Semantics and Necessary Truth

AN INQUIRY INTO THE FOUNDATIONS

OF ANALYTIC PHILOSOPHY

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CHAPTER 3. Locke

A. The Ground of Necessary Truth: Immutable Relations between Ideas

> If, then, the perception that the same ideas will eternally have the same habitudes and relations be not a sufficient ground of knowledge, there could be no knowledge of general propositions in mathematics; for no mathematical demonstration would be other than particular: and when a man had demonstrated any proposition concerning one triangle or circle his knowledge would not reach beyond that particular diagram.
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> —*Essay Concerning Human Understanding*, Bk. IV, chap. 1, sec. 9.

As the above quotation shows, Locke held that the certainty of our knowledge of universal propositions is due to our (intellectual) "perception" of eternally fixed relations between the constituent concepts.[1] We can be sure that any particular which exemplifies C_1 also exemplifies C_2 because we know that C_1RC_2, says Locke. What is this R? Presumably it is entailment, although Locke speaks vaguely of "agreement" between two ideas (which may be intuitively or discursively perceived) and so uses this vague word that compatibility is a kind of "agreement": for, if R above is the latter relation, then we could hardly infer that x exemplifies C_2 from the fact that x exemplifies C_1! But how can we perceive that C_1 will at all times have R to C_2? Locke must have meant that we feel certain of this invariance, but then the very way in which he accounted for the certainty of universal propositions such as those of mathematics could invite a Cartesian skeptic to ask: how do you know that relations between concepts are immutable? Is not this itself a universal proposition, that C_1RC_2 *at all times?*

Actually the only way of answering such a piece of skepticism is

[1] Locke's usage of "idea" in the *Essay* is, of course, notoriously ambiguous. He verbally disagrees with the realists who hold that there are universals, extramental entities, *in rebus*, but it is clear that when he speaks of relations between ideas, such as incompatibility and "coexistence," he is speaking not of mental images but of universals or properties. I shall therefore employ the modern terminology of "properties" or "concepts" (in what Carnap calls the "objective" sense), which is less ambiguous.

to point out that a sentence of the form "C_1 has R to C_2 at time t," or "C_1 has R to C_2 at all times" does not make sense; logical relations do not hold between temporal entities (particulars), hence it does not make sense to say that they hold at some time or at all times between given terms (cf. below, pp. 121, 186-7, 413). However, a more serious shortcoming of Locke's explanation of the possibility of certain knowledge of universal propositions is the following. He attempts to deduce universal propositions about particulars, specifically those of the form "all A are B," from propositions about relations between universals, yet he never fully clarifies just what those relations are. One of the types of "agreement" between universals that he enumerates is "coexistence," illustrated by the proposition "all gold is fixed." But the statement, verbally about universals, "being gold coexists with being fixed" can only mean that anything which has the first property also has the second property, hence it is obscure why coexistence should be counted as a relation between universals. It is not a relation between universals in the sense in which diversity, one of the forms of "disagreement" betwen ideas, is a relation between universals: "squareness is distinct from roundness" is not translatable into the statement about particulars "whatever is square is not round," for the same rule of translation would lead us from the truth "squareness is distinct from whiteness" to the falsehood "whatever is square is not white."

In fact, the only kinds of "agreement" between distinct ideas mentioned by Locke which could be interpreted as relations between universals are *entailment* and *compatibility*. The latter relation may be disregarded in this context, because Locke refers to perceptions of relations between ideas as the source of knowledge of *universal* propositions, and surely a proposition of the form "C_1 is compatible with C_2" cannot warrant a universal proposition about particulars. His view, then, must have been that it is because we perceive, e. g., that triangularity entails a sum of angles equal to 180 degrees, that we may predict with absolute certainty that any existent triangle that might ever be found has the latter property (Q). But, as he well knew, triangularity does not entail Q in the sense in which, say, squareness entails foursidedness. We cannot by mere analysis of the concept "triangle" discover Q but have to presuppose geometrical axioms (of which we have, according to Locke, intuitive knowledge). And one of those axioms is a famous assertion of *existence*, viz. that for any straight line S and a point P outside of S there exists exactly one straight line passing through P and parallel to S. How, then, could Locke maintain that the necessary connection between triangularity and Q is knowable by mere reflection upon "ideas" without concerning oneself with questions of existence? The obscurity of Locke's conception of "perception of agreement" between ideas as the source of a priori knowledge of universal propositions will be felt further if we try to apply it to Euclid's axiom "equals added to equals yield equals." This axiom asserts: for any magnitudes a, b, c, d, if $a = b$ and $c = d$, then $a + c = b + d$. Here the antecedent may be said to entail the consequent, but surely the terms of the relation of entailment are not universals or properties; they are rather propositional functions. Locke was apparently thinking of the subject and predicate of Aristotelian A-propositions when he wrote of "agreement between two ideas," and hence it is no surprise that his theory of a priori knowledge becomes unintelligible when it is applied to propositions, such as the above, which are obviously not of subject-predicate form.

Unlike Kant and Leibniz, Locke nowhere speaks of "necessary" truths (or propositions) in contrast to "contingent" ones, but instead contrasts *certainty* of knowledge with mere probability—and following tradition reserves the honorific term "knowledge" for certain knowledge. It is, of course, important to distinguish the propositions "it is known with certainty that p" and "p is a necessary truth." For example, if Fermat's theorem, that famous undecided proposition of algebra, is true, then it is a necessary truth; yet mathematicians so far can only conjecture that it is true, they do not know for certain. Again, "being known with certainty" is a time-dependent predicate, in the sense that a proposition may not be known with certainty at one time but known with certainty at another time, while it would not make sense to ask *at what time* a given proposition is a necessary truth.[2] However, it seems that Locke was aware, though none too clearly, of this distinction, for he allows that there may objectively exist an "agreement" between two ideas although we do not perceive it:

> it is fit to observe that certainty is twofold; certainty of truth, and certainty of knowledge. Certainty of truth is, when words

[2] See below, p. 121.

> are so put together in propositions as exactly to express the agreement or disagreement of the ideas they stand for, as really it is. Certainty of knowledge is, to perceive the agreement or disagreement of ideas, as expressed in any proposition [Bk. IV, chap. 6, sec. 3].

Thus, with reference to Fermat's theorem, " $n > 2$ " may *really entail* " there is no solution for the equation: $x^n + y^n = z^n$," though we do not perceive the entailment. By " certain truth," then, Locke meant in this context nothing else than " necessary truth." And his theory of necessary truth [3] might be reconstructed as follows: a necessary truth is a true proposition of the form " xRy," where x and y are universals (" concepts " in the objective sense) or propositional functions, and R is one of the following relations: entailment, incompatibility, compatibility, identity, difference; further any proposition about particulars which follows from such a proposition (as " no squares are circles " follows from " squareness is incompatible with circularity ") is a necessary truth. By including functions in the range of x and y [4] we make it possible to apply Locke's vague concept of " agreement (or disagreement) of *ideas* " to propositions which do not have subject-predicate form, like the cited axiom of Euclid and Fermat's theorem. If R is identity or difference, then, according to Locke, we have *intuitive* knowledge of the truth of xRy—and this, he says, is the summit of certainty (and, we might add, of triviality) ; but if R is entailment or incompatibility, xRy may be but indirectly known, by inference from propositions that are intuitively known—this is *demonstrative* knowledge, which is less certain than intuitive knowledge.

What Locke called " intuitive knowledge of identity and diversity (of ideas) " raises some interesting questions. Although the examples

[3] The theory (or " explanation ") of a priori knowledge which Russell offered in his lucid *The Problems of Philosophy* (London, 1912), ch. 10, seems to be essentially the same as Locke's theory, though Russell was not aware of it (indeed, the only reference to Locke is a reference to him as one of those " empiricists " who held that " all knowledge is derived from experience ") .

[4] The propositional functions to be substituted may be quite complex, e. g. they may be conjunctions of propositional functions. Thus the parallel axiom, which Locke must have regarded as intuitively evident, can be pressed into the form " x entails y " only if for " x " we substitute " x is a straight line and y is a point outside of x " and for " y " we substitute " there is exactly one straight line passing through y and parallel to x."

of pairs of "diverse" ideas he gave were pairs of *incompatible* ideas (whiteness and blackness, triangularity and circularity), he could not have failed to distinguish mere difference from incompatibility: whiteness is distinct from roundness, yet the same particular may exemplify both universals. The difference between these two relations is reflected also by the fact that a statement of incompatibility of universals entails a universal proposition about particulars while a statement of mere difference does not: "whiteness is incompatible with blackness" entails "no surface is simultaneously both black and white all over," but "whiteness is different from roundness" surely does not entail "nothing is both white and round." Now, many contemporary empiricists would hold that a nominalistic language is in principle adequate for the expression of all genuine knowledge, though the translation of intensional statements into such a language may present practical difficulties.[5] By a nominalistic language I mean a language containing no other names than names of particulars and only individual variables (and accordingly corresponding to the language structure of the lower functional calculus enriched with names of particulars and predicates applicable to particulars); and by an intensional statement I mean a statement containing names of, or variables ranging over, such abstract entities as universals or propositions, like "roundness," "that the sun is larger than the moon" (the latter expression is not a sentence but a description of a state of affairs, occurring in such nonextensional contexts as "he believes that the sun is larger than the moon"). But let us see now whether intensional statements—"roundness is different from whiteness," "roundness is incompatible with squareness"—are even in principle translatable into such a language. For the latter statement one might propose the translation "the sentence 'for any x, if x is round then x is not square' is necessary." If it should turn out, indeed, that a metalinguistic statement of the form "S is necessary" (where S is the name of a sentence) can only mean "the proposition expressed by S is necessary," then the success of the translation would be merely deception. But as reasons against the translatability of modal statements into an extensional metalanguage will be presented later,[6] the point will not be pressed now;

[5] See N. Goodman and W. V. Quine, "Steps towards a Constructive Nominalism," *Journal of Symbolic Logic, 12* (1947).

[6] Cf. below, pp. 121, 181. 195 f.

instead we ask how such a translation would fit into Locke's theory of necessary truth. It should be obvious that it just could not be fitted in. For Locke explains the necessary truth of the universal proposition "nothing which is round is square" with reference to the truth of the intensional statement "roundness is incompatible with squareness." The perception of the incompatibility of the attributes is, acording to him, the "ground" of our certain knowledge of the proposition expressed by the extensional statement.

But a "nominalist" will have even greater difficulty with the translation of "whiteness is different from roundness" into his favored language. Surely, neither "nothing which is white is round" nor "some things which are white are not round" would be adequate: some white things *are* round, and it is not self-contradictory to suppose that all white things are round even though whiteness and roundness are distinct attributes. Presumably he would again try a translation into the metalanguage: "white" is not synonymous with "round." But thus translated, the statement would express a *contingent* truth: it is clearly conceivable that these two predicates should be used synonymously, even though in fact they are not so used. Furthermore, on this interpretation the intensional statement in question would be specifically a statement about the English language whose correct translation into, say, German would be, not "das Weisse ist verschieden von dem Runden" but "'white' ist nicht gleichbedeutend mit 'round.'"[7] Again, the law of identity for universals, "every universal (attribute) is self-identical," would on this interpretation turn into the empirical generalization, which is surely false, that any two tokens of the same predicate have the same meaning. And consistency would compel a similar interpretation of the law of identity for individuals. Thus Leibniz' criterion *par excellence* of necessary truth would cease to be a necessary truth and turn into a contingent falsehood!

It should be evident, then, that if knowledge of necessary truths is considered "genuine knowledge," then Locke's theory of necessary truth is incompatible with the nominalists' claim that all genuine knowledge is in principle expressible in a nominalistic language. For the intensional statements which according to Locke are the ground of the necessity of extensional statements (specifically

[7] The whole problem of identity of attributes, or of synonymy, here touched upon, will receive detailed discussion in Chap. 10.

of universal statements about particulars) do not seem to be adequately translatable into such a language. And before rejecting Locke's theory for that very reason, a nominalist would do well to re-examine his faith in the adequacy of a nominalistic language. Consider, for example, the simple intensional statement "B_2 is darker than B_1," where "B_2" and "B_1" are names of distinct shades of blue and thus names of universals. We might translate this statement into a universal statement about particulars by constructing the relational predicate of the first level "being-darker-in-color-than": for any x and y, if x is an instance of B_2 and y an instance of B_1, then x is darker-in-color than y. But suppose we were asked why we are so sure that there are no exceptions to this generalization. Would it not be plausible to reply, with Locke, that we are so confident because we perceive that the *universals* B_2 and B_1 stand in the specified relation and *must* stand in that relation as long as they remain self-identical? [8] And are we not thus forced back into the intensional language?

B. *The Contingent Universality of Laws of Nature*

One of the main themes running through Locke's *Essay* is that our knowledge of nature is severely limited, since we do not see any necessary connections between different secondary qualities, or between secondary qualities and powers, or between the primary qualities of the "insensible particles" of substances and the secondary qualities and powers of the substances themselves. By "powers" Locke meant all dispositional properties (as we say nowadays) except those whose direct manifestation consists in the occurrence of sensations; thus he calls solubility in a given liquid a "power," but not so colors, although he calls colors (secondary) "qualities" and defines qualities as powers of objects to produce specific kinds of "ideas" (sensations). The well-known (and well-worn) confusions involved in the distinction between primary and secondary qualities will not be discussed in this context, since our analysis will be focused on Locke's conception of "necessary connection." For this purpose it will be sufficient to identify the three types of properties of substances and of the items of which they

[8] In the terminology of the British idealists, the relation is *internal*. For a critical comment on the concept of "internal relation," see below, pp. 73-5.

are composed by enumeration: primary qualities are such qualities as mass, size, shape, state of motion; secondary qualities are such qualities as colors, tastes, smells, temperatures; and powers are such properties as solubility, malleability, thermal and electrical conductivity. Locke's thesis might now be stated as follows: we do not know with certainty any proposition of the form "every instance of natural kind *K* has property *P*" unless *P* is contained in the nominal essence of *K* (in which case the proposition would be "trifling," uninformative); and by "nominal essence" of *K* Locke meant a complex idea composed of ideas of observable qualities of observable things, in contrast to the "real essence" which is the sum total of the primary qualities of the insensible particles of which instances of *K* are composed and which *determine* the observable qualities of the latter (for example, the color of a metal is part of its nominal essence, the atomic weight is part of its real essence).

But Locke does not say or imply that such propositions about natural kinds *are* contingent. In other words, he does not say that *there is no* necessary connection between the concept "being gold" and the concept "being soluble in aqua regia"; he only says that *we do not see* any such necessary connections, and that this is the reason why we cannot be certain of the truth of informative universal propositions about natural kinds. More than that, he held that if we knew the real essences of substances and saw necessary connections between the primary qualities of the particles and the observable qualities of the perceptual objects, then we would see a necessary connection between being gold and being soluble in aqua regia, just as we do see a necessary connection between being a Euclidean triangle and having an angle sum of 180 degrees.[9] If we use the Kantian terminology, this means nothing less than that according to Locke true generalizations about natural kinds are *synthetic a priori* propositions, though in most cases (exceptions will be considered presently) the finite human mind has, for lack of adequate ideas of natural kinds, only *a posteriori* knowledge of such propositions. In other words, Locke held that if the informative proposition "all gold is soluble in aqua regia" is true (which we cannot know), then it is just as much an a priori truth as the proposition about the sum of the angles of a triangle, although our

[9] This clearly nonempiricist aspect of Locke's epistemology is noted by W. Kneale, in *Probability and Induction* (Oxford, 1949), p. 71.

knowledge of this proposition is of the same imperfect kind as would be our knowledge of the proposition about the triangle if it were based on repeated measurements of the interior angles of particular triangles.

As it will be argued in the sequel that this very view convicts Locke of serious confusion about the notion of necessary truth, it is wise first to support the above interpretation by citations. After referring to Newton's "corpuscularian hypothesis" as "that which is thought to go farthest in an intelligible explication of the qualities of bodies," he says "whichever hypothesis be clearest and truest . . . our knowledge concerning corporeal substances will be very little advanced by any of them, till we are *made to see what qualities and powers of bodies have a necessary connexion or repugnance one with another*; which, *in the present state of philosophy*, I think, we know but to a very small degree: and I doubt whether, *with those faculties we have*, we shall ever be able to carry our general knowledge (I say not particular experience) in this part much farther" (Bk. IV, chap. 3, sec. 16, italics mine). And in section 25 of the same chapter: "I doubt not but if we could discover the figure, size, texture, and motion of the minute constituent parts of any bodies, we should know without trial several of their operations one upon another, as we do now the properties of a square or a triangle." Locke emphasizes, however, that even if we knew the real essences of substances, our knowledge of nature would still fall short of the ideal of mathematical certainty, since *we cannot discover* any "connexion" between primary qualities and the secondary qualities (or sensations—Locke notoriously confused qualities, originally defined as powers of producing "ideas," with the ideas which are their manifestations) "determined" by them. And a careful analysis of his language reveals that he did not deny the existence of such "connexions" but only the possibility, for the finite human mind, of discovering them:

> It is evident that the bulk, figure, and motion of several bodies about us, produce in us several sensations, as of colours, sounds, taste, smell, pleasure, and pain, etc. These mechanical affections of bodies having no affinity at all with those ideas they produce in us (there being no *conceivable connexion* between any impulse of any sort of body, and any perception of a colour or smell

> which we find in our minds), we can have no distinct knowledge of such operations beyond our experience . . . As the ideas of sensible secondary qualities which we have in our minds, can *by us be no way deduced* from bodily causes, nor any correspondence or connexion be found between them and those primary qualities which experience shows us produce them in us; so, on the other side, the operation of our minds upon our bodies is as unconceivable. [Bk. IV, chap. 3, sec. 28. Italics mine.]

The context leaves no doubt that by "conceivable," in the expression "no conceivable connexion," Locke meant "conceivable by us." It seems that for Locke the contingency of a proposition was not absolute but relative to cognitive powers. In this respect he falls in line with Leibniz, who, as we have seen, held that singular propositions about individual substances are only contingent relative to the finite human mind with its imperfect concepts of individual substances. But let us examine, now, the question of just *why* Locke should have thought that we would have demonstratively certain knowledge of generalizations about natural kinds if the two conditions he mentioned were satisfied, viz. (1) knowledge of the real essences of natural kinds, and (2) insight into necessary connections between primary and secondary qualities.

Consider Locke's example "gold is soluble in aqua regia." Let "Q_g" denote the set of primary qualities of gold (= the qualities of the atoms of the substance, e. g. mass of the gold atom and shape and size of the gold atom) which Locke calls its "real essence"; and analogously "Q_a" is to denote the primary qualities of aqua regia. Then the above proposition would be deducible from the following premises:

(1) gold has Q_g, (2) aqua regia has Q_a, (3) if a solid characterized by Q_g is immersed in a liquid characterized by Q_a, there results a solution. The third premise states a connection between primary qualities and a secondary quality, for "being a solution" must be counted as a secondary quality of a liquid since secondary qualities are those qualities of observable substances which are not predicable of the insensible particles, and surely "being a solution" is not predicable of an atom.[10] Now, if the fact that a proposition is a necessary

[10] Besides, Locke *contrasts* primary qualities with powers—though in this he is

consequence of a set of premises is to warrant the judgment that it is itself necessary, it must first be shown that the premises are themselves necessary propositions. Yet how could (3) be anything but a contingent generalization? It is a proposition of the same kind as "whenever nitrogen and hydrogen combine in the volume proportion 1: 3, there results a gas with the unpleasant odor of ammonia." What would it be like for superior intellects to perceive a necessary conection between antecedent and consequent? Locke might reply that just as a mathematically untrained human intellect might not see that "x is a triangle" entails "x has an angle sum of 180°" even though the entailment objectively holds, so here the antecedent may, for all we know, entail the consequent, though the human intellect is too weak to see this. However, what the mathematical demonstration enables us to see is that the proposition about triangles is entailed by the axioms of the geometry (specifically, the parallel axiom), which leaves it an open question whether "x is a triangle" entails "x has an angle sum of 180°." What the proof establishes is only that the conjunction of "x is a triangle" with the axioms of the geometrical system entails "x has an angle sum of 180°." Moreover, if a geometry in the sense of an empirical theory of physical space (not in the sense of a pure calculus) is conceived as a hypothetico-deductive system, then the very evidence on which the axioms are accepted consists in the truth of the theorems they imply; the parallel axiom, for example, would be accepted because it is found that the angle sum of a triangle *is* 180°. It follows that if the theorems are initially regarded as contingent truths, then the axioms must likewise be regarded as contingent: if q is contingent, then p can be a *reason* for q only if it is itself contingent (no contingent propositions are deducible from necessary propositions). But then we might say to Locke that just because such generalizations as "gold is soluble in aqua regia" are contingent, any premises that might explain this uniformity would themselves have to be contingent. Indeed, Locke's claim that we are ignorant of any necessary connection between being gold and being soluble in aqua regia was probably meant in the sense of ignorance of the *explanation* of the observed uniformity. And

inconsistent with (a) his definition of "quality" as a power of producing ideas, and (b) his inclusion of "mobility" in the list of primary qualities.

Locke, to be sure, nowhere even attempts to analyze the notion of "necessary connexion." And it is hard to anticipate how he would have reacted to Hume's claim that the "necessary connexion" between impact of A on B and subsequent motion of B is nothing but our firm expectation of the second event when we witness the first event, the expectation itself being the result of observation of the "constant conjunction" of two kinds of events. But since he uses the same expression, "necessary connexion," when referring to the certainty of the propositions of mathematics, and there is no indication of deliberately ambiguous usage, it is natural to interpret Locke to have believed, just as Kant did, that the axioms of mechanics have a certainty which such inductive generalizations as "gold is soluble in aqua regia" lack. Presumably Locke thought that, if A is the event of a body impinging on a mobile body of equal or less mass, and B the event of the latter body subsequently moving, one could foresee *a priori* that B would happen if A happened. It is strange that he did not even try to give an argument supporting this claim of a priori predictability. For unless this claim is made good, there is no reason whatever to suppose that we would be able to predict a priori the powers of substances (such as the solubility in aqua regia of gold) if only we knew their real essences.

C. *Trifling Propositions and Genuine Knowledge*

Kant's conception of an analytic judgment as a judgment whose predicate is contained in the subject was clearly anticipated by Locke; he called them "trifling propositions." That all gold is malleable, "is a very certain proposition, if malleableness be a part of the complex idea the word 'gold' stands for. But then here is nothing affirmed of gold, but that that sound stands for an idea in which malleableness is contained; and such a sort of truth and certainty as this it is to say, 'a centaur is four-footed'" (Bk. IV, chap. 6, sec. 9). But if "analytic" is used in this sense, then it is equally clear that Locke anticipated Kant's claim that the propositions of mathematics are *synthetic a priori* truths:

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unknown substratum which we call 'substance'; but what other qualities necessarily co-exist with such combinations, we cannot certainly know, unless we can discover their natural dependence; *which in their primary qualities we can go but a very little way in . . .*" (chap. 6, sec. 7, italics mine).

we can know the truth (of?), and so may be certain in propositions which affirm something of another, which is a necessary consequence of its precise complex idea, but not contained in it: as that "the external angle of all triangles is bigger than either of the opposite internal angles"; which relation of the outward angles to either of the opposite internal angles, making no part of the complex idea signified by the name "triangle," this is a real truth, and conveys with it instructive real knowledge [Bk. IV, chap. 8, sec. 8].

That such a truth is not analytic in the Kantian sense seems indeed undeniable. No contradiction is deducible from its negation without the use of extralogical (specifically geometrical) postulates. Locke could have given even more striking examples of necessary geometrical truths which are not formally demonstrable. Consider the proposition: "Given a straight line *S* and a circle *C* such that *S* has more than one point in common with *C*, then *S* has exactly two points in common with *C*." If anyone claims that this proposition is demonstrable on the basis solely of definitions of the geometrical terms involved, let him produce the goods! And if he should reply that it is nevertheless systematically analytic, i. e. demonstrable within an adequately formalized Euclidean geometry, he would throw himself open to a double retort: (1) What is the criterion of "adequate formalization"? Presumably that enough postulates should be explicitly listed to enable one to produce a purely *formal* proof, independent of spatial intuition, of the theorems of the system. But then the claim of the systematic analyticity of our proposition is trivial. (2) In the same sense of "systematically analytic," empirical generalizations like the laws of mechanics would become systematically analytic the moment the empirical science in question were cast into the form of a deductive system. But surely no law of mechanics has the sort of self-evidence which the cited geometrical proposition has. Besides, even if a formal demonstration on the basis of explicit definitions were possible, it would be deception to suppose that for this reason spatial intuition plays no part in geometrical knowledge. For such definitions as "straight line = line uniquely determined by two points," "circle = closed line all of whose points are equidistant from a given point" are arrived at in no other way than by

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analyzing spatial intuitions (and, as was noted in our discussion of Kant, it is for this reason that Kant could not be forced to concede by exhibiting a formal proof based on such definitions). A logical empiricist would no doubt reply that if our proposition belongs to formal geometry, then, the geometrical terms being uninterpreted, it is not a proposition at all but a propositional function, and if it is a proposition about physical space, "straight line" being interpreted to mean the path of a light ray, then it may well have exceptions in view of the "bending" of light rays in intense gravitational fields. But this objection is irrelevant if we take the proposition to make an assertion about visual space.[12]

At any rate it is clear that if the thesis of the analyticity of all a priori knowledge is part of what is meant by "empiricism," then Locke was no empiricist at all. "To the extent that knowledge is a priori it is not about reality" is a battle cry of modern logical empiricism. Does this mean that from an a priori truth nothing can be deduced as to the properties of empirically given objects? But surely the a priori knowledge that "'x is a cube' entails 'x has 12 edges'" warrants the deduction that all the cubical pieces of sugar in that sugar bowl have 12 edges! Locke notes that in precisely this sense all mathematical knowledge is *real*: "Is it true of the idea of a triangle, that its three angles are equal to two right ones? It is true also of a triangle wherever it really exists. Whatever other figure exists that is not exactly answerable to that idea of a triangle in his mind, is not at all concerned in that proposition" (Bk. IV, chap. 4, sec. 6). Of course, no logical empiricist would deny that we have a priori knowledge of the cited proposition about the contents of the sugar bowl. But what, then, is he denying when he avers that this is not knowledge "about reality"? Anything more than that the proposition which is known a priori is contingent? But is it not trivial to insist that whatever is deducible from an a priori truth is itself an a priori truth? Perhaps the empiricist battle cry could be interpreted as the exhortation "let's not confuse a priori knowledge and empirical (factual) knowledge; they are quite distinct, and the methods for acquiring knowledge of mathematical truths are no more suitable for acquiring knowledge of empirical truths than the methods for catching lions are

[12] The logical empiricists' attempt to explain geometry without admitting synthetic a priori propositions will be discussed in more detail below, Chap. 8.

appropriate for catching fish." But then the dictum of the "factual emptiness" of a priori knowledge becomes redundant once the distinction between a priori and empirical (factual) knowledge is recognized, and it is none too clear just who the philosophers are that need the exhortation. Even "rationalists" like Leibniz, who attempted to deduce propositions of physics like the law of inertia from an a priori principle like the principle of insufficient reason, did not attempt the self-contradictory feat of deducing a contingent truth from an a priori truth: what they tried to show was that physical propositions commonly accepted on empirical evidence admitted of a priori deduction from a self-evident principle, just as Euclid performed an a priori deduction from (to him) self-evident axioms of such empirically confirmed propositions as the Pythagorean theorem. It is one thing to attempt a deduction from self-evident principles of propositions that *seem* to be contingent, another thing to attempt such a demonstration of propositions that *are* contingent. Unfortunately the modern philosophers who frequently sound this "empiricist" battle cry seem to think that to believe in synthetic a priori knowledge is *equivalent* to confusing a priori and empirical knowledge. But this equivalence would hold only—as will be argued in detail in Chapter 5—if "synthetic" were synonymous with "empirical" (and accordingly, if "analytic" and "a priori" were synonymous). And it is significant in this connection that Locke *both* distinguished genuine knowledge from pseudo-knowledge (knowledge of synthetic propositions and knowledge of "trifling" propositions) within the genus "a priori knowledge" *and* discussed at considerable length the difference between a priori knowledge and empirical knowledge.

D. *Simplicity of Ideas*

We have seen, while examining the doctrines of Leibniz and Kant, that the concept of *unanalyzable* (simple) ideas plays an important role in connection with the question of whether a given self-evident proposition is analytic. Locke has some highly significant things to say on the subject of simple ideas, and in fact it is impossible to state the precise sense in which he was an empiricist without speaking of simple ideas: what contrasts his epistemology with that of the rationalists Leibniz and Kant is not the denial of synthetic appropriate for catching fish." But then the dictum of the "factual emptiness" of a priori knowledge becomes redundant once the distinction between a priori and empirical (factual) knowledge is recognized, and it is none too clear just who the philosophers are that need the exhortation. Even "rationalists" like Leibniz, who attempted to deduce propositions of physics like the law of inertia from an a priori principle like the principle of insufficient reason, did not attempt the self-contradictory feat of deducing a contingent truth from an a priori truth: what they tried to show was that physical propositions commonly accepted on empirical evidence admitted of a priori deduction from a self-evident principle, just as Euclid performed an a priori deduction from (to him) self-evident axioms of such empirically confirmed propositions as the Pythagorean theorem. It is one thing to attempt a deduction from self-evident principles of propositions that *seem* to be contingent, another thing to attempt such a demonstration of propositions that *are* contingent. Unfortunately the modern philosophers who frequently sound this "empiricist" battle cry seem to think that to believe in synthetic a priori knowledge is *equivalent* to confusing a priori and empirical knowledge. But this equivalence would hold only—as will be argued in detail in Chapter 5—if "synthetic" were synonymous with "empirical" (and accordingly, if "analytic" and "a priori" were synonymous). And it is significant in this connection that Locke *both* distinguished genuine knowledge from pseudo-knowledge (knowledge of synthetic propositions and knowledge of "trifling" propositions) within the genus "a priori knowledge" *and* discussed at considerable length the difference between a priori knowledge and empirical knowledge.

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> A studious blind man, who had mightily beat his head about visible objects, and made use of the explication of his books and friends to understand those names of light and colors which often came in his way, bragged one day, that he now understood what "scarlet" signified. Upon which his friend demanding, what scarlet was? the blind man answered "It was like the sound of a trumpet." Just such an understanding of the name of any other simple idea will he have who hopes to get it only from a definition, or other words made use of to explain it [Bk. III, chap. 4, sec. 11].

But although Locke does offer a definition of "simple idea" relative to which his empiricist thesis is not a tautology, the definition offered raises a formidable difficulty for his claim that simple ideas are undefinable. The definition occurs implicitly in the following passage: " there is nothing can be plainer to a man than the clear and distinct perception he has of those simple ideas; which, being each in itself uncompounded, contains in it nothing but one uniform appearance or conception in the mind, and is not distinguishable into different ideas " (Bk. II, chap. 2, sec. 1). Surely, if the ideas of colors, and of the determinable " color " itself, are " in themselves uncompounded," so are such ideas of shape (Locke speaks of " figure ") as straightness and circularity. Now, Locke says that the names of simple ideas are " incapable of being defined," " the reason whereof is this, that the several terms of a definition signifying several ideas, they can all together by no means represent an idea which has no composition at all " (Bk. III, chap. 4, sec. 7). But what about such definitions as " the straight line is the shortest distance between two points," " the straight line is the line which is uniquely determined by two points," " a circle is a closed line all of whose points are equidistant from a given point "? Again, Locke lists the idea of number (the determinable, not ideas of determinate numbers) as simple, as, likewise, the idea of the number one. (Presumably he thought that all the natural numbers larger than one are definable, as the immediate successors of their immediate predecessors, but not unity itself.) What would he have said about the Frege-Russell definitions of number and of the number one in particular? Again, infinity as predicated of the series of natural numbers is considered a simple idea by Locke; what would he have said of the modern definition in terms of one-to-one correspondence? Again, in the section where he urges us to observe the limits of definability (" if all terms were definable, it would be a process in infinitum "), he ridicules scholastic as well as " modern " definitions of " motion." Since Locke's parody raises a vexing problem which is still with us nowadays as we ask what, after all, an *analysis* of a concept is, it is worth quoting in full:

> The atomists, who define motion to be " a passage from one place to another ": what do they more than put one synonymous word for another? For what is " passage " other than motion? And if they were asked what " passage " was, how would they better define it than by " motion "? For is it not at least as

> proper and significant to say "Passage is a motion from one place to another," as to say "Motion is a passage?" etc. *This is to translate, and not to define,* when we change two words of the same signification one for another [Bk. III, chap. 4, sec. 9, italics mine].

Actually, if the criticized definition were rewritten by putting "*change* of place" in place of "passage from one place to another," there would be nothing circular about it and it would be a perfectly good example of traditional definition *per genus et differentiam*: bodies change in many different respects—such as color, shape, temperature, degree of solidity—and change of place is one among them; the notion of change is thus more general than the notion of motion. Nevertheless, if we can attach any meaning to Locke's definiens for "simple idea" at all, we will agree that the idea of motion *is* simple, just like ideas of colors. Of course, if it should be maintained that the more generic idea of change is "distinguishable" within the idea of motion, one could only reply that in the same sense the generic idea of color is "distinguishable" within the idea of redness, or the generic idea of taste within the idea of sweetness—and one would just have to confess that Locke's definition of simplicity of ideas is too vague to be of any use at all. The point of chief importance in the above quotation, however, is the distinction between *translation* and *definition* there drawn. By saying that the names of simple ideas are undefinable Locke evidently meant, not that no synonyms could be produced, or that no other expression of equal extension with the defined expression could be produced, but that no complex of words would have the power to evoke the idea in question in a man who had not antecedently perceived instances of the idea. Thus he would presumably have maintained that one who understood the meanings of "change" and "place" still could not be made to understand the meaning of "motion" by pronouncing the definiens "change of place" unless he had witnessed instances of motion in the first place; and similarly that one who had never seen a circle would not be enabled to imagine a circle, or to identify visually a presented circle as a circle, by being told that a circle is a closed line all of whose points are equidistant from a given point. Now it turns out, however, that there is no a priori guarantee whatever that all simple ideas (in the vague sense of Locke's definition) should be undefinable or that all definable ideas should be complex. It is entirely conceivable that a man who had never perceived an instance of orange, yet had perceived instances of yellow and of red, and who was also acquainted ostensively with the relation of betweenness (in the sense in which a shade of a given color is between two neighboring shades), could imagine an orange patch and/or identify an orange patch if he were told "orange is the color between yellow and red." And it is likewise conceivable that a person who was ostensively acquainted with the meanings of "straight line," "right angle," and "equal" but who had never seen squares, would *not* be able to imagine a square if he were told "a square is a closed figure bounded by equal straight lines in such a way that adjacent sides always form right angles"; but surely the idea of squareness is *complex* in the sense of being "distinguishable into different ideas."

Two important consequences follow. (1) In view of the vagueness of Locke's definition of "simple idea," the only tolerably clear criterion of simplicity to be extracted from the *Essay* is the psychological-genetic criterion of undefinability (in the specified sense), and therefore the *general* empiricist thesis "all simple ideas are caused by perception (or introspection) of instances" does resolve into a tautology. Only *applicative* statements of the form "this is a simple idea (e. g. the idea of redness is simple)" would have factual content. But any such statement is, whether true or false, a hypothesis of empirical psychology which it may, moreover, be practically impossible to subject to controlled experimental test. No general *epistemological* thesis seems to remain that would serve to identify Locke as an empiricist. If the thesis is "*some* ideas are caused by perception (or introspection) of instances, and could not have originated otherwise" (though it may be doubtful *which* they are), then it would be difficult to find a rationalist who would disagree. But more important in connection with our topic is (2) that a simple idea in Locke's psychological-genetic sense could hardly be identified with an *unanalyzable* idea and, correspondingly, Locke's notion of "definition" cannot be identified with the notion of *analysis*. When a philosopher claims that such geometrical concepts as circle, ellipse, and parabola are *analyzable*, he surely does not commit himself to any such dubious causal laws of psychology as the claim of definability in Locke's sense would amount to. When Russell offers "class of similar classes" as the analysis of the concept of number, he surely does not mean to imply that one who had not antecedently acquainted himself with particular numbers through usual counting procedures but who understood the meaning of "similar classes" would come to understand the meaning of "number" by just being told "that's a class of similar classes."

But what, then, do philosophers mean by the contrast simple-analyzable (concepts)? It seems fair to say that even contemporary analysts whose language and thought is immeasurably more precise than Locke's are no less uncertain than Locke on this point. A look at recent statements by Carnap about simplicity of predicates should suffice to substantiate this charge (or rehabilitation of Locke, depending on one's point of view). As was pointed out in the discussion of Leibniz, the desire to guarantee the consistency of any given state description that a given language allows to be constructed led Carnap to lay down a requirement of simplicity: only *simple* predicates are eligible as primitive predicates. Carnap confesses "that an exact explication of the concept of simplicity cannot easily be given" but thinks that "nevertheless, the concept seems clear enough for many practical purposes" ("On the Application of Inductive Logic," p. 137). As an example of a simple property he gives "a certain shade of blue," and answers the objection that an analysis of this property is possible as follows: "The spectral analysis of this blue into spectral colors as its components or the physical analysis of it in terms of electro-magnetic waves are, of course, not analyses in the present sense; *they do not show the experience to be composite but rather establish, by way of induction, certain correlations between this color blue and other experiences.*" (My italics) It seems that a criterion of the simplicity of P used by Carnap is that the *experience* consisting in the seeing of an instance of P should be unanalyzable, unified. But what if in observing a patch of a given shade of blue I discriminate the shade from the hue? Is the truly simple property, then, the shade in abstraction from any hue? But a shade can be perceived only as a shade *of* a given color! It is similar with perception of sounds: every sound has both a determinate pitch and a determinate loudness; if, let us say, middle C were a simple quality in Carnap's sense then a perception of an instance of middle C should not be analyzable into pitch and loudness components; yet there could be no perception of pitch abstracted from loudness. The difficulty comes out clearly also if we consider predicates designating shapes, like "circular" and "elliptical." If the sight of a given shade of blue is simple, so is the sight of, say, a circular disk. But even if we abstract from the color components of the visual sense datum, we can still discriminate between the shape and the size of the sense datum; and if it be replied that no particular size is *designated* by the predicate "circular," it must be said that nonetheless any observable instance of the designated shape also has size (necessarily) and that the perception of a determinate size, accordingly, is a component of the perception of the round disk. Now, what would Carnap say if it were maintained that the meaning of "circular" is analyzable by means of either the definition of synthetic geometry or the definition of analytic geometry? Would he not have to say, to be consistent, that "these are not *analyses* in the present sense; they do not show the experience (of seeing a circular shape) to be composite but rather establish, by way of induction, certain correlations between this shape of circularity and other experiences"? Would it be so unplausible to regard it as an *inductive* conclusion that a figure which looks, as viewed normally, circular turns out to satisfy an equation of the form "$x^2 + y^2 = k$" when embedded in a Cartesian coordinate system? It must be confessed, then, that Carnap has not elucidated the *relevant* sense of "analyzable" in which such properties as determinate shades of color or determinate shapes are not analyzable. Consequently the concept of simple descriptive predicates, which is part and parcel of the new foundations of deductive and inductive logic laid by Carnap at the present time, must be declared as no clearer than Locke's concept of simple ideas.