other action, whether it concerns the most essential or the most insignificant matter in our lives. If you have to decide whether you will take a certain medicine, your decision will depend on two things: on whether you want to recover your health and on whether taking the medicine is a means appropriate to this end. If you have to decide the choice of a profession, your decision will depend on your personal desires as to shaping your life and on the question whether the profession intended will involve the satisfaction of these desires. Every action presupposes both a volitional decision and a certain kind of knowledge con-

cerning future events which cannot be furnished by a verified sentence but only by a sentence with an appraised weight.

It may be that the physical conditions involved are similar to former ones and that analogous sentences have been verified before; but the very sentence in question must concern a future event and, therefore, has not yet been verified. It may be true that every day at nine o'clock I found the train at the station and that it took me to my place of work; but, if I want to take it this morning I must know if the same will be true today. A determination of the weight, therefore, is not restricted to occasional predictions of wide bearing which cannot be based on similar antecedents; it is needed as well for the hundreds of insignificant predictions of everyday life.

In the examples given the unverified sentence concerns a future event; in such cases the weight may be considered as the predictional value of the sentence, i.e., as its value in so far as its quality as a prediction is concerned. The concept of weight, however, is not restricted to future events; it applies to past events as well and is, in so far, of a wider extension. The facts of history are not always verified, and some of them possess only a moderate weight. Whether Julius Caesar was in Britain is not certain and can only be stated with a degree of probability. The "facts" of geology and of archeology are rather doubtful as compared with facts of modern history; but even in modern history there are uncertain statements. In daily life uncertain statements concerning the past also occur and may even be important for actions. Did my friend post my letter to the bookseller yesterday so that I may expect the book to arrive tomorrow? There are friends for whom this proposition possesses a rather low weight.

This example shows a close connection between the

weights of propositions concerning past events and predictions: their weights enter into the calculations of predictional values of future events which are in causal connection with the past event. This is an important relation; it is to play a role in the logical theory of weights. We may therefore apply the name "predictional value" to the weights both of future and of past events and distinguish the two subcases as direct and indirect predictional values, if such a distinction is necessary. In this interpretation predictional value is a predicate of propositions of every type.

There is one apparent difference between truth-value and weight. Whether a sentence is true depends on the sentence alone, or rather on the facts concerned. The weight, on the contrary, is conferred upon a sentence by the state of our knowledge and may therefore vary according to a change in knowledge. That Julius Caesar was in Britain is either true or false; but the probability of our statement about this depends on what we know from historians and may be altered by further discoveries of old manuscripts. That there will be a world-war next year is either true or false; if we have only a certain probability for the proposition, this is simply due to the mediocre state of sociological prediction, and perhaps some day a more scientific sociology will give better forecasts of sociological weather. Truth-value, therefore, is an absolute predicate of propositions, and weight a relative predicate.

To summarize the results of our inquiry into the general features of language, as far as we have advanced, let us put together the following points. Language is built up of certain physical things, called symbols because they have a meaning. Meaning is a certain correspondence of these physical things to other physical things; this correspondence is established by certain rules, called the rules of lan-

guage. Symbols do not form a continuous series but are grouped in an atomistic structure: the basic elements of language are propositions. So meaning becomes a predicate of propositions. There are, in addition, two other predicates of propositions: their truth-value, i.e., their being true or false, and their predictional value or weight, i.e., a substitute for their truth-value as long as this is unknown. This triplet of predicates represents those properties of propositions on which logical inquiry is to be based.

§ 4. The language of chess as an example, and the two principles of the truth theory of meaning

We shall now illustrate our theory of language by an example. This example allows a very simple form of language and will therefore show in a very clear way the three predicates of propositions. We shall also use this example to make a further advance in the theory of the three predicates.

We take the game of chess and the well-known rules in use for the notation of the positions, pieces, and moves. This notation is based on a system of two-dimensional co-ordinates containing the letters *a*, *b*, *c*, ... , *h*, for one dimension, and the numbers 1, 2, ... , 8, for the other one; the pieces are indicated usually by the initials of their names. A set of symbols

$$Kt\ c\ 3$$

represents a sentence; it says, "There is a knight on the square of co-ordinates *c* and 3." Similarly, the set of symbols

$$Kt\ c\ 3—e\ 4$$

describes a move; it reads, "The knight is moved from the square *c* 3 to *e* 4."

Now let us raise the question of the application of the first two predicates of propositions, meaning and truth-value. The simplicity of our example permits us to discover a close connection between these two predicates: the given sentences of our language have a meaning because they are verifiable as true or false. Indeed, that we accept the set of symbols "*Kt c* 3" as a sentence is only due to the fact that we may control its truth. "*Kt c* 3" would remain a sentence in our language even were there no knight on *c* 3; it would then be a false sentence, but still a sentence. On the other hand, a group of symbols

$$Kt\ c\ g$$

would be meaningless because it cannot be determined as true or false. Therefore we would not call it a proposition; it would be a group of signs without meaning. A meaningless set of signs is to be recognized by the fact that the addition of the negation sign does not transform it into a true sentence. Let us apply the sign $\sim$ for negation; then the set

$$\sim Kt\ c\ g$$

is as meaningless as the foregoing one. A false sentence, however, is changed into a true one by adding the negation sign. So, if there is no knight on the square *c* 3, the set of symbols

$$\sim Kt\ c\ 3$$

would be a true sentence.

These reflections are of importance because they show a relation between meaning and verifiability. The concept of truth appears as the primary concept to which the concept of meaning can be reduced; a proposition has meaning be-

cause it is verifiable, and it is meaningless in case it is not verifiable.

This relation between meaning and verifiability has been pointed out by positivism and pragmatism. We will not enter for the present into a discussion of these ideas; we want to present these ideas before criticizing them. Let us call this theory the *truth theory of meaning*. We shall summarize it in the form of two principles.

First principle of the truth theory of meaning: a proposition has meaning if, and only if, it is verifiable as true or false.

By this stipulation, the two terms "having meaning" and "being verifiable" become equivalent. But, although this is a far-reaching determination of the concept of meaning, it is not a sufficient one. If we know that a proposition is verifiable, we know that it has meaning; but we do not yet know what meaning it has. This is not altered even if we know what truth-value the proposition has. The meaning of a sentence is not determined by its truth-value; i.e., the meaning is not known if the truth-value is given, nor is the meaning changed if the truth-value is changed. We need, therefore, another determination which concerns the content of meaning. This intension of a proposition is not an additional property which we must give separately; the intension is given with the proposition. But there is a formal restriction which we must add, by definition, concerning the intension, and without which the intension would not be fixed. This additional definition is performed by means of the concept of "the same meaning." All sentences have meaning; but they do not all have the same meaning. The individual separation of different meanings is achieved if we add a principle which determines the same meaning.

To introduce this concept we must alter our chess language in a certain way. Our language is as yet very rigid,

i.e., built up on very rigorous prescriptions; we shall now introduce certain mitigations. We may admit a change of the order of letters and numbers: the capital designating the piece may be put at the end; an arrow may be used instead of the dash, etc. Then we can express the same meaning by different sentences; thus the two sentences

$$Kt\ c\ 3\text{—}e\ 4$$
$$c\ 3\ Kt\text{→}4\ e$$

have the same meaning. Why do we speak here of the same meaning? A *necessary* criterion for the same meaning can easily be given: the sentences must be connected in such a way that, if any observation makes one sentence true, the other is also made true, and, if it makes one sentence false, the other is also made false. It is held by positivists that this is also a *sufficient* criterion. We formulate, therefore, the

Second principle of the truth theory of meaning: two sentences have the same meaning if they obtain the same determination as true or false by every possible observation.

Let us turn now to the question of truth. When do we call a sentence true? We demand in this case that the symbols should be in a certain correspondence to their objects; the nature of this correspondence is prescribed by the rules of language. If we examine the sentence *Kt c* 3, we look to that square which has the co-ordinates *c* and 3; and, if there is a knight at this place, the sentence is true. Verification, therefore, is an act of comparison between the objects and the symbols. It is, however, not a "naïve comparison," such as a comparison which would demand a certain similarity between objects and symbols. It is an "intellectual comparison"—a comparison in which we must apply the rules of language, understanding their

contents. We must know for this comparison that the capital denotes the piece, that the letter co-ordinate denotes the column, etc. So the comparison is, in itself, an act of thought. What it deals with, however, is not an imaginary "content" of the symbols but the symbols themselves, as physical entities. The ink marks "*Kt c* 3" stand in a certain relation to the pieces on the chessboard; therefore these marks form a true sentence. Truth, therefore, is a physical property of physical things, called symbols; it consists in a relation between these things, the symbols, and other things, the objects.

It is important that such a physical theory of truth can be given. We need not split the proposition into its "mental meaning" and into its "physical expression," as idealistic philosophers do, and attach truth to the "mental meaning" only. Truth is not a function of meaning but of the physical signs; conversely, meaning is a function of truth, as we noted before. The origin of the idealistic theory of truth may be sought in the fact that a judgment about truth presupposes thinking; but it does not concern thinking. The statement, "The proposition *a* is true," concerns a physical fact, which consists in a correspondence of the set of signs included in *a*, and certain physical objects.

Let us now ask about the third predicate of propositions within our language. We always meet predictional values when actions are in question; so they must appear when the game of chess is actually played. Indeed, the players of the game are continuously in a situation demanding the determination of a weight. They want to move their pieces in such a way as to attain a certain arrangement of the pieces on the board, called "mate"; and to reach that end they must foresee the moves of the opponent. So each player assigns weights to propositions expressing future

moves of his opponent, and it is just the quality of a good player to find good weights, i.e., to regard as likely those moves of the opponent which afterward occur. This illustration corresponds to our exposition of the concept of weight: we see that the weight becomes superfluous if a verification is attained but that it is indispensable as long as a verification is not at hand. A player who used only meaning and truth as predicates of his chess propositions would never win the game; when the unknown becomes known to him, it is too late for interference. The predictional value is the bridge between the known and the unknown; that is why it is the basis of action.

Although predictional values are used by everyone, it is very difficult to clarify how they are calculated. In this respect, the determination of the weight of a proposition differs greatly from that of truth. We showed that, for our language, truth could be defined in a relatively simple way. We cannot do the same for the weight. The weight of the future moves is not a question of the physical state of the pieces alone, but it includes considerations about the psychical states of the player. This case is too complicated, therefore, to serve as an example for the development of the theory of predictional values. As we previously said, we shall postpone this development to a later part of our inquiry. Until then, let us regard the possibility of the determination of a weight as a given fact.

§ 5. Extension of the physical theory of truth to observation propositions of ordinary language

The truth theory of meaning is based on the assumption that propositions can be verified as true or false. We developed this theory, therefore, for an example in which the question of verifiability can easily be settled. Propositions of ordinary language, however, are of many different

types, and it may be questioned, at least for some of these types, if verification is possible at all. If we want to extend the truth theory of meaning and the physical theory of truth to ordinary language, it will be reasonable to begin with a type of proposition for which verification contains no difficulties.

This rather simple type of proposition is given by sentences of the kind, "There is a table," "This steamer has two funnels," or "The thermometer indicates 15° centigrade." We shall call them *observation propositions* because they concern facts accessible to direct observation—in the current sense of this word. Later on this question will be more precisely examined; it will be shown that to speak of direct verification for these propositions presupposes a certain idealization of the actual conditions. However, it is a good method to begin with a certain approximation to the actual situation and not with the problem of knowledge in all its complexity; for the present we shall start therefore with the presupposition that for observation sentences absolute verification is possible, and we shall maintain this presupposition throughout the present chapter of our inquiry.

We begin with the question of the physical theory of truth and shall postpone the problem of meaning to the following section. This order of the inquiry is dictated by the result of the preceding section, which showed that meaning is a function of truth; so we had better begin with the question of truth.

We can indeed apply our idea that truth is a correspondence between symbols and facts established by the rules of language; but this correspondence is not always easily seen. Only to the extent to which terms occur which denote physical objects is the correspondence obvious. This is evident from the method in which such terms are

defined. For this purpose, we might imagine a "dictionary" which gives on one side the words, on the other side samples of the real things, so that this dictionary would resemble a collection of specimens, like a zoo, rather than a book. It is more difficult to establish the correspondence for logical terms such as numbers. We mentioned the example, "This steamer has two funnels." As for the terms "steamer" and "funnel," the corresponding objects will be found in our collection of specimens—but what of "two"? In such a case, we must look for the definition of the term and substitute it in place of the term. This is a rather complicated matter; but modern logic shows in principle how to perform it. We cannot enter here into a detailed description and can only summarize the method developed in textbooks of logistic. It is shown that a sentence containing "two" has to be transformed into an "existence proposition" containing the variables x and y; and, if we introduce this definition into our original sentence concerning the steamer, we shall finally find a correspondence between the funnels and these symbols y and x. So the term "two" is also reduced to a correspondence.

There is still the term "has." This is a propositional function expressing possession. Propositional functions of such a simple type may be imagined as contained in our collection of specimens. They are relations, and relations are given there by examples which represent them. So the relation "possession" might be expressed, say, by a man wearing a hat, a child holding an apple, a church having a tower, etc. This method of definition is not so stupid as it at first appears. It corresponds to the actual way in which the meaning of words is learned by a child. Children learn to talk by hearing words in immediate connection with the things or facts to which they belong; and they learn to understand the word "has" because this word is

used on such occasions as those described. Our collection of specimens corresponds to the grand zoölogical garden of life through which children are guided by their parents.

We see that the correspondence between the sentence and the fact can be established if the sentence is true. It clearly presupposes the rules of language; but it presupposes more: it requires thought. The judgment, "The sentence is true," cannot be performed without understanding the rules of language. This is necessary because any correspondence is a correspondence only with respect to certain rules. To speak of the correspondence between men's bodies and men's suits presupposes a rule of comparison; for there are many points in which suits and men differ entirely. What can be said here is that applying certain rules—in the case of this example, geometrical rules—we find a correspondence between these two kinds of objects. The same is valid for the comparison between symbols and objects, and therefore this comparison needs thought. So the physical theory of truth cannot free us from thought. What is to be thought, however, is not the original sentence *a*, but the sentence, "The sentence *a* is true." It may be admitted that this is a psychological question and that it is perhaps psychologically impossible to separate thinking of *a* and of "*a* is true"; only for a very complicated sentence *a* might this separation be possible. To get rid of this psychological puzzle, we may state our conception in the following way: a sentence of the type, "This proposition is true," concerns a physical fact, namely, a certain relation between the symbols, as physical things, and the objects, as physical things. To give an example: the proposition, "This steamer has two funnels," concerns a physical fact; the proposition *A*, reading, "The proposition, 'This steamer has two funnels,' is true," concerns another physical fact which includes the group of

signs, "This steamer has two funnels." That is why we call our theory the physical theory of truth. But this theory does not aim to make thinking superfluous; what it maintains is only that the object of a proposition stating truth is itself a physical object.

The physical theory of truth involves difficulties which can only be solved within a theory of types. One of the puzzles occurring here is the following: if the sentence *a* is true, this implies that the sentence *A*, reading, "The sentence *a* is true," is true also, and vice versa; thus *a* and *A* have the same meaning, according to the second principle of the truth theory of meaning. But the physical theory of truth distinguishes both sentences as concerning different facts. To justify this distinction we have to assume that both sentences are of different types and that the truth theory of meaning applies to sentences of equal type only. The sentence *a* cannot concern a fact comprehending the sentence *a;* that we may infer from *a* to *A* is possible only because the sentence *a* in being put before us shows itself to us and furnishes new material which may be considered in the sentence *A* of a higher level. Reflections of this kind have led Tarski[5] to the strict proof that a theory of truth cannot be given within the language concerned, but demands a language of a higher level; by this analysis some doubts[6] uttered against the physical theory of truth could be dissolved.

§ 6. Extension of the truth theory of meaning to observation propositions of ordinary language

Having shown that observation sentences of ordinary language fit in with the physical theory of truth, we shall try now to extend also the truth theory of meaning to this kind of proposition. This extension demands some preliminary analysis concerning the concepts occurring in the theory of meaning as developed.

[5] A. Tarski, "Der Wahrheitsbegriff in den formalisierten Sprachen," *Studia Philosophica* (Warsaw, 1935); cf. also *Actes du Congrès International de Philosophie Scientifique* (Paris: Hermann & Cie., 1936), Vol. III: *Langage*, containing contributions of A. Tarski and Marja Kokoszynska concerning the same subject. Another contribution of Marja Kokoszynska is to be found in *Erkenntnis*, VI (1936), 143 ff.

[6] C. G. Hempel, "On the Logical Positivist's Theory of Truth," *Analysis*, II, No. 4 (1935), 50.

We begin with the first principle. It states that meaning is tied to verifiability. We said above that we would take for granted the possibility of verification, and we shall continue to maintain this presupposition in the present section. But that is to mean only that we shall put aside objections against the term "verification"; we must, however, now analyze the term "possibility."

Before entering upon this analysis, we have to notice that the possibility which we demand does not concern the assumption in question but only the method of its verification.[7] The assumption itself may be impossible; then the verification will furnish the result that the proposition is false. This is allowable because verification has a neutral meaning for us: it signifies determination as true or false. So the proposition, "Hercules is able to bear the terrestrial globe on his shoulders," is verifiable if there is any Hercules before us raising such pretensions; although we are sure that the realization of his contention is not possible, the verification is possible and will show his contention to be false.

We must ask now what is meant by possibility of verification. The term "possibility" is ambiguous because there are different concepts of possibility; we must therefore add a definition of possibility.

First, there is the concept of *technical* possibility. This concerns facts the realization of which lies within the power of individuals or of groups of men. It is technically possible to build a bridge across the Hudson; to build a bridge across the Channel, from Calais to Dover, is perhaps already technically impossible, and it is surely technically impossible to build a bridge over the Atlantic.

Second, there is the concept of *physical* possibility. It

[7] This has been recently emphasized by Carnap, "Testability and Meaning," *Philosophy of Science*, III (1936), 420.

demands only that the fact in question be conformable to physical laws, regardless of human power. The construction of a bridge across the Atlantic is physically possible. A visit to the moon is physically possible too. But to construct a perpetual-motion machine constantly furnishing energy is physically impossible; and a visit to the sun would be physically impossible, too, because a man would be burned, together with his space ship, before reaching the sun's surface.

Third, there is the concept of *logical* possibility. It demands still less; it demands only that the fact can be imagined or, strictly speaking, that it involve no contradiction. The *perpetuum mobile* and the visit to the sun are logically possible. It would be logically impossible however, to construct a quadrangular circle, or to find a railway without rails. This third concept of possibility is the widest one; it excludes only contradictions.

Let us now apply these concepts to the question of verifiability. It must be kept in mind that these three concepts of possibility are to be applied to the method of verification and not to the fact described by the proposition.

The concept of technical possibility is usually not meant when we talk of the possibility of verification. On the contrary, it is emphasized that the postulate of verifiability leaves a greater liberty to propositions than technical possibility would allow. The statement, "Measured from the bridge across the Atlantic, the difference of the tides would be about ten meters," is taken as verifiable because such a bridge is physically possible; from this bridge we would have only to drop a plumb line to the surface of the water and could measure in this way the level of the water—which ships cannot do because they must follow the rise and fall in sea-level. We shall, therefore, reject technical possibility as a criterion for verifiability.

The concept of physical possibility furnishes a frame wide enough for statements of the given kind; but there are other statements which are excluded by it. To these belong statements concerning a very remote future. That there will be, two hundred years hence, a world similar to that of today cannot be verified by me; so this would be a meaningless proposition if we accept physical possibility for the definition of verifiability. This difficulty might be overcome by a small change in the definition of verifiability; we could content ourselves with the verification performed by any human being and renounce our playing a personal role in the process. But there are other sentences which still would be meaningless. Such would be a sentence concerning the world after the death of the last representative of mankind. Or take a sentence concerning the interior of the sun; that there are forty million degrees of heat in the sun's center cannot be verified because it is physically impossible to introduce an instrument of measurement into the sun's bulk. To this category belong also sentences concerning the atomistic structure of matter. That the electrons revolve in elliptic orbits around the kernel of the atom, that they have a spin, etc., is physically unverifiable in the strict sense of the term. Let us call *physical meaning* the concept of meaning as defined by the demand of physical possibility of verification. Then the given sentences have no physical meaning.

The concept of logical possibility is the widest of the three concepts; applying it to the definition of verifiability, we obtain the concept of *logical meaning*. All the examples given above have logical meaning. A statement about the world two hundred years hence is meaningful, then, because it is not logically impossible that I should live even then, i.e., to suppose this would be no contradiction. And to talk about the world after my death, or after the death

of the last man, is meaningful because it is not logically impossible that we should have impressions even after our death. I will not say that this concept of meaning presupposes eternal life; it makes use only of the fact that eternal life is no contradiction, and it abstains, prudently, from any presupposition that there be some chance of its being a reality. Similar reflections hold for the example of measurements in the interior of the sun. I can imagine a thermometer of considerable length put into the sun's center, and the mercury column mounting to a degree marked by the figure four with seven zeros; though I do not think that any physicist will ever attempt to construct such a thermometer, there is no logical contradiction in the conception. It contradicts the laws of physics, to be sure; but physical laws are, in the end, matters of fact and not logical necessities. As for statements concerning the structure of the atom I may imagine myself diminished to such a degree that electrons will appear to have the size of tennis balls; if anybody raised an objection to this, I would be able to answer him that such a presupposition involves no contradiction.

If we are now to make a choice between these two definitions of physical meaning and logical meaning, we must clearly keep in our mind that this is a question for a volitional decision and not a question of truth-character. It would be entirely erroneous to ask: What is the true conception of meaning? or which conception *must* I choose? Such questions would be meaningless because meaning can only be determined by a definition. What we could do would be to propose the acceptance of this decision. There are, however, two questions of truth-character connected with the decision. As we showed in § 1, these are the questions as to the decision actually used in science and as to the entailed decisions of each decision. Let us begin here

with the latter; instead of suggesting proposals we prefer the method of erecting logical signposts showing the necessary connections for every possible choice.

We see already from the given examples that both definitions of meaning suffer from grave disadvantages. The conception of physical meaning is too narrow; it excludes many sentences which science and daily life obviously accept as meaningful. The conception of logical meaning is better in this respect; but there is the opposite danger that this conception is too tolerant and may include sentences as meaningful which its adherents do not like to see endorsed within this category.

Such sentences indeed exist. The most important type are sentences including an infinity of observation sentences. Take propositions containing the word "all," referring to an infinite number of arguments; or propositions concerning the limit of the frequency in an infinite series of events, as they occur in statistics. It is no contradiction to imagine an observer of eternal life who counts such a series. But the defenders of the truth theory of meaning have a natural aversion to propositions of this type; and they justify this by insisting that such propositions have no meaning. We see that they presuppose, then, the concept of physical meaning. This concept, on the other hand, seems too narrow; we want to remain in agreement with physics and would not like to be obliged to reject such sentences as those concerning the structure of atoms, or the interior of the sun.

Our analysis, therefore, does not lead to a preference for one of the two conceptions. It leads to a "neither-nor"; or, better, to a "both." Indeed, both conceptions are of a certain value and may be used; what is to be demanded is only a clear statement, in every case, as to which of the two conceptions we have in mind.

This corresponds also to the procedure of actual science. There are many famous examples in modern physics of the application of the concept of physical meaning. Einstein's rejection of absolute simultaneity is of this kind; it is based on the impossibility of signals moving faster than light, and this, of course, is only physical impossibility. Applying instead the concept of logical meaning we can say that absolute simultaneity is meaningful because it can be imagined that there is no limit for increasing the speed of signals. The difference of these two concepts of meaning has been formulated as follows: for our world absolute simultaneity has no meaning, but for another world it might have a meaning. The qualification "for our world" expresses the acknowledgment of physical laws for the definition of the possibility of verification. In the same sense, it is impossible only for our world to observe the interior of the electron, and so propositions concerning this subject are meaningless for our world only. If such a clear terminology is used, ambiguity is avoided, and the two conceptions may both be tolerated.

Let us now proceed to an examination of the second principle of the truth theory of meaning in its application to observation sentences. This principle determines that two given sentences have the same meaning when any possible fact will lead to the same truth-value for both the sentences in question. The bearing of this determination must be considered now.

When we introduced the second principle in the example of the game of chess, the full bearing of the principle could not be recognized because the language in question was very simple and concerned only simple objects. In the language of science, however, the second principle obtains a very wide bearing. It happens frequently that certain sentences appear to have a very different meaning, whereas

later examination shows that they are verified by the same observations. An example would be the concept of motion. When we say that the body A moves toward the body B, we believe that we are stating a fact different from the case in which B moves toward A. It can be shown, however, that both sentences are verified, respectively, by the same observational facts. Einstein's famous theory of relativity can be conceived as a consequence following from the second positivistic principle of meaning. It is the function of this principle to suppress what we might call the subjective intension of meaning and, instead, to determine meaning in an objective way. It is by the addition of this principle only that the antimetaphysical attitude of positivism is completed, having been inaugurated with the first principle.

Some remarks must be added concerning the term "possibility" within the formulation of the second principle—remarks which make use of our distinctions regarding the definition of possibility.

To avoid contradictions, we use for the second principle the same definition of possibility as for the first. Thus for physical meaning the second principle is to be conceived as prescribing the same meaning to two propositions if it is not physically possible to observe facts which furnish a different verification for the two propositions in question; for logical meaning, accordingly, the equality of meaning is dependent on the logical impossibility of finding different verifications. Our example concerning the relativity of motion corresponds to physical meaning. It is physically impossible to find facts which confirm the statement, "A moves toward B," and do not confirm the statement, "B moves toward A"—this is the content of Einstein's principle of relativity. Einstein does not speak of a logical necessity here; on the contrary, he emphasizes the empirical origin of his principle, and it is just the words "physically impossible" in which this empirical origin becomes manifest. Analysis has shown that it is logically possible to imagine facts which distinguish the two sentences in question; so it is logically possible to imagine a world in which the principle of relativity does not hold.[8] The concept of absolute motion, therefore, has logical meaning. It is only for our world that it does not apply.

We do not intend to enter into a more detailed analysis of these questions here. The function of the second principle is dependent on the conception of the first one; we shall, therefore, now continue our exposition of the first principle and enter upon a necessary critique of it.

Our discussion of this principle was not satisfactory. We arrived at two definitions of meaning and showed that both could be tolerated; but our subjective feelings are in favor of one of them, namely, of that definition which demands physical possibility of verification, and which accordingly furnishes the more rigorous concept of meaning. The concept of physical meaning looks sounder than that of logical meaning, and the epistemological progress of physics in recent times is indeed due to emphasizing this conception. Einstein's purification of space-time doctrines, the elucidation of the theory of atoms by the quantum theory, and many other similar clarifications have been carried through by the use of the rigorous concept of physical meaning. The advantage of this concept lies in its healthy appeal for restricting sense to descriptions of practicable operations. We spoke of the concept of technical possibility; if this concept is rejected for the definition of verifiability, it is because it cannot be demarcated sharply and would change with the advance of the technical abilities of mankind. The domain of the technically

[8] Cf. the author's "*Philosophie der Raum-Zeit-Lehre*, § 34.

possible has as its upper limit physical possibility; in this sense, we might say that the decision to adopt physical meaning is the decision as to practicable operations. It would therefore be the aim of epistemology to build up a theory of physics in which all propositions concerning our world were justified by physical meaning and did not need to be supported by the concept of logical meaning.

This postulate is not satisfied by the considerations previously developed. We found that sentences concerning events of the remote future, or concerning the structure of the atom, presuppose logical meaning because they cannot be verified if the laws of physics hold. But though this be true, we have the feeling that such a justification by logical meaning does violence to what we really think. We do not agree that we accept a sentence about the temperature in the interior of the sun only because we can imagine a thermometer which obediently continues to perform its functions in conditions under which all other bodies are vaporized. We do not believe that physical statements concerning the structure of the atom have meaning only because we can imagine our own body diminished to atomic dimensions, watching the movement of the electrons as we watch the sun's rising. There must be something wrong in our theory of meaning; and we will try to discover what it is.

§ 7. The meaning of indirect propositions, and the two principles of the probability theory of meaning

A way out of this difficulty has been indicated by pragmatism and positivism. It consists in introducing a second type of verification, which we will call *indirect verification*. There are propositions which cannot be directly verified, but which can be reduced in a certain way to other propositions capable of direct verification. Let us call

propositions of this kind *indirect propositions;* accordingly, observation propositions may be called *direct propositions*.

Using these concepts, we construct a solution in the following way. We retain the demand of physical possibility, thus using the concept of physical meaning alone. But those propositions which turn out to be unverifiable on this definition are no longer considered as observation propositions; they change from direct propositions to indirect propositions. So they acquire an indirect meaning; and the occurrence of such propositions in physics is no longer in contradiction to the postulate of physical meaning.

Before entering into a detailed analysis of this plan, let us add a remark. The question whether or not a proposition is a direct one cannot be answered unambiguously; the answer depends on the definition of meaning. Take our proposition concerning the temperature in the interior of the sun; from the standpoint of logical meaning it is direct, from that of physical meaning it is not. The same holds for the term "observation proposition." This term seems to have a clear meaning; but we find that it depends on the definition of the possibility of observation. To observe the temperature in the interior of the sun, in the same sense as we observe the temperature of our chamber, is logically possible but not physically. So all these categories of sentences have no absolute meaning but vary with the definition of meaning.

Let us now take up the question of indirect verification. The determination of this term is suggested by the method of verification used in the practice of science. The sun's temperature is measured in a very complicated way. Physicists observe the energy contained in light rays of different colors emitted from the sun; and, comparing the obtained distribution to analogous observations on terrestrial light rays, they calculate the temperature of the sun's

surface. The regularities presupposed in this measurement are involved in the laws of radiation. After determining the temperature on the surface of the sun, physicists, by rather vague and speculative calculations, arrive finally at the number of forty million degrees for the interior of the sun; these calculations contain a number of physical observations of all kinds, especially those involved in the theory of atoms.

We find that in this way the indirect sentence is reduced to a class of direct sentences. These direct sentences concern electrical and optical instruments of measurement, thermometers, colors, etc., but all are situated on our earth in the physical laboratories, so that no visit to the sun is needed. It is true that there is such a reduction of indirect sentences to direct sentences. What we have to study is the kind of relation between the two categories.

Pragmatists and positivists have made an attempt to clarify this relation. This attempt is based on the supposition that there is an equivalence between the indirect sentence, on one side, and the class of direct sentences, on the other side. The structure of this class of direct sentences may be rather complicated; it is not simply built up in the form of a conjunction of the direct sentences, i.e., a combination by "and," but it may contain disjunctions, negations, implications, etc. This is obvious even in a simple case: for measuring the temperature of our chamber we may use a mercury thermometer, or an alcohol thermometer, etc. This "or" will be transferred into the class of direct propositions equivalent to the statement concerning the temperature of our chamber. Let us denote the aggregate of direct propositions by $[a_1, a_2, \ldots, a_n]$, the indirect proposition by A; then positivism maintains the equivalence

$$A \equiv [a_1, a_2, \ldots, a_n] \qquad (1)$$

The sign $\equiv$ denotes equality of truth value, i.e., if one side is true, the other side is true too; and if one side is false, then the other side is also false. Applying now the second principle of the truth theory of meaning, we find that the indirect proposition A has the same meaning as the class of direct propositions.

We shall call this method of determining the meaning of indirect propositions the *principle of retrogression*. According to this principle, the meaning of the indirect proposition is obtained by constructing the observation propositions from which the indirect proposition is inferred; the principle of retrogression maintains that this inference is to be interpreted as an equivalence and that the meaning of the conclusion of the inference is the same as the meaning of the premisses of the inference. The meaning of the indirect proposition is accordingly constructed by a retrogression, i.e., by a process inverse to the procedure of the scientist. The scientist advances from observation propositions to the indirect proposition; the philosopher, for the purpose of interpretation, goes backward from the indirect proposition to its premisses. This is the idea expressed by Wittgenstein in his formula: the meaning of a proposition is the method of its verification.[9] Pragmatists have, at an earlier time, expressed the same idea by calling observation propositions the "cash value" of the indirect proposition.[10]

[9] Although this formula is not verbally contained in Wittgenstein's *Tractatus logico-philosophicus* (London, 1922), it expresses his ideas very adequately and has been used, with this intention, within the "Vienna Circle."

[10] Cf. W. James, *Pragmatism* (New York, 1907), Lecture VI: "How will the truth be realized? What experiences will be different from those which would obtain if the belief were false? What, in short, is the truth's cash-value in experiential terms?" This idea goes back to the pragmatic maxim of C. S. Peirce, first pronounced in 1878: "Consider what effects, that might conceivably have practical bearings, we conceive the object of our conception to have. Then, our conception of these effects is the whole of our conception of the object" (*Collected Papers of C. S. Peirce*, V, Cambridge, Mass., 1934, 1). The logical development of the theory inaugurated by this formula is due mainly to James, Dewey, and Schiller.

This equivalence theory of indirect meaning is of seductive power on account of its simplicity and clearness. If it should hold, the theory of knowledge would acquire a very simple form: all that physics states would be a summary of observation propositions. This has been, indeed, emphasized by positivists. But this theory does not survive more rigorous criticism.

It is not true that the class of direct sentences occurring on the right of the equivalence (1) is a finite one. The equivalence sign $\equiv$ means a double implication, i.e., an implication from left to right and another implication from right to left. Hence the propositions $a_1, a_2, \ldots, a_n$, comprehend the whole series of propositions from which A can be inferred and at the same time all propositions which can be inferred from A. But this is not a finite class; or, at least, it is a practically infinite class, i.e., a class which never can be exhaustively given to human beings. Take as an example the sentence A concerning the temperature of the sun. Among $a_1, a_2, \ldots, a_n$ we have, then, observations concerning radiation of sunbeams and hot bodies, observations concerning spectral lines, etc. It is true that the class of propositions from which we start in order to infer A is a finite one, and even a practically finite one; for what we have is always a finite number of propositions. But the class of propositions which we can infer from A is not finite. We may infer from A that the temperature of a certain body, brought within a short distance r from the sun, would be T degrees; we cannot perform this experiment because we cannot leave the earth's surface. There is an infinite class of such sentences; by making r run through all possible numerical values this class would be infinite. It is therefore a grave mistake to think that the right side of (1) can ever be practically given.

This needs an additional remark. There is one case in

which the infinity of consequences drawn from A would present no difficulties: this would be so if the same consequences could be inferred from the finite set $[a_1, a_2, \ldots, a_n]$. In this case, our knowledge of the set $[a_1, a_2, \ldots, a_n]$ would enable us to assert the whole class of consequences drawn from A; there would be no surplus meaning in A, compared to the set $[a_1, a_2, \ldots, a_n]$. But this is obviously not the case in physics. For physical propositions the proposition A has a surplus meaning; and the consequences inferred from A cannot be drawn from the set $[a_1, a_2, \ldots, a_n]$. That the temperature at a distance r from the sun has a determinate value T cannot logically be inferred from $[a_1, a_2, a_3, \ldots, a_n]$; it is logically possible that future observation at a place distant r from the sun would furnish a value different from T in spite of the formerly observed set $[a_1, a_2, \ldots, a_n]$. This is due to the independence of empirical observations; there is no logical compulsion that a future observation should correspond to former ones, or to any expected result. It is because the physical statement A includes predictions for future observations that it contains a surplus meaning compared with the set $[a_1, a_2, \ldots, a_n]$; and it is the indeterminateness of the future which baffles the equivalence theory of positivism concerning indirect sentences.

The real connections are of a more complicated character. We start from a finite class of propositions $[a_1, a_2, \ldots, a_n]$; but from this class there is no logical implication to A. What we have is only a *probability implication*.[11] Let us denote the probability implication by the sign $\rightarrow$; then we have to write

$$[a_1, a_2, \ldots, a_n] \rightarrow A \qquad (2)$$

[11] As to the rules of the probability implication, see the author s *Wahrscheinlichkeitslehre* (Leiden: Sijthoff, 1935), § 9.

On the other hand, even the inferences from A to a_1, a_2, ..., a_n, are not absolutely sure; for it may happen that A is true, whereas a_1, a_2, ..., a_n are not true—although this is very improbable. So we have also a probability implication, and not a logical implication, from A to a_1, a_2, ..., a_n:

$$A \rightarrow [a_1, a_2, \ldots, a_n] \tag{3}$$

The logical equivalence is defined by the double implication; let us accordingly introduce a new term for the mutual probability implication and call it *probability connection*. Using the sign $\ominus$ for this relation, we have

$$A \ominus [a_1, a_2, \ldots, a_n] \tag{4}$$

This probability connection takes the place of the equivalence (1).

The rejection of the equivalence (1) was based on the idea that the class of observation sentences which may be co-ordinated with A is not finite. It may be asked now whether there is at least an infinite class of observation sentences such that it is equivalent to A. This question will be examined later (§§ 15–17); for the present it may be sufficient to say that, if there is such an equivalent class, it is infinite.

Now it is true that the control of an infinite set of observation sentences, one after the other, is only physically impossible, not logically impossible. Thus, if we put aside, for a moment, all other difficulties in the determination of the equivalent class and leave the discussion of these for later consideration, we might say that the admission of logical meaning would enable us to reduce an indirect sentence to an equivalent set of observation sentences. But we must realize that with this interpretation of indirect sentences most propositions of physics are endowed with meaning only because it is not logically impossible to count, term after term, an infinite series. I do not think that such reasoning will convince anyone. Nobody would take such a formal possibility into actual consideration; it is not this logical possibility which leads us to accept the indirect sentences as meaningful. Substantiating the equivalence theory of indirect sentences by reference to the logical possibility of controlling an infinite set of observations would be to destroy the connection between rational reconstruction and actual science and would annihilate the very basis of positivism and pragmatism.

This result expresses the definite failure of the truth theory of meaning. It is not possible to maintain the postulate of strict verifiability for indirect sentences; sentences of this kind are not strictly verifiable because they are not equivalent to a finite class of direct sentences. The principle of retrogression does not hold because the inference from the premises to the indirect sentence is not a tautological transformation but a probability inference. We are forced, therefore, to make a decision: either to renounce indirect sentences and consider them as meaningless or to renounce absolute verifiability as the criterion of meaning. The choice, I think, cannot be difficult, as it has been already decided by the practice of science. Science has never renounced indirect sentences; it has shown instead, the way to define meaning by means other than absolute verifiability.

This means is furnished by the predicate of weight. We showed in § 3 that, in all cases in which the truth-value of a proposition is not known, the predictional value takes the place of the truth-value. So it may perform the same function for indirect sentences. The truth theory of meaning, therefore, has to be abandoned and to be replaced by the probability theory of meaning. We formulate the

First principle of the probability theory of meaning: a proposition has meaning if it is possible to determine a weight, i.e., a degree of probability, for the proposition.

For the definition of the "possibility" occurring here we accept physical possibility. It can easily be shown that this is sufficient to grant meaning to all the examples with which we have dealt; we need not introduce logical possibility because those propositions which demanded logical possibility for obtaining meaning within the truth theory receive meaning within the probability theory as indirect propositions. This becomes obvious if we regard such examples as the statement concerning the temperature of the sun. It is physically possible to ascribe a probability to this statement. It is true that in this case we cannot determine the exact degree of probability, but this is due only to technical obstacles. We have at least an appraisal of the probability; this is shown by the fact that physicists accept the statement as fairly reliable and would never agree to statements ascribing to the sun a temperature of, say, some hundreds of degrees only. It will be our task, of course, to discuss this question of the determination of the probability in a more detailed way; and we shall do that later on. For the present, this preliminary remark may suffice.

The second principle of the truth theory of meaning is now replaced by the following one:

Second principle of the probability theory of meaning: two sentences have the same meaning if they obtain the same weight, or degree of probability, by every possible observation.

As before, the concept of possibility occurring here is the same as for the first principle; so it is once more physical possibility which we accept for our definition.

Let us call the meaning defined by these two principles *probability meaning;* the previously developed concept of

meaning may then be called *truth meaning.* By the distinction between physical and logical possibility, truth meaning bifurcates into *physical truth meaning* and *logical truth meaning.* It might be asked whether there is the same bifurcation for probability meaning. Such a distinction turns out to be superfluous because the combination of logical possibility with weight does not furnish a concept distinct from logical truth meaning; if it is logically possible to obtain a weight for a sentence, it is also logically possible to obtain a verification. Only physical reasons can exclude verification and at the same time permit the determination of a weight; if we disregard the laws of physics, we are in imagination free from physical experiments and need not distinguish the possibility of a determination of the weight and of verification. Thus logical probability meaning and logical truth meaning are identical. Probability meaning, therefore, is always physical probability meaning. We may therefore drop the addition "physical" and speak simply of probability meaning; both probability meaning and physical truth meaning may be comprehended by the name *physical meaning.*

The probability theory of meaning may be considered as an expansion of the truth theory of physical meaning in which the postulate of verifiability is taken in a wider sense, including the physical possibility of determining either the truth-value or a weight. We shall therefore include both theories under the name *verifiability theory of meaning.* The narrower sense of verification will be expressed by "absolute verification."

The justification of this expansion is given by the fact that this theory, and only this theory, corresponds to the practice of science. When a man of science speaks of the temperature of the sun, he does not take his sentences as meaningful because there is a logical possibility of direct

verification but because there is a physical possibility of inferring the temperature of the sun from terrestrial observations. The man of science also knows that this inference is not a logical inference but a probability inference. It may happen that all his premises $a_1, a_2, \ldots, a_n$ are true but that the result A of his inference is false; therefore he can maintain A only with a certain probability.

Some additional remarks must be added. We introduced the concept of "indirect proposition" to obtain meaning for sentences which had none under the presupposition of a certain definition of meaning, but which had meaning under another definition of meaning, being then observation propositions. There are, in addition, other propositions which are in no case observation propositions for any of the definitions of meaning, and which must be conceived as indirect propositions for every theory of meaning. Such are propositions concerning the development of mankind, concerning biological species, concerning the planetary system—in general, sentences the objects of which are so large, or so temporally extended, that a direct view of them is in no case possible. To these propositions belong, in addition, statements concerning abstract matters, such as the spirit of the Renaissance, the egoistic character of a certain person, and the like. All these propositions have to be treated as indirect.

For these propositions also our contention is valid that there is, in general, no logical equivalence between the general or abstract proposition and the aggregate of observation propositions on which they are based. This is obvious from the fact that we are never absolutely sure of the indirect proposition, although the basic propositions may be of the highest certainty. The facts from which we infer the egoistic character of a man may be undoubtedly certain; but that does not exclude our observing at some later

date some actions of the man which are not compatible with the hypothesis of egoism. Propositions of this kind demand the same expansion of the concept of meaning as was given before; it is only the probability theory of meaning which can do justice to them, without doing violence to the actual use of such propositions in science or in daily life. So we cannot accept the positivistic interpretation that these propositions are equivalent to a finite set of verifiable propositions; we take them as meaningful only because they possess a certain weight derived from observations.

§ 8. Discussion of the verifiability theory of meaning

We have now to consider some objections which may be raised against the verifiability theory of meaning. Since this term is to include both truth theory and probability theory of meaning, we are speaking here of objections raised against both theories in common; such a common discussion is possible because the probability theory is a continuous expansion of the truth theory of meaning.

The usual objections start from the fact that the concept of meaning is frequently used without special reference to verification. Poets talk of ancient myths, religious men of God and the heavens, scientific men of the possible origin of the world, without being interested in the question of verification. They may agree that in these cases verification lies beyond human power; but they are convinced that in spite of this their ideas at least have meaning. They even see images with the "mind's eye" and feel sure that they have a clear idea of what they intend. Is not this psychological fact a proof against the connection of meaning and verifiability?

To this we must answer that the cases considered are not of a uniform character and must be carefully classified.

There are many cases in which not the verifiability but the truth is to be denied. Stories invented by poets, and old myths, are surely not true; and just on this account they are verifiable, this term denoting only the neutral quality that a determination as true or false is possible. So these cases are not examples of a separation of meaning and verifiability. On the other hand there are cases in which the verifiability is questioned indeed, as in the case of many religious statements which their adherents frequently advance with the pretension that no human knowledge can ever verify their truth.

We are referring here mainly to religious mysticism, which in all times has exercised a great influence upon men, but whose doctrines cannot be measured in the scale of scientific truth. The utterances of religious prophets are frequently of such a kind that strangers do not understand them at all, whereas the believers are raised to the highest exaltation; or, if there is an ordinary sense in the words used, it is maintained by the adherents that this verifiable part of the doctrine is not the essential meaning—that there is a "higher" meaning which has nothing to do with verifiability.

Before entering into an analysis of this conception, we may make a general remark. If we intend to contest the right of mystics to speak of their speech as meaningful, this is not to question the relevance their utterances may have for themselves or for their auditors. It would be a naïve intellectualism to contest the moral and esthetical value which mysticism may have and actually has had in the history of the human spirit. But, if mystic utterances may have significance, this does not imply that they also have signification. Music too has an effect of the highest order on men and may be one of the best means of spiritual and moral education. But we do not speak of the meaning

of music. In this case the lack of the property "meaning" is obvious because music does not possess the external forms of language. Mystic utterances, however, show such forms; this is the reason why the emotional and educational character of such utterances may be confounded with what we call "meaning."

It is a matter of fact that language is not always used with the intention of communicating something to other persons. Language may be used for the purpose of influencing persons, of raising in them certain states of feelings which we want to have produced in them; and language may be a good instrument for this, even better sometimes than music, which if not accompanied by speech may have incomplete effects only. A good preacher may raise the feelings of devotion, penitence, contrition, or the impulse for a life according to the moral conceptions of the church by means of his sermon; and the effect of the accompanying chants may be confined to a subordinate role in comparison with his speech. A politician, by means of his speech, can force his opinion upon a meeting even in case rational reflections should refute his views. Colloquial language also is never entirely free from such a suggestive component—be it the suggestion contained in a salesman's speech to a customer, or in a teacher's speech to his pupil, or in the speech of friend to friend. But the *suggestive* function of language must be logically separated from its *communicative* function, i.e., its function of informing other persons about certain facts or relations between facts.

There is still a third function of language which must be distinguished from its communicative function. Language may release us from an inner constraint, may slacken a tensed mind—be it the oppression caused by physical or psychical pains, or the delightful tension of joy, or the nervous constraint of productive situations of a creative

mind. The *relaxive* function expresses itself in a whole range of diverse forms—the "Oh" uttered when a needle pricks our finger, a tune whistled to one's self, the verses releasing the emotional tension of a poet. This relaxive function of language is as different from the communicative function as is the suggestive function; it may show relations to the latter in assuming an autosuggestive function, such as in the case of a child's talking loudly in entering a dark chamber alone. We may combine these two functions, the suggestive and the relaxive function, in the term *emotional* functions, indicating that it is the emotional sphere which is concerned, and leaving open the possibility of adding other functions of a similar character.[12]

It is not our task here to point out why emotional functions are so well performed by the use of utterances which at the same time have a communicative character; what interests us is the question of the logical determination of the communicative function. This determination is not free from arbitrariness; but it seems to me that there are two factors indispensable for any such definition if it is to correspond to the use of speech in practical life.

The first is that a communicative function begins only when there are certain rules established for the use of the terms. We spoke of the relaxive function the word "Oh" may have for a person pricked by a needle; now imagine a person sitting in a dentist's chair and receiving the order to indicate any feeling of pain caused by the drill. The "Oh" uttered in such a case—though not losing, happily, its relaxive function—possesses at the same time a communicative function; it communicates to the dentist the fact that his drill has pierced the thin surface of the tooth's

[12] We follow, in the exposition of the different functions of language, ideas developed by Ogden, Bühler, and Carnap.

enamel. This "Oh" is a sentence endowed with meaning; it is so because it is an utterance in correspondence with the rules established by the dentist's order. It is the adaptation to certain rules which transforms an utterance with a relaxive character into one with communicative character, i.e., into a proposition (cf. also § 2).

The rules we speak of are arbitrary within wide limits; but there is one property—and this is the second essential factor we wish to indicate—which we demand if they may be called rules determining a meaning. This property is the occurrence of something such as a truth-value. For this we do not demand absolute truth; our predicate of weight is a sufficient representative of what is to be demanded here. But some such determination must occur; we must be able to assent to, or to deny, a sentence, or at least to assent to it in some degree. There never was, indeed, a theory of meaning which contradicted this postulate. Mystic utterances are set forth by their adherents with such a claim, even with pretensions of an extremely high degree of truth; for mystics talk of the absolute truth of their doctrines. This is just why they distinguish their discourse from emotional stimuli such as music. Music, though it may be suggestive, exciting, powerful, is not true, whereas the speech of a mystic pretends to be true, absolutely true.

If the verifiability theory of meaning is then questioned by philosophers who want to support mysticism, or any kind of "nonphysical" truth, it is not the predicate of truth-value which is attacked by them. What they attack, instead, is the verifiability of such propositions; they do not acknowledge that it must always be possible to determine the truth-value by observational methods. The religious man maintains his statements concerning God, the Judgment Day, etc., as true but admits that there is no

possibility of proving their truth empirically. It is, therefore, the difference between existence of the truth-value and empirical determinability of a truth-value which constitutes the subject of every discussion concerning the verifiability theory of meaning.

With this formulation the problem of the definition of meaning acquires a more definite form. We have distinguished three kinds of meaning which we called *physical truth meaning*, *probability meaning*, and *logical meaning*. Let us introduce a fourth term for the kind of meaning presupposed in religious or mystic speech; let us call it *super-empirical meaning*. The adherents of this kind of meaning, we said, do not contradict the idea that a statement is to be true or false; they do not admit, however, that the usual methods of empirical science are the only means to determine a truth-value. They oppose, therefore, super-empirical meaning to empirical meaning, combining in the latter term the three other kinds of meaning mentioned. The logical order of the four kinds of meaning may be indicated by the diagram in Figure 1; if we consider the classes of propositions admitted as meaningful by each of these definitions, their extensions form domains which include or are included in one another.

We must now analyze the question as to the choice between empirical and super-empirical meaning. This question, we must admit, cannot be raised in the form of whether we are forbidden or allowed to decide for super-empirical meaning. We have made clear that the question of meaning is not a matter of truth-character but of definition and, therefore, a volitional decision; thus a question as to our being forbidden or allowed one usage or another cannot be raised. As we pointed out in § 1, instead there are two questions of truth-character connected with the decision. They concern the decision actually used in science, and what we call the "entailed decisions." The first of these questions does not interest us at the moment; we wish to make a choice, to decide on a definition. It is therefore the second question, the question of the entailed decisions, which we have to raise; it is only in answering

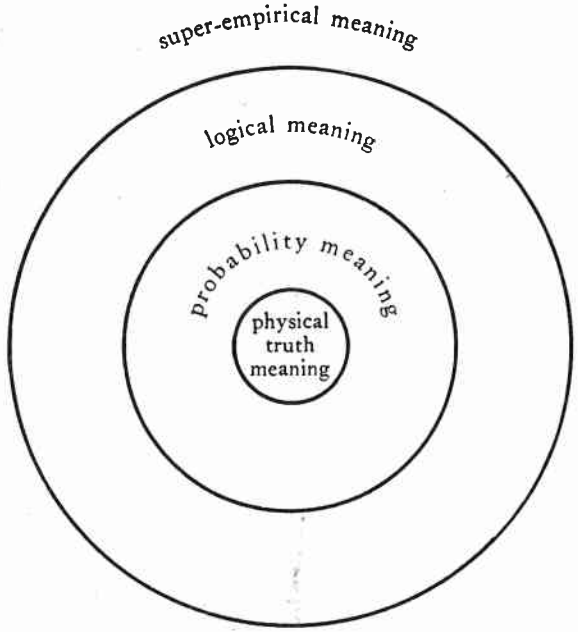

FIG. 1.—The different kinds of meaning

this question that we shall find a basis for settling the question of the connection of meaning and verifiability.

Positivists have advanced the idea that statements which have super-empirical meaning are empty; we pretend, it is said, to mean something, but we do not mean anything. I do not think that this is a clear refutation. It is difficult to convince a person that his words mean nothing; this is because the acknowledgment of this con-

tention depends on the definition of the terms "meaning something" and "meaning nothing." Under what conditions is a statement empty? If this is the case when a statement is not verifiable, then of course super-empirical meaning is empty; but how could we convince a person that he should accept this definition of emptiness? Arguments of this kind are *argumenta ad hominem;* they may persuade certain persons, but they do not clarify the problem.

The question of the entailed decisions is clear and unambiguous. It leads to an indubitable distinction between the issues relevant to the decisions in favor of empirical or super-empirical meaning.

To carry through this analysis, we must first introduce a classification of super-empirical statements. Let us group into one class all those statements for which it is maintained that we have no means at all for knowing their truth-value; into the other class we put all those statements the truth-value of which is known, but by super-empirical methods.

As to the first of these two classes, we may now indicate a property which distinguishes it from empirical statements. This property concerns the applicability of such statements for the purpose of actions. If we want to make use of a statement in the pursuit of a certain action, we must know its truth-value, or at least, its weight. We do not intend to say that statements of known truth-value are a sufficient basis of actions; we explained previously (§ 3) that an action always presupposes a volitional decision concerning an aim. But besides this fixing of the aim, we need a certain knowledge, i.e., statements with a truth-character, to attain the aim; they indicate the way of the realization. Now it is obvious that this function can only be performed by statements the truth-value or weight of

which is known. It follows that the statements of our first class of super-empirical statements never can be used as bases of actions.

Let us proceed now to the second class. It seems that for these statements an inapplicability for actions cannot be maintained. Religious belief has been historically the source of many actions, and even of actions of the greatest import. The ideas that the world is a creation of God, that God is omnipotent and omnipresent, that there is a life after death, etc., have played a great role in human history. It is admitted that empirical proofs of these statements cannot be given; but there were at all times adherents of such ideas so highly convinced of their super-empirical truth that they did not hesitate to lead wars, to kill people, or to sacrifice their own lives, when the acknowledgment of such statements demanded it.

To analyze this problem, we must first point out that not all religious statements are divested of empirical meaning. The statement of life after death involves future experiences similar to those we have in ordinary life; if we must contest its physical truth meaning, we cannot deny its logical meaning. Such statements may become bases of actions if they are supposed to be true; for, if a statement is to become a basis of actions, it is sufficient if we think it to be true. If the primitive man puts jars with food and water into the tombs of his friends, this action is correctly derived from his belief that his friends will continue to live after death. In such a case, our inquiry has to take another direction; we have to ask whether there are methods for discovering the truth-value of statements having logical meaning. The answer is given in the discussion of scientific methods; it is shown there that this is possible only if there are at least probability inferences to such statements, that is, if they belong to that part of logical

meaning which coincides with probability meaning. And if so, we cannot admit that there is a super-empirical determination of their weight different from a determination by empirical methods. For the other part, for the domain of merely logical meaning, there is no possibility of determining the truth-value, or weight; it follows that the inferences derived from such statements and leading to actions are false—that they are simply a false substantiation of actions. This does not mean that the statement is false but that the substantiation is false; the truth-value of the statement is unknown and precisely for this reason no inferences concerning actions can be deduced from it. The status of this kind of statement, therefore, is settled by reflections belonging to the discussion of science, and we may abandon further discussion here.

What is of a greater significance to us is the discussion of genuinely super-empirical statements—statements which have not even logical meaning. It is the second class of these statements, those which are considered as true, which we must now consider.

Let us ask for the relation of such statements to actions. It seems that such statements may be applied to actions; we cannot demonstrate, as for statements having logical meanings, that their truth-value must necessarily remain unknown—we cannot because they are not submitted to the methods of probability calculations. If some people believe that the cat is a divine animal, they do not claim to be able to prove this empirically; in spite of that, such a belief may determine their actions. It may, for instance, prevent them from killing cats. In this case, a super-empirical statement may become relevant for actions.

To analyze this problem, let us proceed to a closer analysis of the given example. We may first ask our cat worshiper for the reasons of his belief. He may answer that

there are some indications of divine character in cats, such as the sparkling of their eyes, but that a full proof cannot be given empirically; he knows directly, he says, about the divine character of cats because they raise in him a certain feeling of awe—in short, he feels the cat's divinity. It is this immediate knowledge which determines him never to kill a cat.

It is not our intention to dissuade our cat worshiper from his belief. What we oppose to his religious conviction is a statement of a very modest type. What he calls a divine animal, we say, may be called by us an animal which raises, in certain people, feelings of awe—in short, an "emotion-producing" animal. To his super-empirical concept "divine" we co-ordinate in this way the empirical concept "emotion-producing"; it is empirical because it is defined by the occurrence of certain psychological reactions in man, belonging to the sphere of observational facts.[13] Our co-ordinated concept is equivalent to his in the following sense: every action which he may derive from his super-empirical meaning may be derived from our co-ordinated empirical meaning as well. His principle, e.g., that divine animals must not be killed, reads with us: emotion-producing animals must not be killed.

Our opponent may object that this equivalence does not hold for him. He observed frequently, he says, that people are persuaded if someone tells them that "divine animals must not be killed"; but the profane words, "emotion-producing animals must not be killed," do not convert them. This may be true; yet it proves nothing but a special suggestive influence attached to the word "divine"—no more. We spoke above about the suggestive function of language; we see now that two propositions which logically determine

[13] We invoke here psychological facts but leave the question as to the character of psychological facts to a later investigation (cf. § 26).

the same consequences may differ as to their suggestive effects. The super-empirical meaning therefore reduces to a surplus suggestive effect; it does not lead us, however, to actions different from those determined by empirical meaning, if the volitional decisions are made in a corresponding way.

We do not forbid anyone to decide for super-empirical meaning; but he cannot rid himself of the consequence that we may co-ordinate to his propositions others of empirical meaning which have the same bearing upon our actions. The "super-empirical content" of the proposition, therefore, is not utilizable, not convertible; super-empirical propositions are like inconvertible papers which we keep in our safe without the possibility of any future realization. This is the result of our critical analysis of the different definitions of meaning, carried through by means of the question of the entailed decisions.

The expediency of this characterization may be questioned by pointing out the fact that there are many verifiable statements, and even statements known as true, which we never use as a basis of actions. This is true; it is due to the fact that our knowledge is much larger than the domain of practically useful sentences. We know that Charlemagne died in 814, or that the moon is at a distance of 238,840 miles from the earth, or that the English language has about 400,000 words; and indeed we make no practical use of this knowledge. But we might do so; and it may happen that some day we shall be placed in some situation which demands the utilization of this knowledge. With regard to Charlemagne, it might happen that a quarrel concerning an inheritance, or the right to bear a certain title, depended on the year of his death; the moon's distance will gain practical importance at the moment when space navigation is rendered practicable, and the size of

the vocabulary of the English language has its practical bearing at the moment when a complete English dictionary is to be constructed. I do not say that meaning *is* utility, or that truth *is* utility; I only say that sentences having empirical meaning *may* become useful. Neither do I say that they are true because they may become useful; I say that they may become useful because they are verifiable. It is not the definition of truth or of weight which is to be given here; these concepts are presupposed in the present discussion. It is the definition of meaning which we discuss, and the question whether this term is to be made a function of truth or of weight; we base this decision on the fact that the verifiability definition of meaning leads to a combination of meaning and utilizability, and determines meaningful propositions as those which may be used as the basis of actions.

Is this pragmatism? The answer may be determined by those who have a better knowledge of pragmatism than I have. For the theory developed here it is essential that meaning is not defined in terms of utility but in terms of truth and weight; only the argument for this choice of the definition is furnished by its relation to utilization. This relation is in itself a statement which we maintain as true; it may be seen from this that theories about the combination of meaning and utilizability presuppose the concept of truth and that truth cannot be defined by utilizability. As far as I see, pragmatists did not clarify these rather complex relations. But our conception may perhaps be taken as a further development of ideas which originated in pragmatism. It was the great merit of the founders of pragmatism to have upheld an antimetaphysical theory of meaning at a time when the logical instruments for a theory of knowledge were not yet developed to such a high degree as in our own day.

It is the advantage of our characterization of the verifiability theory of meaning that it does not prescribe the verifiability definition of meaning but that it clarifies this definition together with its entailed decisions. It is the method of the logical signpost which we apply here, leaving the decision to everyone as his personal matter. If we decide, personally, for the verifiability theory, this is because its consequences, the combination of meaning and action, appear to us so important that we do not want to miss them.

We must ask, however, whether the substantiation we give here for empirical meaning applies to each of the three kinds of meaning comprised by us in the concept of empirical meaning. In entering into this inquiry, we shall meet with remarkable results.

We have already pointed out that the domain of merely logical meaning includes propositions which can never be used for action. This is because their truth-value is not accessible to us. Thus this domain turns out to be of a kind similar in this respect to super-empirical meaning; propositions of merely logical meaning as well as super-empirical propositions are inconvertible, are not utilizable for actions.

On the other hand, if we regard physical truth meaning, we find that this definition cannot be justified by utilizability either. In § 3 we discussed the difference between truth and weight, and we showed that truth can only be determined for sentences concerning the past; whereas sentences concerning the future can be ranged only within the scale of weight, their truth-value being unknown to us. We added that this entails a preponderance of weight, in opposition to truth, as soon as the viewpoint of action is introduced; for actions are to be based on statements concerning the future. Statements concerning past events assume importance for actions only in so far as they lead to statements concerning the future, i.e., in so far as they furnish a basis for a determination of the weight of statements. The problem of these inferences to statements concerning the future embraces the problem of induction and will be analyzed later; independently of the result of such analysis, it is obvious that only sentences with an appraised weight furnish the direct basis for actions, not sentences known as true. The argument which we gave in favor of the verifiability theory of meaning—that those sentences which can furnish a basis for actions are to be regarded as meaningful—turns out, therefore, to be an argument in favor of the probability theory of meaning, and to distinguish it from the truth theory. The truth theory is too narrow; it takes as meaningful only a part of the propositions used as a basis of action, and only that part which furnishes the indirect basis, needing in each case the completion by propositions of another class, of the class of sentences with an appraised weight. It would be erroneous to say that these sentences are a possible basis for action only because they will eventually be verified as true or false; for as soon as they are so verified they are no longer a basis for action—the events described in the sentences being then passed and no longer accessible to actions. It is therefore just the predicate of weight which indicates the link between statement and action.

So our analysis leads us to ascribe a unique position to the probability theory of meaning. It is just this theory of meaning which is distinguished by the postulate of a relation between meaning and action. The line of separation in the domain of meaning, as far as it is determined by the postulate of utilizability of statements, cuts through the domain of empirical meaning; it leaves the merely logical meaning on the same side as super-empirical mean-

ing, determining both as comprehending inconvertible statements. On the other side of the line, we find both physical truth meaning and probability meaning; but the first only because it is connected with the second—only because true sentences may lead to sentences having a weight, can they serve as a basis for action. Combining, as in § 7, both physical truth meaning and probability meaning under the name of *physical meaning*, we may say that the domain of physical meaning is the utilizable domain. Therefore it is the probability theory of meaning alone which allows us to satisfy the postulate connecting meaning and utilizability.[14]

This is of importance in respect to a criticism of positivism. Positivists have defended their concept of meaning by insisting that only theirs has meaning; we found that this is an unwarranted absolutism, and that the question of the entailed decisions of the given definition of meaning had to be raised. We tried to show that there is a distinction in favor of a definition which connects meaning with verifiability, but we discover now, on a more exact consideration, that this distinction is opposed to a theory which restricts meaning to absolutely verifiable sentences

[14] Among my former publications concerning the probability theory of meaning, I may mention the following. The idea that empirical propositions are not to be conceived as two-valued entities but are to be dealt with as having a "truth-value" within a continuous scale of probability (a view which demands that they be considered within a probability logic) was first expounded by me at the first congress of "Erkenntnislehre der exakten Wissenschaften" in Prague in 1929 (cf. *Erkenntnis*, I [1930], 170–73). A continuation of these ideas was presented to the following congress, held in Königsberg in 1930 (cf. *ibid.*, II [1931], 156–71). The construction of the probability logic demanded by me has been carried through, in the form of a logistic calculus (including the theory of modalities), in my paper "Wahrscheinlichkeitslogik," *Berichte der Berliner Akademie Wissenschaften* (math.-phys. Kl. [1932]); cf. also my book *Wahrscheinlichkeitslehre*. The two principles of the probability theory of meaning given in § 7 were first formulated in "Logistic Empiricism in Germany and the Present State of Its Problems," *Journal of Philosophy*, XXXIII, No. 6 (March 12, 1936), 147–48 and 154.

—sentences verifiable as true or false only. In our search for tenable arguments in favor of the verifiability theory of meaning, we find therefore that these arguments lead to an expansion of this theory; they should incline the positivist to connect meaning with the wider concept of weight and not with the concept of truth.

Our theory of meaning may therefore be called a further development of positivism, as well as being conceived as a further development of pragmatism. This connection with positivism has a psychological foundation. It seems to me that the psychological motives which led positivists to their theory of meaning are to be sought in the connection between meaning and action and that it was the postulate of utilizability which always stood behind the positivistic theory of meaning, as well as behind the pragmatic theory, where indeed it was explicitly stated. Yet what was overlooked, at least by positivists, was the fact that no true statements concerning the future can ever be attained. This corresponds to the state of epistemology at the time of the foundation of positivism. The probability character of knowledge was not recognized; the laws of physics were taken to be strictly valid for empirical phenomena, and it was tacitly supposed that they furnish statements concerning the future which are to be taken as absolutely true. We read in the books of the older positivists that the object of science is to foresee the future and that this constitutes the very significance of science. This was said, however, without considering the fact that predicting the future presupposes inductions and that the problem of induction must be solved before a theory of meaning can be given which includes the predictive function of science. Although the problem of induction had been unfolded in all its rigor by Hume, its relevance was not seen, and a naïve absolutism concerning future-propo-

sitions was joined to the verifiability conception of meaning. But on account of this very combination, the latter conception did not lead to far-reaching restrictions of the content of science.

A more critical attitude was developed in the second phase of positivism—its critical phase. Hume's skeptical objections against induction were accepted, and the failure of any attempt to arrive at a logical solution of induction became more obvious in terms of the pretensions of precision developed in logistics. The impossibility of obtaining certain knowledge about future events was recognized, and this cognizance led, in combination with the postulate of logic as two-valued, to the repudiation of every attempt to interpret scientific propositions as forecasts of future experience. Thus resulted the modern positivistic theory, a strange combination of common-sense elements with a doctrinaire radicalism, which contradicted every unbiased view of the intentions of science. The postulate of absolute verifiability, when pronounced within science, has been mitigated by inconsequent application and therefore could do no harm; but in the hands of philosophers it was exaggerated to a radicalism which questioned the legitimacy of the very aim of science—the prevision of the future. Wittgenstein, the most radical mind among modern positivists, writes: "That the sun will rise to-morrow is a hypothesis; and that means that we do not *know* whether it will rise."[15] He does not realize that there are degrees in the domain of the unknown, such as we have expressed by the predicate of weight. Keeping strictly to the postulate of absolute verifiability, he arrives at the conclusion that nothing can be said about the future.

This does not imply for him that future propositions are meaningless; they have meaning, but their truth-value is

[15] *Op. cit.*, p. 181.

unknown. It indicates, however, that he cannot construct a connection between meaning and action. If we divest his theory of its dogmatical attire, and apply our test of the decisions entailed, we come to the following determination: for Wittgenstein a sentence is meaningful when we can wait for its verification. The stress is on the term "wait for"; we cannot actively utilize the proposition—we can only passively wait for knowledge about it. It is obvious that for this purpose his definition of meaning as verifiability is sufficient. But it is obvious also that in this way an important and healthy tendency of the older positivism has been abandoned—the tendency to combine meaning and action. The decomposing process of analysis has not been accompanied in this case by a constructive process; the possibility of basing meaning on the predicate of weight has been overlooked because a satisfactory interpretation of this predicate could not be developed. The key to a theory of meaning corresponding to the intentions of physics lies in the probability problem. It has been the fate of the positivistic doctrines that they have been driven by logical criticism into an intellectual asceticism which has suppressed all understanding of the "bridging" task of science—the task of constructing a bridge from the known to the unknown, from the past to the future. The cause for this unhealthy doctrinairism is to be found in underestimating the concept of probability. Probability is not an invention made for the sport of gamblers, or for the business of social statistics; it is the essential form of every judgment concerning the future and the representative of truth for any case where absolute truth cannot be obtained.

A further consequence of this lack of insight into the significance of the concept of probability becomes manifest in the erroneous interpretation of the relation between direct and indirect sentences. The principle of retrogres-

sion has its origin in mistaking the probability relation between these two kinds of sentences and in replacing it by an equivalence. This principle may therefore be considered as the typical expression of the too narrow logicism which characterizes this form of positivism, of the unwarranted simplification which does violence to the actual structure of science. The positivism of the radical sort cannot be considered as an interpretation of indirect sentences corresponding to the practice of physics.

The more tolerant representatives of positivism recognized this discrepancy between their theory and actual science; and so they looked for an expansion of the narrow definition of meaning previously accepted. Carnap in some recent publications[16] has developed an expansion of the criterion of the meaningful in which the idea of absolute verification is abandoned; he introduces instead the concept of "degree of confirmation," which furnishes a graduated series of propositions, and which is to apply to predictions as well as to propositions concerning past events. This "degree of confirmation" corresponds, in many respects, to our "weight"; with the difference, however, that Carnap doubts whether it is identical with "probability." It seems to me a sign of great progress that with this new theory of Carnap the development of the conceptions of the Vienna Circle turns in a direction leading to a closer connection with physics and to a better approximation to the actual state of knowledge; with this change an old difference between Carnap's conceptions and mine, which was the subject of many a discussion,[17] is considerably reduced. A discussion of Carnap's new conception must,

[16] "Wahrheit und Bewährung," *Actes du Congrès International de Philosophie Scientifique, 1935* (Paris, 1936), IV, 18; "Testability and Meaning," *Philosophy of Science*, III (1936), 420, and *ibid.*, IV (1937), 1.

[17] Cf. the discussion on the congress of Prague, 1929, reported in *Erkenntnis*, I (1930), 268–70.

however, be postponed until he has given some additional information concerning a determination of his "degree of confirmation" and the rules of operating with it. From our point of view, all these questions are answered by the theory of probability, and chapter v will present our answers in detail; but, if the interpretation in terms of probability is not accepted by Carnap, he must develop a theory of his own about degrees of confirmation. The main difficulty of such a theory will lie in the problem of the application of the degree of confirmation to actions; the problem of induction will arise for Carnap in a new form if the solution of this problem within the frame of a logic of probability, such as developed by me, is not considered as applicable to his interpretation of the "weight" of propositions.

Let us add some words concerning the second principle of the verifiability theory of meaning. As we showed, it is the logical function of this principle to cut off any surplus meaning which might be supposed in a proposition beyond its verifiable content. It performs this function in a very "polite" way: it does not forbid "metaphysical" concepts, like forces, tendencies, essences, and deities, but it states: if there is an equivalent nonmetaphysical proposition, i.e., a proposition which does not use these terms, but which has the same truth-value as the first one for all possible facts, then both propositions have the same meaning. Thus the "metaphysical" proposition is deprived of its pretended surplus meaning and reduced to an equivalent nonmetaphysical proposition. This process of cutting off metaphysical claims was first insisted upon by the nominalists of the Middle Ages. William of Ockham pronounced the principle in the form, "entia non sunt multiplicanda praeter necessitatem," and since that time "Ockham's razor" has been the program of every consequent empiri-

cism or logicism. Leibnitz' "principium identitatis indiscernibilium" and its application to the problems of space and motion, Hume's reduction of causality to an invariable succession in time, and Mach's criticism of the concept of force and of Newton's theory of space constitute examples of the application of the second principle of the verifiability theory of meaning, i.e., of Ockham's principle; in modern physics, it was above all Einstein's theory of relativity which opened to Ockham's principle a new domain of application. It is not only the relativity of motion which we must mention here; there are also many other parts of Einstein's theories, such as his conception of simultaneity and his principle of equivalence of gravitation and acceleration, which are to be conceived as an outcome of the second principle of the verifiability theory of meaning. This principle may therefore be called the very basis of an antimetaphysical attitude.

What we said about the necessary expansion of the first principle of the verifiability theory of meaning is, however, valid for the second principle as well. Our insisting on the postulate of absolute verifiability would lead us to renounce any application of the principle because there are no sentences which can be absolutely verified. If we want to be able to point out sentences which have equal meaning, we must content ourselves with showing that they obtain an equal weight by all observable facts. We need not enter into a further investigation of this, as the discussion would only repeat the arguments of the analysis of the first principle.

As to the first principle, it was the effect of the transition from the postulate of absolute verification to the postulate of determinability of a weight that the domain of physical meaning was enlarged; propositions which had no meaning for the first conception obtained meaning for

the second. Correspondingly, the same transition for the second principle implies an increase of the differences of meaning; propositions which have the same meaning within the physical truth theory of meaning may have different meaning within the probability theory of meaning. This occurs when the facts needed for the absolute verification of a proposition are not realizable for physical reasons, whereas there are facts physically possible which furnish different degrees of probability to the proposition in question. In our later investigations we shall discuss some examples of this kind (§ 14); they will show the importance which such a refinement of our logical instruments may obtain in the pursuit of the interpretation of the language of science and daily life.

If our expansion of the concept of meaning should be attacked on the ground that our wider concept of meaning might open the door to metaphysics, this would be entirely erroneous. Our theory of meaning is able to adopt Ockham's razor in a fitting form; the formulation we gave to the second principle cuts off all empty additions to sentences as well as does the formulation within the truth theory of meaning. The probability theory of meaning therefore maintains the antimetaphysical position of positivism and pragmatism, without taking over the too narrow conception of meaning from which these theories suffer if they are interpreted according to the strict wording of their programs.

Conversely, we must say that it is the probability theory of meaning alone which may give a satisfactory substantiation to the second principle of the verifiability theory of meaning. We pointed out that a substantiation of the verifiability theory of meaning consists in a relation between meaning and action; our example of the "divine animal" showed that we may co-ordinate to a given

"super-empirical" proposition an empirical one which leads to the same actions. The second principle does nothing but formulate the consequence which this idea implies for a theory of meaning based on the relation of meaning to action. We may state it in the form: if two sentences will lead us under all possible conditions to the same actions, they have the same meaning. However, this formulation is possible only within the probability theory of meaning; for only if we introduce the predicate of weight can the relation of meaning and action be demonstrated. On the other hand, it becomes obvious from this formulation that the antimetaphysical function of the principle is kept. In our formulation also the principle denies any "super-empirical meaning" and states: *there is as much meaning in a proposition as can be utilized for action.* With this formulation, the close relation of the probability theory of meaning to pragmatism becomes still more obvious; we think, though, that our theory, by using the concepts of probability and weight, may furnish a better justification of the relation between meaning and action than pragmatism is able to give. This outcome of the probability theory of meaning—the connection of meaning and action—seems to me the best guaranty of its correspondence to empirical science and to the intention of language in actual life.

CHAPTER II

IMPRESSIONS AND THE EXTERNAL WORLD