

READINGS IN PHILOSOPHICAL ANALYSIS

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Truth by Convention *

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The less a science has advanced, the more its terminology tends to rest on an uncritical assumption of mutual understanding. With increase of rigor this basis is replaced piecemeal by the introduction of definitions. The interrelationships recruited for these definitions gain the status of analytic principles; what was once regarded as a theory about the world becomes reconstrued as a convention of language. Thus it is that some flow from the theoretical to the conventional is an adjunct of progress in the logical foundations of any science. The concept of simultaneity at a distance affords a stock example of such development: in supplanting the uncritical use of this phrase by a definition, Einstein so chose the definitive relationship as to verify conventionally the previously paradoxical principle of the absoluteness of the speed of light. But whereas the physical sciences are generally recognized as capable only of incomplete evolution in this direction, and as destined to retain always a nonconventional kernel of doctrine, developments of the past few decades have led to a widespread conviction that logic and mathematics are purely analytic or conventional. It is less the purpose of the present inquiry to question the validity of this contrast than to question its sense.

I

A definition, strictly, is a convention of notational abbreviation.¹ A *simple* definition introduces some specific expression, e.g., 'kilometer', or 'e', called the *definiendum*, as arbitrary shorthand for some complex expression, e.g., 'a thousand meters' or ' $\lim_{n \rightarrow \infty} (1 + \frac{1}{n})^n$ ', called the *definiens*. A

contextual definition sets up indefinitely many mutually analogous pairs of definienda and definientia according to some general scheme; an example is the definition whereby expressions of the form ' $\frac{\sin---}{\cos---}$ ', are abbreviated as 'tan---'. From a formal standpoint the signs thus introduced are wholly

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¹ Cf. Russell, *Principles of Mathematics* (Cambridge, 1903), p. 429.

arbitrary; all that is required of a definition is that it be theoretically immaterial, i.e., that the shorthand which it introduces admit in every case of unambiguous elimination in favor of the antecedent longhand.²

Functionally a definition is not a premiss to theory, but a license for re-writing theory by putting definiens for definiendum or vice versa. By allowing such replacements a definition transmits truth: it allows true statements to be translated into new statements which are true by the same token. Given the truth of the statement 'The altitude of Kibo exceeds six thousand meters', the definition of 'kilometer' makes for the truth of the statement 'The altitude of Kibo exceeds six kilometers'; given the truth of the statement ' $\frac{\sin \pi}{\cos \pi} = \frac{\sin \pi}{\cos \pi}$ ', of which logic assures us in its earliest pages, the contextual definition cited above makes for the truth of the statement ' $\tan \pi = \frac{\sin \pi}{\cos \pi}$ '. In each case the statement inferred through the definition is true only because it is shorthand for another statement which was true independently of the definition. Considered in isolation from all doctrine, including logic, a definition is incapable of grounding the most trivial statement; even ' $\tan \pi = \frac{\sin \pi}{\cos \pi}$ ' is a definitional transformation of an antecedent self-identity, rather than a spontaneous consequence of the definition.

What is loosely called a logical consequence of definitions is therefore more exactly describable as a logical truth definitionally abbreviated: a statement which becomes a truth of logic when definienda are replaced by definienda. In this sense ' $\tan \pi = \frac{\sin \pi}{\cos \pi}$ ' is a logical consequence of the contextual definition of the tangent. 'The altitude of Kibo exceeds six kilometers' is not *ipso facto* a logical consequence of the given definition of 'kilometer'; on the other hand it would be a logical consequence of a quite suitable but unlikely definition introducing 'Kibo' as an abbreviation of the phrase 'the totality of such African terrain as exceeds six kilometers in altitude', for under this definition the statement in question is an abbreviation of a truth of logic, viz., 'The altitude of the totality of such African terrain as exceeds six kilometers in altitude exceeds six kilometers.'

Whatever may be agreed upon as the exact scope of logic, we may expect definitional abbreviations of logical truths to be reckoned as logical rather than extralogical truths. This being the case, the preceding conclusion shows logical consequences of definitions to be themselves truths of logic.

² From the present point of view a contextual definition may be recursive, but can then count among its definienda only those expressions in which the argument of recursion has a constant value, since otherwise the requirement of eliminability is violated. Such considerations are of little consequence, however, since any recursive definition can be turned into a direct one by purely logical methods. Cf. Carnap, *Logische Syntax der Sprache*, Vienna, 1934, pp. 23, 79.

To claim that mathematical truths are conventional in the sense of following logically from definitions is therefore to claim that mathematics is part of logic. The latter claim does not represent an arbitrary extension of the term 'logic' to include mathematics; agreement as to what belongs to logic and what belongs to mathematics is supposed at the outset, and it is then claimed that definitions of mathematical expressions can so be framed on the basis of logical ones that all mathematical truths become abbreviations of logical ones.

Although signs introduced by definition are formally arbitrary, more than such arbitrary notational convention is involved in questions of definability; otherwise any expression might be said to be definable on the basis of any expressions whatever. When we speak of definability, or of finding a definition for a given sign, we have in mind some traditional usage of the sign antecedent to the definition in question. To be satisfactory in this sense a definition of the sign not only must fulfill the formal requirement of unambiguous eliminability, but must also conform to the traditional usage in question. For such conformity it is necessary and sufficient that every context of the sign which was true and every context which was false under traditional usage be construed by the definition as an abbreviation of some other statement which is correspondingly true or false under the established meanings of its signs. Thus when definitions of mathematical expressions on the basis of logical ones are said to have been framed, what is meant is that definitions have been set up whereby every statement which so involves those mathematical expressions as to be recognized traditionally as true, or as false, is construed as an abbreviation of another correspondingly true or false statement which lacks those mathematical expressions and exhibits only logical expressions in their stead.³

An expression will be said to occur *vacuously* in a given statement if its replacement therein by any and every other grammatically admissible expression leaves the truth or falsehood of the statement unchanged. Thus for any statement containing some expressions vacuously there is a class of statements, describable as *vacuous variants* of the given statement, which are like it in point of truth or falsehood, like it also in point of a certain skeleton of symbolic make-up, but diverse in exhibiting all grammatically possible variations upon the vacuous constituents of the given statement. An expression will be said to occur *essentially* in a statement if it occurs in all the vacuous variants of the statement, i.e., if it forms part of the aforementioned skeleton. (Note that though an expression occur non-vacuously in a statement it may fail of essential occurrence because some of its parts occur vacuously in the statement.)

³ Note that an expression is said to be defined, in terms, e.g., of logic, not only when it is a single sign whose elimination from a context in favor of logical expressions is accomplished by a single application of one definition, but also when it is a complex expression whose elimination calls for successive application of many definitions.

Now let S be a truth, let the expressions E_i occur vacuously in S , and let the statements S_i be the vacuous variants of S . Thus the S_i will likewise be true. On the sole basis of the expressions belonging to a certain class α , let us frame a definition for one of the expressions F occurring in S outside the E_i . S and the S_i thereby become abbreviations of certain statements S' and S'_i which exhibit only members of α instead of those occurrences of F , but which remain so related that the S'_i are all the results of replacing the E_i in S' by any other grammatically admissible expressions. Now since our definition of F is supposed to conform to usage, S' and the S'_i will, like S and the S_i be uniformly true; hence the S'_i will be vacuous variants of S' , and the occurrences of the E_i in S' will be vacuous. The definition thus makes S an abbreviation of a truth S' which, like S , involves the E_i vacuously, but which differs from S in exhibiting only members of α instead of the occurrences of F outside the E_i . Now it is obvious that an expression cannot occur essentially in a statement if it occurs only within expressions which occur vacuously in the statement; consequently F , occurring in S' as it does only within the E_i if at all, does not occur essentially in S' ; members of α occur essentially in its stead. Thus if we take F as any non-member of α occurring essentially in S , and repeat the above reasoning for each such expression, we see that, through definitions of all such expressions in terms of members of α , S becomes an abbreviation of a truth S'' involving only members of α essentially.

Thus if in particular we take α as the class of all logical expressions, the above tells us that if logical definitions be framed for all non-logical expressions occurring essentially in the true statement S , S becomes an abbreviation of a truth S'' involving only logical expressions essentially. But if S'' involves only logical expressions essentially, and hence remains true when everything except that skeleton of logical expressions is changed in all grammatically possible ways, then S'' depends for its truth upon those logical constituents alone, and is thus a truth of logic. It is therefore established that if all non-logical expressions occurring essentially in a true statement S be given definitions on the basis solely of logic, then S becomes an abbreviation of a truth S'' of logic. In particular, then, if all mathematical expressions be defined in terms of logic, all truths involving only mathematical and logical expressions essentially become definitional abbreviations of truths of logic.

Now a mathematical truth, e.g., 'Smith's age plus Brown's equals Brown's age plus Smith's,' may contain non-logical, non-mathematical expressions. Still any such mathematical truth, or another whereof it is a definitional abbreviation, will consist of a skeleton of mathematical or logical expressions filled in with non-logical, non-mathematical expressions all of which occur vacuously. Thus every mathematical truth either is a truth in which only mathematical and logical expressions occur essentially, or is a definitional abbreviation of such a truth. Hence, granted definitions of all mathe-

mathematical expressions in terms of logic, the preceding conclusion shows that all mathematical truths become definitional abbreviations of truths of logic—therefore truths of logic in turn. For the thesis that mathematics is logic it is thus sufficient that all mathematical notation be defined on the basis of logical notation.

If on the other hand some mathematical expressions resist definition on the basis of logical ones, then every mathematical truth containing such recalcitrant expressions must contain them only inessentially, or be a definitional abbreviation of a truth containing such expressions only inessentially, if all mathematics is to be logic: for though a logical truth, e.g., the above one about Africa, may involve non-logical expressions, it or some other logical truth whereof it is an abbreviation must involve only logical expressions essentially. It is of this alternative that those⁴ avail themselves who regard mathematical truths, insofar as they depend upon non-logical notions, as elliptical for hypothetical statements containing as tacit hypotheses all the postulates of the branch of mathematics in question. Thus, suppose the geometrical terms 'sphere' and 'includes' to be undefined on the basis of logical expressions, and suppose all further geometrical expressions defined on the basis of logical expressions together with 'sphere' and 'includes', as with Huntington.⁵ Let Huntington's postulates for (Euclidean) geometry, and all the theorems, be expanded by thoroughgoing replacement of definienda by definientia, so that they come to contain only logical expressions and 'sphere' and 'includes', and let the conjunction of the thus expanded postulates be represented as 'Hunt (sphere, includes).' Then, where ' Φ (sphere, includes)' is any of the theorems, similarly expanded into primitive terms, the point of view under consideration is that ' Φ (sphere, includes),' insofar as it is conceived as a mathematical truth, is to be construed as an ellipsis for 'If Hunt (sphere, includes) then Φ (sphere, includes).' Since ' Φ (sphere, includes)' is a logical consequence of Huntington's postulates, the above hypothetical statement is a truth of logic; it involves the expressions 'sphere' and 'includes' inessentially, in fact vacuously, since the logical deducibility of the theorems from the postulates is independent of the meanings of 'sphere' and 'includes' and survives the replacement of those expressions by any other grammatically admissible expressions whatever. Since, granted the fitness of Huntington's postulates, all and only those geometrical statements are truths of geometry which are logical consequences in this fashion of 'Hunt (sphere, includes),' all geometry becomes logic when interpreted in the above manner as a conventional ellipsis for a body of hypothetical statements.

But if, as a truth of mathematics, ' Φ (sphere, includes)' is short for 'If

⁴ E.g. Russell, *op. cit.*, pp. 429-430; Behmann, "Sind die mathematischen Urteile analytisch oder synthetisch?" *Erkenntnis*, 4 (1934), pp. 8-10.

⁵ "A Set of Postulates for Abstract Geometry", *Mathematische Annalen*, 73 (1913), pp. 522-559.

Hunt (sphere, includes) then Φ (sphere, includes),’ still there remains, as part of this expanded statement, the original statement ‘ Φ (sphere, includes)’; this remains as a presumably true statement within some body of doctrine, say for the moment “non-mathematical geometry”, even if the title of mathematical truth be restricted to the entire hypothetical statement in question. The body of all such hypothetical statements, describable as the “theory of deduction of non-mathematical geometry,” is of course a part of logic; but the same is true of any “theory of deduction of sociology,” “theory of deduction of Greek mythology,” etc., which we might construct in parallel fashion with the aid of any set of postulates suited to sociology or to Greek mythology. The point of view toward geometry which is under consideration thus reduces merely to an exclusion of geometry from mathematics, a relegation of geometry to the status of sociology or Greek mythology; the labelling of the “theory of deduction of non-mathematical geometry” as “mathematical geometry” is a verbal *tour de force* which is equally applicable in the case of sociology or Greek mythology. To incorporate mathematics into logic by regarding all recalcitrant mathematical truths as elliptical hypothetical statements is thus in effect merely to restrict the term ‘mathematics’ to exclude those recalcitrant branches. But we are not interested in renaming. Those disciplines, geometry and the rest, which have traditionally been grouped under mathematics are the objects of the present discussion, and it is with the doctrine that mathematics in this sense is logic that we are here concerned.⁶

Discarding this alternative and returning, then, we see that if some mathematical expressions resist definition on the basis of logical ones, mathematics will reduce to logic only if, under a literal reading and without the gratuitous annexation of hypotheses, every mathematical truth contains (or is an abbreviation of one which contains) such recalcitrant expressions only inessentially if at all. But a mathematical expression sufficiently troublesome to have resisted trivial contextual definition in terms of logic can hardly be expected to occur thus idly in all its mathematical contexts. It would thus appear that for the tenability of the thesis that mathematics is logic it is not only sufficient but also necessary that all mathematical expressions be capable of definition on the basis solely of logical ones.

Though in framing logical definitions of mathematical expressions the ultimate objective be to make all mathematical truths logical truths, attention is not to be confined to mathematical and logical truths in testing the conformity of the definitions to usage. Mathematical expressions belong to the general language, and they are to be so defined that all statements containing them, whether mathematical truths, historical truths, or falsehoods under traditional usage, come to be construed as abbreviations of other statements which are correspondingly true or false. The definition intro-

⁶ Obviously the foregoing discussion has no bearing upon postulate method as such, nor upon Huntington’s work.

ducing 'plus' must be such that the mathematical truth 'Smith's age plus Brown's equals Brown's age plus Smith's' becomes an abbreviation of a logical truth, as observed earlier; but it must also be such that 'Smith's age plus Brown's age equals Jones' age' becomes an abbreviation of a statement which is empirically true or false in conformity with the county records and the traditional usage of 'plus'. A definition which fails in this latter respect is no less Pickwickian than one which fails in the former; in either case nothing is achieved beyond the transient pleasure of a verbal recreation.

But for these considerations, contextual definitions of any mathematical expressions whatever could be framed immediately in purely logical terms, on the basis of any set of postulates adequate to the branch of mathematics in question. Thus, consider again Huntington's systematization of geometry. It was remarked that, granted the fitness of Huntington's postulates, a statement will be a truth of geometry if and only if it is logically deducible from 'Hunt (sphere, includes)' without regard to the meanings of 'sphere' and 'includes'. Thus ' Φ (sphere, includes)' will be a truth of geometry if and only if the following is a truth of logic: 'If a is any class and R any relation such that Hunt (a, R), then Φ (a, R).' For 'sphere' and 'includes' we might then adopt the following contextual definition: Where '---' is any statement containing ' a ' or ' R ' or both, let the statement 'If a is any class and R any relation such that Hunt (a, R), then ---' be abbreviated as that expression which is got from '---' by putting 'sphere' for ' a ' and 'includes' for ' R ' throughout. (In the case of a compound statement involving 'sphere' and 'includes', this definition does not specify whether it is the entire statement or each of its constituent statements that is to be accounted as shorthand in the described fashion; but this ambiguity can be eliminated by stipulating that the convention apply only to whole contexts.) 'Sphere' and 'includes' thus receive contextual definition in terms exclusively of logic, for any statement containing one or both of those expressions is construed by the definition as an abbreviation of a statement containing only logical expressions (plus whatever expressions the original statement may have contained other than 'sphere' and 'includes'). The definition satisfies past usage of 'sphere' and 'includes' to the extent of verifying all truths and falsifying all falsehoods of geometry; all those statements of geometry which are true, and only those, become abbreviations of truths of logic.

The same procedure could be followed in any other branch of mathematics, with the help of a satisfactory set of postulates for the branch. Thus nothing further would appear to be wanting for the thesis that mathematics is logic. And the royal road runs beyond that thesis, for the described method of logicizing a mathematical discipline can be applied likewise to any non-mathematical theory. But the whole procedure rests on failure to conform the definitions to usage; what is logicized is not the in-

tended subject-matter. It is readily seen, e.g., that the suggested contextual definition of 'sphere' and 'includes', though transforming purely geometrical truths and falsehoods respectively into logical truths and falsehoods, transforms certain empirical truths into falsehoods and vice versa. Consider, e.g., the true statement 'A baseball is roughly a sphere,' more rigorously 'The whole of a baseball, except for a certain very thin, irregular peripheral layer, constitutes a sphere.' According to the contextual definition, this statement is an abbreviation for the following: 'If a is any class and R any relation such that Hunt (a, R), then the whole of a baseball, except for a thin peripheral layer, constitutes an [a member of] a .' This tells us that the whole of a baseball, except for a thin peripheral layer, belongs to every class a for which a relation R can be found such that Huntington's postulates are true of a and R . Now it happens that 'Hunt (a , includes)' is true not only when a is taken as the class of all spheres, but also when a is restricted to the class of spheres a foot or more in diameter;⁷ yet the whole of a baseball, except for a thin peripheral layer, can hardly be said to constitute a sphere a foot or more in diameter. The statement is therefore false, whereas the preceding statement, supposedly an abbreviation of this one, was true under ordinary usage of words. The thus logicized rendering of any other discipline can be shown in analogous fashion to yield the sort of discrepancy observed just now for geometry, provided only that the postulates of the discipline admit, like those of geometry, of alternative applications; and such multiple applicability is to be expected of any postulate set.⁸

Definition of mathematical notions on the basis of logical ones is thus a more arduous undertaking than would appear from a consideration solely of the truths and falsehoods of pure mathematics. Viewed *in vacuo*, mathematics is trivially reducible to logic through erection of postulate systems into contextual definitions; but "cette science n'a pas uniquement pour objet de contempler éternellement son propre nombril."⁹ When mathematics is recognized as capable of use, and as forming an integral part of general language, the definition of mathematical notions in terms of logic becomes a task whose completion, if theoretically possible at all, calls for mathematical genius of a high order. It was primarily to this task that Whitehead and Russell addressed themselves in their *Principia Mathematica*. They adopt a meager logical language as primitive, and on its basis alone they undertake to endow mathematical expressions with definitions which conform to usage in the full sense described above: definitions which not only reduce mathematical truths and falsehoods to logical ones, but reduce *all* statements, containing the mathematical expressions in question, to

⁷ Cf. Huntington, *op. cit.*, p. 540.

⁸ Note that a postulate set is superfluous if it *demonstrably* admits of one and only one application: for it then embodies an adequate defining property for each of its constituent primitive terms. Cf. Tarski, "Einige methodologische Untersuchungen über die Definierbarkeit der Begriffe," *Erkenntnis*, 5 (1934), p. 85 (Satz 2).

⁹ Poincaré, *Science et Méthode*, Paris, 1908, p. 199.

equivalent statements involving logical expressions instead of the mathematical ones. Within *Principia* the program has been advanced to such a point as to suggest that no fundamental difficulties stand in the way of completing the process. The foundations of arithmetic are developed in *Principia*, and therewith those branches of mathematics are accommodated which, like analysis and theory of number, spring from arithmetic. Abstract algebra proceeds readily from the relation theory of *Principia*. Only geometry remains untouched, and this field can be brought into line simply by identifying n -dimensional figures with those n -adic arithmetical relations ("equations in n variables") with which they are correlated through analytic geometry.¹⁰ Some question Whitehead and Russell's reduction of mathematics to logic,¹¹ on grounds for whose exposition and criticism there is not space; the thesis that all mathematics reduces to logic is, however, substantiated by *Principia* to a degree satisfactory to most of us. There is no need here to adopt a final stand in the matter.

If for the moment we grant that all mathematics is thus definitionally constructible from logic, then mathematics becomes true by convention in a relative sense: mathematical truths become conventional transcriptions of logical truths. Perhaps this is all that many of us mean to assert when we assert that mathematics is true by convention; at least, an *analytic* statement is commonly explained merely as one which proceeds from logic and definitions, or as one which, on replacement of definienda by definienda, becomes a truth of logic.¹² But in strictness we cannot regard mathematics as true purely by convention unless all those logical principles to which mathematics is supposed to reduce are likewise true by convention. And the doctrine that mathematics is *analytic* accomplishes a less fundamental simplification for philosophy than would at first appear, if it asserts only that mathematics is a conventional transcription of logic and not that logic is convention in turn: for if in the end we are to countenance any *a priori* principles at all which are independent of convention, we should not scruple to admit a few more, nor attribute crucial importance to conventions which serve only to diminish the number of such principles by reducing some to others.

But if we are to construe logic also as true by convention, we must rest logic ultimately upon some manner of convention other than definition: for it was noted earlier that definitions are available only for transforming truths, not for founding them. The same applies to any truths of

¹⁰ Cf. Study, *Die realistische Weltansicht und die Lehre vom Raume*, Brunswick, 1914, pp. 86-92.

¹¹ Cf. e.g., Dubislav, "Ueber das Verhältnis der Logik zur Mathematik", *Annalen der Philosophie*, 5 (1925), pp. 193-208; Hilbert, *Die Grundlagen der Mathematik*, Leipzig, 1928, pp. 12, 21.

¹² Cf. Frege, *Grundlagen der Arithmetik*, Breslau, 1884, p. 4; Behmann, *op. cit.*, p. 5. Carnap, *op. cit.*, uses the term in essentially the same sense but subject to more subtle and rigorous treatment.

mathematics which, contrary to the supposition of a moment ago, may resist definitional reduction to logic; if such truths are to proceed from convention, without merely being reduced to antecedent truths, they must proceed from conventions other than definitions. Such a second sort of convention, generating truths rather than merely transforming them, has long been recognized in the use of postulates.¹⁸ Application of this method to logic will occupy the next section; customary ways of rendering postulates and rules of inference will be departed from, however, in favor of giving the whole scheme the explicit form of linguistic convention.

II

Let us suppose an approximate maximum of definition to have been accomplished for logic, so that we are left with about as meager as possible an array of primitive notational devices. There are indefinitely many ways of framing the definitions, all conforming to the same usage of the expressions in question; apart from the objective of defining much in terms of little, choice among these ways is guided by convenience or chance. Different choices involve different sets of primitives. Let us suppose our procedure to be such as to reckon among the primitive devices the *not*-idiom, the *if*-idiom ('If . . . then . . .'), the *every*-idiom ('No matter what x may be, --- x ---'), and one or two more as required. On the basis of this much, then, all further logical notation is to be supposed defined; all statements involving any further logical notation become construed as abbreviations of statements whose logical constituents are limited to those primitives.

'Or', as a connective joining statements to form new statements, is amenable to the following contextual definition in terms of the *not*-idiom and the *if*-idiom: A pair of statements with 'or' between is an abbreviation of the statement made up successively of these ingredients: first, 'If'; second, the first statement of the pair, with 'not' inserted to govern the main verb (or, with 'it is false that' prefixed); third, 'then'; fourth, the second statement of the pair. The convention becomes clearer if we use the prefix ' \sim ' as an artificial notation for denial, thus writing ' \sim ice is hot' instead of 'Ice is not hot' or 'It is false that ice is hot.' Where '---' and '——' are any statements, our definition then introduces '--- or ——' as an abbreviation of 'If \sim --- then ——.' Again 'and', as a connective joining statements, can be defined contextually by construing '--- and ——' as an abbreviation for ' \sim if --- then \sim ——.' Every such idiom is what is known as a *truth-function*, and is characterized by the fact that the truth or falsehood of the complex statement which it generates is uniquely determined by the truth or falsehood of the several statements which it combines. All truth-functions

¹⁸ The function of postulates as conventions seems to have been first recognized by Gergonne, "Essai sur la théorie des définitions", *Annales des mathématiques pures et appliquées* (1819). His designation of them as "implicit definitions", which has had some following in the literature, is avoided here.

are known to be constructible in terms of the *not*- and *if*-idioms as in the above examples.¹⁴ On the basis of the truth-functions, then, together with our further primitives—the *every*-idiom *et al.*—all further logical devices are supposed defined.

A word may, through historical or other accidents, evoke a train of ideas bearing no relevance to the truth or falsehood of its context; in point of *meaning*, however, as distinct from connotation, a word may be said to be determined to whatever extent the truth or falsehood of its contexts is determined. Such determination of truth or falsehood may be outright, and to that extent the meaning of the word is absolutely determined; or it may be relative to the truth or falsehood of statements containing other words, and to that extent the meaning of the word is determined relatively to those other words. A definition endows a word with complete determinacy of meaning relative to other words. But the alternative is open to us, on introducing a new word, of determining its meaning *absolutely* to whatever extent we like by specifying contexts which are to be true and contexts which are to be false. In fact, we need specify only the former: for falsehood may be regarded as a derivative property depending on the word ' \sim ', in such wise that falsehood of ' $---$ ' means simply truth of ' $\sim---$.' Since all contexts of our new word are meaningless to begin with, neither true nor false, we are free to run through the list of such contexts and pick out as true such ones as we like; those selected become true by fiat, by linguistic convention. For those who would question them we have always the same answer, 'You use the word differently.' The reader may protest that our arbitrary selection of contexts as true is subject to restrictions imposed by the requirement of *consistency*—e.g., that we must not select both ' $---$ ' and ' $\sim---$ '; but this consideration, which will receive a clearer status a few pages hence, will be passed over for the moment.

Now suppose in particular that we abstract from existing usage of the locutions 'if-then', 'not' (or ' \sim '), and the rest of our logical primitives, so that for the time being these become meaningless marks, and the erstwhile statements containing them lose their status as statements and become likewise meaningless, neither true nor false; and suppose we run through all those erstwhile statements, or as many of them as we like, segregating various of them arbitrarily as true. To whatever extent we carry this process, we to that extent determine meaning for the initially meaningless marks 'if', 'then', ' \sim ', and the rest. Such contexts as we render true are true by convention.

We saw earlier that if all expressions occurring essentially in a true state-

¹⁴ Sheffer ("A Set of Five Independent Postulates for Boolean Algebras", *Trans. Amer. Math. Soc.*, 14 (1913), pp. 481-488) has shown ways of constructing these two, in turn, in terms of one; strictly, therefore, such a one should supplant the two in our ostensibly minimal set of logical primitives. Exposition will be facilitated, however, by retaining the redundancy.

ment S and not belonging to a class a are given definitions in terms solely of members of a , than S becomes a definitional abbreviation of a truth S'' involving only members of a essentially. Now let a comprise just our logical primitives, and let S be a statement which, under ordinary usage, is true and involves only logical expressions essentially. Since all logical expressions other than the primitives are defined in terms of the primitives, it then follows that S is an abbreviation of a truth S'' involving only the primitives essentially. But if one statement S is a definitional abbreviation of another S'' , the truth of S proceeds wholly from linguistic convention if the truth of S'' does so. Hence if, in the above process of arbitrarily segregating statements as true by way of endowing our logical primitives with meaning, *we assign truth to those statements which, according to ordinary usage, are true and involve only our primitives essentially*, then not only will the latter statements be true by convention, but so will all statements which are true under ordinary usage and involve only logical expressions essentially. Since, as remarked earlier, every logical truth involves (or is an abbreviation of another which involves) only logical expressions essentially, the described scheme of assigning truth makes all logic true by convention.

Not only does such assignment of truth suffice to make all those statements true by convention which are true under ordinary usage and involve only logical expressions essentially, but it serves also to make all those statements false by convention which are false under ordinary usage and involve only logical expressions essentially. This follows from our explanation of the falsehood of '---' as the truth of ' \sim ---', since '---' will be false under ordinary usage if and only if ' \sim ---' is true under ordinary usage. The described assignment of truth thus goes far toward fixing all logical expressions in point of meaning, and fixing them in conformity with usage. Still many statements containing logical expressions remain unaffected by the described assignments: all those statements which, from the standpoint of ordinary usage, involve some non-logical expressions essentially. There is hence room for supplementary conventions of one sort or another, over and above the described truth-assignments, by way of completely fixing the meanings of our primitives—and fixing them, it is to be hoped, in conformity with ordinary usage. Such supplementation need not concern us now; the described truth-assignments provide partial determinations which, as far as they go, conform to usage, and which go far enough to make all logic true by convention.

But we must not be deceived by schematism. It would appear that we sit down to a list of expressions and check off as arbitrarily true all those which, under ordinary usage, are true statements involving only our logical primitives essentially; but this picture wanes when we reflect that the number of such statements is infinite. If the convention whereby those statements are singled out as true is to be formulated in finite terms, we

must avail ourselves of conditions finite in length which determine infinite classes of expressions.¹⁵

Such conditions are ready at hand. One, determining an infinite class of expressions all of which, under ordinary usage, are true statements involving only our primitive *if*-idiom essentially, is the condition of being obtainable from

(1) 'If if p then q then if if q then r then if p then r '

by putting a statement for ' p ', a statement for ' q ', and a statement for ' r '. In more customary language the form (1) would be expanded, for clarity, in some such fashion as this: 'If it is the case that if p then q , then, if it is the case further that if q then r , then, if p , r .' The form (1) is thus seen to be the principle of the syllogism. Obviously it is true under ordinary usage for all substitutions of statements for ' p ', ' q ', and ' r '; hence such results of substitution are, under ordinary usage, true statements involving only the *if*-idiom essentially. One infinite part of our program of assigning truth to all expressions which, under ordinary usage, are true statements involving only our logical primitives essentially, is thus accomplished by the following convention:

(I) Let all results of putting a statement for ' p ', a statement for ' q ', and a statement for ' r ' in (1) be true.

Another infinite part of the program is disposed of by adding this convention:

(II) Let any expression be true which yields a truth when put for ' q ' in the result of putting a truth for ' p ' in 'If p then q .'

Given truths '---' and 'If --- then ---,' (II) yields the truth of '—'. That (II) conforms to usage, i.e., that from statements which are true under ordinary usage (II) leads only to statements which are likewise true under ordinary usage, is seen from the fact that under ordinary usage a statement '—' is always true if statements '---' and 'If --- then ---' are true. Given all the truths yielded by (I), (II) yields another infinity of truths which, like the former, are under ordinary usage truths involving only the *if*-idiom essentially. How this comes about is seen roughly as follows. The truths yielded by (I), being of the form of (1), are complex statements of the form 'If --- then ---.' The statement '---' here may in particular be of the form (1) in turn, and hence likewise be true according to (I). Then, by (II), '—' becomes true. In general '—' will not be of the form (1), hence would not have been obtainable by (I) alone. Still '—' will in every such case be a statement which, under

¹⁵ Such a condition is all that constitutes a *formal system*. Usually we assign such meanings to the signs as to construe the expressions of the class as statements, specifically true statements, theorems; but this is neither intrinsic to the system nor necessary in all cases for a useful application of the system.

ordinary usage, is true and involves only the *if*-idiom essentially; this follows from the observed conformity of (I) and (II) to usage, together with the fact that the above derivation of '—' demands nothing of '—' beyond proper structure in terms of 'if-then'. Now our stock of truths embraces not only those yielded by (I) alone, i.e., those having the form (1), but also all those thence derivable by (II) in the manner in which '—' has just now been supposed derived.¹⁶ From this increased stock we can derive yet further ones by (II), and these likewise will, under ordinary usage, be true and involve only the *if*-idiom essentially. The generation proceeds in this fashion *ad infinitum*.

When provided only with (I) as an auxiliary source of truth, (II) thus yields only truths which under ordinary usage are truths involving only the *if*-idiom essentially. When provided with further auxiliary sources of truths, however, e.g., the convention (III) which is to follow, (II) yields truths involving further locutions essentially. Indeed, the effect of (II) is not even confined to statements which, under ordinary usage, involve only logical locutions essentially; (II) also legislates regarding other statements, to the extent of specifying that no two statements '---' and 'If --- then —' can both be true unless '—' is true. But this overflow need not disturb us, since it also conforms to ordinary usage. In fact, it was remarked earlier that room remained for supplementary conventions, over and above the described truth-assignments, by way of further determining the meanings of our primitives. This overflow accomplishes just that for the *if*-idiom; it provides, with regard even to a statement 'If --- then —' which from the standpoint of ordinary usage involves non-logical expressions essentially, that the statement is not to be true if '---' is true and '—' not.

But present concern is with statements which, under ordinary usage, involve only our logical primitives essentially; by (I) and (II) we have provided for the truth of an infinite number of such statements, but by no means all. The following convention provides for the truth of another infinite set of such statements; these, in contrast to the preceding, involve not only the *if*-idiom but also the *not*-idiom essentially (under ordinary usage).

- (III) Let all results of putting a statement for '*p*' and a statement for '*q*', in 'If *p* then if $\sim p$ then *q*' or 'If if $\sim p$ then *p* then *p*,' be true.¹⁷

Statements generated thus by substitution in 'If *p* then if $\sim p$ then *q*' are statements of hypothetical form in which two mutually contradictory

¹⁶ The latter in fact comprise all and only those statements which have the form 'If if if *q* then *r* then if *p* then *r* then *s* then if if *p* then *q* then *s*'.

¹⁷ (1) and the two formulae in (III) are Łukasiewicz's three postulates for the propositional calculus.

statements occur as premisses; obviously such statements are trivially true, under ordinary usage, no matter what may figure as conclusion. Statements generated by substitution in 'If [it is the case that] if $\sim p$ then p , then p ' are likewise true under ordinary usage, for one reason as follows: Grant the hypothesis, viz., that if $\sim p$ then p ; then we must admit the conclusion, viz., that p , since even denying it we admit it. Thus all the results of substitution referred to in (III) are true under ordinary usage no matter what the substituted statements may be; hence such results of substitution are, under ordinary usage, true statements involving nothing essentially beyond the *if*-idiom and the *not*-idiom (' \sim ').

From the infinity of truths adopted in (III), together with those already at hand from (I) and (II), infinitely more truths are generated by (II). It happens, curiously enough, that (III) adds even to our stock of statements which involve only the *if*-idiom essentially (under ordinary usage); there are truths of that description which, though lacking the *not*-idiom, are reached by (I)-(III) and not by (I) and (II). This is true, e.g., of any instance of the principle of identity, say

(2) 'If time is money then time is money.'

It will be instructive to derive (2) from (I)-(III), as an illustration of the general manner in which truths are generated by those conventions. (III), to begin with, directs that we adopt these statements as true:

(3) 'If time is money then if time is not money then time is money.'

(4) 'If if time is not money then time is money then time is money.'

(I) directs that we adopt this as true:

(5) 'If if time is money then if time is not money then time is money then if if if time is not money then time is money then time is money then if time is money then time is money.'

(II) tells us that, in view of the truth of (5) and (3), this is true:

(6) 'If if if time is not money then time is money then time is money then if time is money then time is money.'

Finally (II) tells us that, in view of the truth of (6) and (4), (2) is true.

If a statement S is generated by (I)-(III), obviously only the structure of S in terms of 'if-then' and ' \sim ' was relevant to the generation; hence all those variants S_i of S which are obtainable by any grammatically admissible substitutions upon constituents of S not containing 'if', 'then', or ' \sim ', are likewise generated by (I)-(III). Now it has been observed that (I)-(III) conform to usage, i.e., generate only statements which are true under ordinary usage; hence S and all the S_i are uniformly true under ordinary usage, the S_i are therefore vacuous variants of S , and hence only

'if', 'then', and '~' occur essentially in S. Thus (I)-(III) generate only statements which under ordinary usage are truths involving only the *if*-idiom and the *not*-idiom essentially.

It can be shown also that (I)-(III) generate *all* such statements.¹⁸ Consequently (I)-(III), aided by our definitions of logical locutions in terms of our primitives, are adequate to the generation of all statements which under ordinary usage are truths which involve any of the so-called truth-functions but nothing else essentially: for it has been remarked that all the truth-functions are definable on the basis of the *if*-idiom and the *not*-idiom. All such truths thus become true by convention. They comprise all those statements which are instances of any of the principles of the so-called propositional calculus.

To (I)-(III) we may now add a further convention or two to cover another of our logical primitives—say the *every*-idiom. A little more in this direction, by way of providing for our remaining primitives, and the program is completed; all statements which under ordinary usage are truths involving only our logical primitives essentially become true by convention. Therewith, as observed earlier, all logic becomes true by convention. The conventions with which (I)-(III) are thus to be supplemented will be more complex than (I)-(III), and considerable space would be needed to present them. But there is no need to do so, for (I)-(III) provide adequate illustration of the method; the complete set of conventions would be an adaptation of one of various existing systematizations of general logic, in the same way in which (I)-(III) are an adaptation of a systematization of the propositional calculus.

Let us now consider the protest which the reader raised earlier, viz., that our freedom in assigning truth by convention is subject to restrictions imposed by the requirement of consistency.¹⁹ Under the fiction, implicit in an earlier stage of our discussion, that we check off our truths one by one in an exhaustive list of expressions, consistency in the assignment of truth is nothing more than a special case of conformity to usage.

¹⁸ The proof rests essentially upon Łukasiewicz's proof (in his *Elementy logiki matematycznej*, Warsaw, 1929) that his three postulates for the propositional calculus, viz., (I) and the formulae in (III), are *complete*. Adaptation of his result to present purposes depends upon the fact, readily established, that any formula generable by his two rules of inference (the so-called rule of substitution and a rule answering to (II)) can be generated by applying the rules in such order that all applications of the rule of substitution precede all applications of the other rule. This fact is relevant because of the manner in which the rule of substitution has been absorbed, here, into (I) and (III). The adaptation involves also two further steps, which however present no difficulty: we must make connection between Łukasiewicz's formulae, containing variables 'p', 'q', etc., and the concrete statements which constitute the present subject-matter; also between *completeness*, in the sense (Post's) in which Łukasiewicz uses the term, and the generability of all statements which under ordinary usage are truths involving only the *if*-idiom or the *not*-idiom essentially.

¹⁹ So, e.g., Poincaré, *op. cit.*, pp. 162-163, 195-198; Schlick, *Allgemeine Erkenntnislehre*, Berlin, 1925, pp. 36, 327.

If we make a mark in the margin opposite an expression '---', and another opposite ' \sim ---', we sin only against the established usage of ' \sim ' as a denial sign. Under the latter usage '---' and ' \sim ---' are not both true; in taking them both by convention as true we merely endow the sign ' \sim ', roughly speaking, with a meaning other than denial. Indeed, we might so conduct our assignments of truth as to allow no sign of our language to behave analogously to the denial locution of ordinary usage; perhaps the resulting language would be inconvenient, but conventions are often inconvenient. It is only the objective of ending up with our mother tongue that dissuades us from marking both '---' and ' \sim ---', and this objective would dissuade us also from marking 'It is always cold on Thursday.'

The requirement of consistency still retains the above status when we assign truth wholesale through general conventions such as (I)-(III). Each such convention assigns truth to an infinite sheaf of the entries in our fictive list, and in this function the conventions cannot conflict; by overlapping in their effects they reinforce one another, by not overlapping they remain indifferent to one another. If some of the conventions specified entries to which truth was *not* to be assigned, genuine conflict might be apprehended; such negative conventions, however, have not been suggested. (II) was, indeed, described earlier as specifying that 'If --- then ---' is not to be true if '---' is true and '---' not; but within the framework of the conventions of truth-assignment this apparent proscription is ineffectual without antecedent proscription of '---'. Thus any inconsistency among the general conventions will be of the sort previously considered, viz., the arbitrary adoption of both '---' and ' \sim ---' as true; and the adoption of these was seen merely to impose some meaning other than denial upon the sign ' \sim '. As theoretical restrictions upon our freedom in the conventional assignment of truth, requirements of consistency thus disappear. Preconceived usage may lead us to stack the cards, but does not enter the rules of the game.

III

Circumscription of our logical primitives in point of meaning, through conventional assignment of truth to various of their contexts, has been seen to render all logic true by convention. Then if we grant the thesis that mathematics is logic, i.e., that all mathematical truths are definitional abbreviations of logical truths, it follows that mathematics is true by convention.

If on the other hand, contrary to the thesis that mathematics is logic, some mathematical expressions resist definition in terms of logical ones, we can extend the foregoing method into the domain of these recalcitrant expressions: we can circumscribe the latter through conventional assignment of truth to various of their contexts, and thus render mathematics

conventionally true in the same fashion in which logic has been rendered so. Thus, suppose some mathematical expressions to resist logical definition, and suppose them to be reduced to as meager as possible a set of mathematical primitives. In terms of these and our logical primitives, then, all further mathematical devices are supposed defined; all statements containing the latter become abbreviations of statements containing by way of mathematical notation only the primitives. Here, as remarked earlier in the case of logic, there are alternative courses of definition and therewith alternative sets of primitives; but suppose our procedure to be such as to count 'sphere' and 'includes' among the mathematical primitives. So far we have a set of conventions, (I)-(III) and a few more, let us call them (IV)-(VII), which together circumscribe our logical primitives and yield all logic. By way of circumscribing the further primitives 'sphere' and 'includes', let us now add this convention to the set:

(VIII) Let 'Hunt (sphere, includes)' be true.

Now we saw earlier that where ' Φ (sphere, includes)' is any truth of geometry, supposed expanded into primitive terms, the statement

(7) 'If Hunt (sphere, includes) then Φ (sphere, includes)'

is a truth of logic. Hence (7) is one of the expressions to which truth is assigned by the conventions (I)-(VII). Now (II) instructs us, in view of convention (VIII) and the truth of (7), to adopt ' Φ (sphere, includes)' as true. In this way each truth of geometry is seen to be present among the statements to which truth is assigned by the conventions (I)-(VII).

We have considered four ways of construing geometry. One way consisted of straightforward definition of geometrical expressions in terms of logical ones, within the direction of development represented by *Principia Mathematica*; this way, presumably, would depend upon identification of geometry with algebra through the correlations of analytic geometry, and definition of algebraic expressions on the basis of logical ones as in *Principia Mathematica*. By way of concession to those who have fault to find with certain technical points in *Principia*, this possibility was allowed to retain a tentative status. The other three ways all made use of Huntington's postulates, but are sharply to be distinguished from one another. The first was to include geometry in logic by construing geometrical truths as elliptical for hypothetical statements bearing 'Hunt (sphere, includes)' as hypothesis; this was seen to be a mere evasion, tantamount, under its verbal disguise, to the concession that geometry is not logic after all. The next procedure was to define 'sphere' and 'includes' contextually in terms of logical expressions by construing ' Φ (sphere, includes)' in every case as an abbreviation of 'If a is any class and R any relation such that Hunt (a, R), then Φ (a, R).' This definition was condemned on the grounds that it fails to yield the intended usage of the defined terms. The last pro-

cedure finally, just now presented, renders geometry true by convention without making it part of logic. Here 'Hunt (sphere, includes)' is made true by fiat, by way of conventionally delimiting the meanings of 'sphere' and 'includes'. The truths of geometry then emerge not as truths of logic, but in parallel fashion to the truths of logic.

This last method of accommodating geometry is available also for any other branch of mathematics which may resist definitional reduction to logic. In each case we merely set up a conjunction of postulates for that branch as true by fiat, as a conventional circumscription of the meanings of the constituent primitives, and all the theorems of the branch thereby become true by convention: the convention thus newly adopted together with the conventions (I)-(VII). In this way all mathematics becomes conventionally true, not by becoming a definitional transcription of logic, but by proceeding from linguistic convention in the same way as does logic.

But the method can even be carried beyond mathematics, into the so-called empirical sciences. Having framed a maximum of definitions in the latter realm, we can circumscribe as many of our "empirical" primitives as we like by adding further conventions to the set adopted for logic and mathematics; a corresponding portion of "empirical" science then becomes conventionally true in precisely the manner observed above for geometry.

The impossibility of defining any of the "empirical" expressions in terms exclusively of logical and mathematical ones may be recognized at the outset: for if any proved to be so definable, there can be no question but that it would thenceforward be recognized as belonging to pure mathematics. On the other hand vast numbers of "empirical" expressions are of course definable on the basis of logical and mathematical ones together with other "empirical" ones. Thus 'momentum' is defined as 'mass times velocity'; 'event' may be defined as 'referent of the *later*-relation', i.e., 'whatever is later than something'; 'instant' may be defined as 'class of events no one of which is later than any other event of the class'; 'time' may be defined as 'the class of all instants'; and so on. In these examples 'momentum' is defined on the basis of mathematical expressions together with the further expressions 'mass' and 'velocity'; 'event', 'instant', and 'time' are all defined on the basis ultimately of logical expressions together with the one further expression 'later than'.

Now suppose definition to have been performed to the utmost among such non-logical, non-mathematical expressions, so that the latter are reduced to as few "empirical" primitives as possible.²⁰ All statements then

²⁰ In *Der Logische Aufbau der Welt*, Berlin, 1928, Carnap has pursued this program with such amazing success as to provide grounds for expecting all the expressions to be definable ultimately in terms of logic and mathematics plus just one "empirical" primitive, representing a certain dyadic relation described as *recollection of resemblance*. But for the present cursory considerations no such spectacular reducibility need be presupposed.

become abbreviations of statements containing nothing beyond the logical and mathematical primitives and these "empirical" ones. Here, as before, there are alternatives of definition and therewith alternative sets of primitives; but suppose our primitives to be such as to include 'later than', and consider the totality of those statements which under ordinary usage are truths involving only 'later than' and mathematical or logical expressions essentially. Examples of such statements are 'Nothing is later than itself'; 'If Pompey died later than Brutus and Brutus died later than Caesar then Pompey died later than Caesar.' All such statements will be either very general principles, like the first example, or else instances of such principles, like the second example. Now it is a simple matter to frame a small set of general statements from which all and only the statements under consideration can be derived by means of logic and mathematics. The conjunction of these few general statements can then be adopted as true by fiat, as 'Hunt (sphere, includes)' was adopted in (VIII); their adoption is a conventional circumscription of the meaning of the primitive 'later than'. Adoption of this convention renders all those statements conventionally true which under ordinary usage are truths essentially involving any logical or mathematical expressions, or 'later than', or any of the expressions which, like 'event', 'instant', and 'time', are defined on the basis of the foregoing, and inessentially involving anything else.

Now we can pick another of our "empirical" primitives, perhaps 'body' or 'mass' or 'energy', and repeat the process. We can continue in this fashion to any desired point, circumscribing one primitive after another by convention, and rendering conventionally true all statements which under ordinary usage are truths essentially involving only the locutions treated up to that point. If in disposing successively of our "empirical" primitives in the above fashion we take them up in an order roughly describable as leading from the general to the special, then as we progress we may expect to have to deal more and more with statements which are true under ordinary usage only with reservations, only with a probability recognized as short of certainty. But such reservations need not deter us from rendering a statement true by convention; so long as under ordinary usage the presumption is rather for than against the statement, our convention conforms to usage in verifying it. In thus elevating the statement from putative to conventional truth, we still retain the right to falsify the statement tomorrow if those events should be observed which would have occasioned its repudiation while it was still putative: for conventions are commonly revised when new observations show the revision to be convenient.

If in describing logic and mathematics as true by convention what is meant is that the primitives *can* be conventionally circumscribed in such fashion as to generate all and only the so-called truths of logic and mathematics, the characterization is empty; our last considerations show that the

same might be said of any other body of doctrine as well. If on the other hand it is meant merely that the speaker adopts such conventions for those fields but not for others, the characterization is uninteresting; while if it is meant that it is a general practice to adopt such conventions explicitly for those fields but not for others, the first part of the characterization is false.

Still, there is the apparent contrast between logico-mathematical truths and others that the former are *a priori*, the latter *a posteriori*; the former have "the character of an inward necessity", in Kant's phrase, the latter do not. Viewed behavioristically and without reference to a metaphysical system, this contrast retains reality as a contrast between more and less firmly accepted statements; and it obtains antecedently to any *post facto* fashioning of conventions. There are statements which we choose to surrender last, if at all, in the course of revamping our sciences in the face of new discoveries; and among these there are some which we will not surrender at all, so basic are they to our whole conceptual scheme. Among the latter are to be counted the so-called truths of logic and mathematics, regardless of what further we may have to say of their status in the course of a subsequent sophisticated philosophy. Now since these statements are destined to be maintained independently of our observations of the world, we may as well make use here of our technique of conventional truth-assignment and thereby forestall awkward metaphysical questions as to our *a priori* insight into necessary truths. On the other hand this purpose would not motivate extension of the truth-assignment process into the realm of erstwhile contingent statements. On such grounds, then, logic and mathematics may be held to be conventional while other fields are not; it may be held that it is philosophically important to circumscribe the logical and mathematical primitives by conventions of truth-assignment which yield all logical and mathematical truths, but that it is idle elaboration to carry the process further. Such a characterization of logic and mathematics is perhaps neither empty nor uninteresting nor false.

In the adoption of the very conventions (I)-(III) etc. whereby logic itself is set up, however, a difficulty remains to be faced. Each of these conventions is general, announcing the truth of every one of an infinity of statements conforming to a certain description; derivation of the truth of any specific statement from the general convention thus requires a logical inference, and this involves us in an infinite regress. E.g., in deriving (6) from (3) and (5) on the authority of (II) we *infer*, from the general announcement (II) and the specific premiss that (3) and (5) are true statements, the conclusion that

(7) (6) is to be true.

An examination of this inference will reveal the regress. For present purposes it will be simpler to rewrite (II) thus:

- (II') No matter what x may be, no matter what y may be, no matter what z may be, if x and z are true [statements] and z is the result of putting x for ' p ' and y for ' q ' in 'If p then q ' then y is to be true.

We are to take (II') as a premiss, then, and in addition the premiss that (3) and (5) are true. We may also grant it as known that (5) is the result of putting (3) for ' p ' and (6) for ' q ' in 'If p then q .' Our second premiss may thus be rendered compositely as follows:

- (8) (3) and (5) are true and (5) is the result of putting (3) for ' p ' and (6) for ' q ' in 'If p then q .'

From these two premisses we propose to infer (7). This inference is obviously sound logic; as logic, however, it involves use of (II') and others of the conventions from which logic is supposed to spring. Let us try to perform the inference on the basis of those conventions. Suppose that our convention (IV), passed over earlier, is such as to enable us to infer specific instances from statements which, like (II'), involve the *every*-idiom; i.e., suppose that (IV) entitles us in general to drop the prefix 'No matter what x [or y , etc.] may be' and simultaneously to introduce a concrete designation instead of ' x ' [or ' y ', etc.] in the sequel. By invoking (IV) three times, then, we can infer the following from (II'):

- (9) If (3) and (5) are true and (5) is the result of putting (3) for ' p ' and (6) for ' q ' in 'If p then q ' then (6) is to be true.

It remains to infer (7) from (8) and (9). But this is an inference of the kind for which (II') is needed; from the fact that

- (10) (8) and (9) are true and (9) is the result of putting (8) for ' p ' and (7) for ' q ' in 'If p then q '

we are to infer (7) with help of (II'). But the task of getting (7) from (10) and (II') is exactly analogous to our original task of getting (6) from (8) and (II'); the regress is thus under way.²¹ (Incidentally the derivation of (9) from (II') by (IV), granted just now for the sake of argument, would encounter a similar obstacle; so also the various unanalyzed steps in the derivation of (8).)

In a word, the difficulty is that if logic is to proceed *mediately* from conventions, logic is needed for inferring logic from the conventions. Alternatively, the difficulty which appears thus as a self-presupposition of doctrine can be framed as turning upon a self-presupposition of primitives. It is supposed that the *if*-idiom, the *not*-idiom, the *every*-idiom, and so on, mean nothing to us initially, and that we adopt the conventions

²¹ Cf. Lewis Carroll, "What the Tortoise Said to Achilles", *Mind*, 4, N. S. (1895), pp. 278-280.

(I)-(VII) by way of circumscribing their meaning; and the difficulty is that communication of (I)-(VII) themselves depends upon free use of those very idioms which we are attempting to circumscribe, and can succeed only if we are already conversant with the idioms. This becomes clear as soon as (I)-(VII) are rephrased in rudimentary language, after the manner of (II').²² It is important to note that this difficulty besets only the method of wholesale truth-assignment, not that of definition. It is true, e.g., that the contextual definition of 'or' presented at the beginning of the second section was communicated with the help of logical and other expressions which cannot be expected to have been endowed with meaning at the stage where logical expressions are first being introduced. But a definition has the peculiarity of being theoretically dispensable; it introduces a scheme of abbreviation, and we are free, if we like, to forego the brevity which it affords until enough primitives have been endowed with meaning, through the method of truth-assignment or otherwise, to accommodate full exposition of the definition. On the other hand the conventions of truth-assignment cannot be thus withheld until preparations are complete, because they are needed in the preparations.

If the truth-assignments were made one by one, rather than an infinite number at a time, the above difficulty would disappear; truths of logic such as (2) would simply be asserted severally by fiat, and the problem of inferring them from more general conventions would not arise. This course was seen to be closed to us, however, by the infinitude of the truths of logic.

It may still be held that the conventions (I)-(VIII) etc. are *observed* from the start, and that logic and mathematics thereby become conventional. It may be held that we can adopt conventions through behavior, without first announcing them in words; and that we can return and formulate our conventions verbally afterward, if we choose, when a full language is at our disposal. It may be held that the verbal formulation of conventions is no more a prerequisite of the adoption of the conventions than the writing of a grammar is a prerequisite of speech; that explicit exposition of conventions is merely one of many important uses of a completed language. So conceived, the conventions no longer involve us in vicious regress. Inference from general conventions is no longer demanded initially, but remains to the subsequent sophisticated stage where we frame

²² Incidentally the conventions presuppose also some further locutions, e.g., 'true' ('a true statement'), 'the result of putting . . . for . . . in . . .', and various nouns formed by displaying expressions in quotation marks. The linguistic presuppositions can of course be reduced to a minimum by careful rephrasing; (II'), e.g., can be improved to the following extent:

(II'') No matter what x may be, no matter what y may be, no matter what z may be, if x is true then if z is true then if z is the result of putting x for ' p ' in the result of putting y for ' q ' in 'If p then q ' then y is true.

This involves just the *every*-idiom, the *if*-idiom, 'is', and the further locutions mentioned above.

general statements of the conventions and show how various specific conventional truths, used all along, fit into the general conventions as thus formulated.

It must be conceded that this account accords well with what we actually do. We discourse without first phrasing the conventions; afterwards, in writings such as this, we formulate them to fit our behavior. On the other hand it is not clear wherein an adoption of the conventions, antecedently to their formulation, consists; such behavior is difficult to distinguish from that in which conventions are disregarded. When we first agree to understand 'Cambridge' as referring to Cambridge in England failing a suffix to the contrary, and then discourse accordingly, the rôle of linguistic convention is intelligible; but when a convention is incapable of being communicated until after its adoption, its rôle is not so clear. In dropping the attributes of deliberateness and explicitness from the notion of linguistic convention we risk depriving the latter of any explanatory force and reducing it to an idle label. We may wonder what one adds to the bare statement that the truths of logic and mathematics are *a priori*, or to the still barer behavioristic statement that they are firmly accepted, when he characterizes them as true by convention in such a sense.

The more restricted thesis discussed in the first section, viz. that mathematics is a conventional transcription of logic, is far from trivial; its demonstration is a highly technical undertaking and an important one, irrespectively of what its relevance may be to fundamental principles of philosophy. It is valuable to show the reducibility of any principle to another through definition of erstwhile primitives, for every such achievement reduces the number of our presuppositions and simplifies and integrates the structure of our theories. But as to the larger thesis that mathematics and logic proceed wholly from linguistic conventions, only further clarification can assure us that this asserts anything at all.