ARISTOTLE'S SYLLOGISTIC

FROM THE STANDPOINT OF MODERN FORMAL LOGIC

BY

JAN LUKASIEWICZ

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PREFACE TO THE SECOND EDITION

THE first edition of this book did not contain an exposition of Aristotle's modal syllogistic. I was not able to examine Aristotle's ideas of necessity and possibility from the standpoint of the known systems of modal logic, as none of them was in my opinion correct. In order to master this difficult subject I had to construct for myself a system of modal logic. The first outlines of this I developed in connexion with Aristotle's ideas in my lectures delivered in the Royal Irish Academy during 1951 and in the Queen's University of Belfast in 1952. The complete system I published in *The Journal of Computing Systems*, 1953. My system of modal logic is different from any other such system, and from its standpoint I was able to explain the difficulties and correct the errors of the Aristotelian modal syllogistic.

My book on *Aristotle's Syllogistic* has met with a favourable reception to my knowledge in more than thirty articles and reviews published over the world in English, French, German, Hebrew, Italian, and Spanish. I have ever since been anxious for an opportunity to discuss some of the critical remarks of my reviewers, but in the present issue it has been possible only to add the chapters on modal logic (as the text of the first edition was already printed). I am most grateful to the Clarendon Press for the chance to do so.

J. Ł.

DUBLIN
30 *June* 1955

PUBLISHER'S NOTE

PROFESSOR Jan Łukasiewicz died in Dublin on the 13th of February, 1956, and thus could not see his book through the Press. This was done by his former pupil, Dr. Czesław Lejewski, who read the proofs of the added chapters and extended the index.

PREFACE TO THE FIRST EDITION

In June 1939 I read a paper at the Polish Academy of Sciences in Cracow on Aristotle's syllogistic. A summary of this paper was printed in the same year, but could not be published because of the war. It appeared after the war, but was dated '1939'. During the summer of 1939 I prepared, in Polish, a more detailed monograph on the same subject, and I had already received the proofs of its first part when in September the printer's office was completely destroyed by bombing and everything was lost. At the same time my whole library together with my manuscripts was bombed and burnt. It was impossible to continue the work during the war.

Not till ten years later did I get a fresh opportunity to take up my investigations into Aristotle's syllogistic, this time in Dublin, where since 1946 I have been lecturing on mathematical logic at the Royal Irish Academy. At the invitation of University College, Dublin, I gave ten lectures on Aristotle's syllogistic in 1949, and the present work is the result of those lectures.

This work is confined to the non-modal or 'assertoric' syllogisms, since the theory of these is the most important part of the Aristotelian logic. A systematic exposition of this theory is contained in chapters 1, 2, and 4–7 of Book I of the *Prior Analytics*. These chapters in Th. Waitz's edition—now more than a century old—are the main source of my exposition. I regret that I could not use the new text of the *Prior Analytics* edited with an introduction and a commentary by Sir David Ross and published in 1949, since the historical part of my work was already finished when this edition appeared. I could only correct my quotations from Aristotle by the text of Sir David Ross. In the English version of the Greek texts of the *Analytics* I adhered as far as possible to the Oxford translation of Aristotle's works. Besides the text of the *Prior Analytics* I took into consideration the ancient commentators, especially Alexander. I may mention here that I owe to an anonymous ancient

commentator the solution of historical problems connected with the alleged invention of the fourth syllogistical figure by Galen.

The present work consists of an historical part, Chapters I–III, and a systematic part, Chapters IV and V. In the historical part I have tried to expound the Aristotelian doctrines following the texts as closely as possible, but everywhere I have been anxious to explain them from the standpoint of modern formal logic. In my opinion there does not exist today a trustworthy exposition of the Aristotelian syllogistic. Until now all expositions have been written not by logicians but by philosophers or philologists who either, like Prantl, could not know or, like Maier, did not know modern formal logic. All these expositions are in my opinion wrong. I could not find, for instance, a single author who realized that there is a fundamental difference between the Aristotelian and the traditional syllogism. It seems to me therefore that my own exposition is entirely new. In the systematic part I have tried to explain some theories of modern formal logic necessary to an understanding of Aristotle's syllogistic, and have tried to complete this syllogistic on the lines laid down by Aristotle himself. I was again anxious to be as clear as possible, so that my exposition could be understood by scholars not trained in symbolic or mathematical thinking. I hope therefore that this part of my work may be used as an introduction to modern formal logic. The most important new results in this part I consider to be the proof of decision, given by my pupil J. Słupecki, and the idea of rejection introduced by Aristotle and applied by myself to the theory of deduction.

I am sincerely grateful to the Royal Irish Academy, which, by giving me a position in Dublin, has enabled me to write this book, and to University College, Dublin, for its kind invitation to deliver lectures on Aristotle's logic. I am grateful to the Professors of University College, Dublin, Father A. Gwynn, S.J., and Monsignor J. Shine, who were kind enough to lend me the necessary books. I owe a debt to Sir David Ross, who read my typescript and made some suggestions I was glad to accept. My special thanks are due to the late Father A. Little, S.J., who, although already dangerously ill, willingly corrected the English of the first chapter, to Victor Meally in Dublin, and in

particular to David Rees of Bangor, who read and corrected the English of the whole work. I am also deeply indebted to the officials of the Clarendon Press for their zeal and courtesy in preparing my typescript for printing. The section on Galen is dedicated to my friend Professor Heinrich Scholz of Münster, Westphalia, who was of great assistance to myself and to my wife during the war, and especially during our stay in Münster in 1944. The whole work I dedicate to my beloved wife, Regina Łukasiewicz *née* Barwińska, who has sacrificed herself that I might live and work. Without her incessant care during the war, and without her continual encouragement and help in the loneliness of our exile after it, I could never have brought the book to an end.

J. Ł.

DUBLIN
7 *May* 1950

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ERRATA

Page 84, line 35: for *h* read *b*

Page 90, last line: exchange *a* and *c*

Page 120, line 29: for *100×*61. *c/b* read *61×*100. *b/c*

Page 129, line 24: for *Icd* read *NIcd*

ANCIENT TEXTS AND COMMENTARIES

Aristoteles Graece, ex recensione Immanuelis Bekkeri, vol. i, Berolini, 1831.

Aristotelis Organon Graece, ed. Th. Waitz, vol. i, Lipsiae, 1844; vol. ii, Lipsiae, 1846.

Aristotle's Prior and Posterior Analytics. A Revised Text with Introduction and Commentary by W. D. Ross, Oxford, 1949.

Alexandri in Aristotelis Analyticorum Priorum Librum I Commentarium, ed. M. Wallies, Berolini, 1883.

Ammonii in Aristotelis Analyticorum Priorum Librum I Commentarium, ed. M. Wallies, Berolini, 1899.

Ioannis Philoponi in Aristotelis Analytica Priora Commentaria, ed. M. Wallies, Berolini, 1905.

The texts of Aristotle are quoted according to Bekker's edition. Example: *An. pr.* i. 4, $25^{b}37$ means: *Analytica priora*, Book I, chapter 4, page 25, column b, line 37. The texts of the commentators are quoted according to the above editions of the Academy of Berlin. Example: *Alexander* 100. 11 means: page 100, line 11.

CHAPTER I

ELEMENTS OF THE SYSTEM

§ 1. *The true form of the Aristotelian syllogism*

In three recently published philosophical works the following is given as an example of the Aristotelian syllogism:[1]

(1) All men are mortal,
Socrates is a man,
therefore
Socrates is mortal.

This example seems to be very old. With a slight modification—'animal' instead of 'mortal'—it is quoted already by Sextus Empiricus as a 'Peripatetic' syllogism.[2] But a Peripatetic syllogism need not be an Aristotelian one. As a matter of fact the example given above differs in two logically important points from the Aristotelian syllogism.

First, the premiss 'Socrates is a man' is a singular proposition, as its subject 'Socrates' is a singular term. Now Aristotle does not introduce singular terms or premisses into his system. The following syllogism would therefore be more Aristotelian:

(2) All men are mortal,
All Greeks are men,
therefore
All Greeks are mortal.[3]

This syllogism, however, is still not Aristotelian. It is an inference, where from two premisses accepted as true, 'All men are mortal' and 'All Greeks are men', is drawn the conclusion 'All Greeks are mortal'. The characteristic sign of an inference is the word

[1] See Ernst Kapp, *Greek Foundations of Traditional Logic*, New York (1942), p. 11; Frederick Copleston, S.J., *A History of Philosophy*, vol. i: *Greece and Rome* (1946), p. 277; Bertrand Russell, *History of Western Philosophy*, London (1946), p. 218.

[2] Sextus Empiricus, *Hyp. Pyrrh.* ii. 164 *Σωκράτης ἄνθρωπος, πᾶς ἄνθρωπος ζῷον, Σωκράτης ἄρα ζῷον.* A few lines earlier Sextus says that he will speak about the so-called categorical syllogisms, *περὶ τῶν κατηγορικῶν καλουμένων συλλογισμῶν*, used chiefly by the Peripatetics, *οἷς χρῶνται μάλιστα οἱ ἀπὸ τοῦ Περιπάτου.* See also ibid. ii. 196, where the same syllogism is cited with the premisses transposed.

[3] B. Russell, op. cit., p. 219, gives form (2) immediately after form (1), adding in brackets the remark: 'Aristotle does not distinguish between these two forms; this, as we shall see later, is a mistake.' Russell is right when he says that these two forms must be distinguished, but his criticism should not be applied to Aristotle.

'therefore' (ἄρα). Now, and this is the second difference, no syllogism is formulated by Aristotle primarily as an inference, but they are all implications having the conjunction of the premisses as the antecedent and the conclusion as the consequent. A true example of an Aristotelian syllogism would be, therefore, the following implication:

(3) If all men are mortal
and all Greeks are men,
then all Greeks are mortal.

This implication is but a modern example of an Aristotelian syllogism and does not exist in the works of Aristotle. It would be better, of course, to have as an example a syllogism given by Aristotle himself. Unfortunately no syllogism with concrete terms is to be found in the *Prior Analytics*. But there are some passages in the *Posterior Analytics* from which a few examples of such syllogisms may be drawn. The simplest of them is this:

(4) If all broad-leaved plants are deciduous
and all vines are broad-leaved plants,
then all vines are deciduous.[1]

All these syllogisms, whether Aristotelian or not, are only examples of some logical forms, but do not belong to logic, because they contain terms not belonging to logic, such as 'man' or 'vine'. Logic is not a science about men or plants, it is simply applicable to these objects just as to any others. In order to get a syllogism within the sphere of pure logic, we must remove from the syllogism what may be called its matter, preserving only its form. This was done by Aristotle, who introduced letters instead of concrete subjects and predicates. Putting in (4) the letter *A* for 'deciduous', the letter *B* for 'broad-leaved plant', the letter *C* for 'vine', and using, as Aristotle does, all these terms in the singular, we get the following syllogistic form:

(5) If all *B* is *A*
and all *C* is *B*,
then all *C* is *A*.

[1] *An. post.* ii. 16, 98^{b}5–10 ἔστω γὰρ τὸ φυλλορροεῖν ἐφ' οὗ *A*, τὸ δὲ πλατύφυλλον ἐφ' οὗ *B*, ἄμπελος δὲ ἐφ' οὗ *Γ*. εἰ δὴ τῷ *B* ὑπάρχει τὸ *A* (πᾶν γὰρ πλατύφυλλον φυλλορροεῖ), τῷ δὲ *Γ* ὑπάρχει τὸ *B* (πᾶσα γὰρ ἄμπελος πλατύφυλλος), τῷ *Γ* ὑπάρχει τὸ *A*, καὶ πᾶσα ἄμπελος φυλλορροεῖ. From this somewhat carelessly written passage—after τῷ *B*, τῷ δὲ *Γ*, and τῷ *Γ*, παντί ought to be inserted—we get the following syllogism in concrete terms: εἰ πᾶν πλατύφυλλον φυλλορροεῖ καὶ πᾶσα ἄμπελος πλατύφυλλος, πᾶσα ἄμπελος φυλλορροεῖ.

This syllogism is one of the logical theorems invented by Aristotle, but even it differs in style from the genuine Aristotelian syllogism. In formulating syllogisms with the help of letters, Aristotle always puts the predicate in the first place and the subject in the second. He never says 'All *B* is *A*', but uses instead the expression '*A* is predicated of all *B*' or more often '*A* belongs to all *B*'.[1] Let us apply the first of these expressions to form (5); we get an exact translation of the most important Aristotelian syllogism, later called 'Barbara':

(6) If *A* is predicated of all *B*
and *B* is predicated of all *C*,
then *A* is predicated of all *C*.[2]

Starting with the unauthentic example (1) we have reached thus by a step-by-step transition the genuine Aristotelian syllogism (6). Let us now explain these steps and establish them on a textual basis.

§ 2. *Premisses and terms*

Every Aristotelian syllogism consists of three propositions called premisses. A premiss (πρότασις) is a sentence affirming or denying something of something.[3] In this sense the conclusion is also a πρότασις, because it states something about something.[4] The two elements involved in a premiss are its subject and predicate. Aristotle calls them 'terms', defining a term (ὅρος) as that into which the premiss is resolved.[5] The original meaning of the Greek ὅρος, as well as of the Latin *terminus*, is 'limit' or 'boundary'. The terms of a premiss, its subject and predicate, are the limits of the premiss, its beginning and end. This is the very meaning of the word ὅρος, and we should be careful not to identify this logical word with such psychological or metaphysical words as 'idea', 'notion', 'concept', or *Begriff* in German.[6]

[1] τὸ *A* κατηγορεῖται κατὰ παντὸς τοῦ *B* or τὸ *A* ὑπάρχει παντὶ τῷ *B*. See also p. 14, n.

[2] *An. pr.* i. 4, 25^{b}37 εἰ γὰρ τὸ *A* κατὰ παντὸς τοῦ *B* καὶ τὸ *B* κατὰ παντὸς τοῦ *Γ*, ἀνάγκη τὸ *A* κατὰ παντὸς τοῦ *Γ* κατηγορεῖσθαι. The word ἀνάγκη omitted in the translation will be explained later.

[3] Ibid. i, 24^{a}16 πρότασις μὲν οὖν ἐστὶ λόγος καταφατικὸς ἢ ἀποφατικὸς τινός κατά τινος.

[4] Ibid. ii. 1, 53^{a}8 τὸ δὲ συμπέρασμα τὶ κατά τινός ἐστιν.

[5] Ibid. i. 1, 24^{b}16 ὅρον δὲ καλῶ εἰς ὃν διαλύεται ἡ πρότασις, οἷον τό τε κατηγορούμενον καὶ τὸ καθ' οὗ κατηγορεῖται.

[6] Aristotle also uses the word ὅρος in the sense of ὁρισμός, i.e. 'definition'.

Every premiss is either universal, particular, or indefinite. 'All' and 'no' added to the subject are the signs of universality, 'some' and 'some not' or 'not all' are the signs of particularity. A premiss without a sign of quantity, i.e. of universality or particularity, is called indefinite, e.g. 'Pleasure is not good'.[1]

Nothing is said in the *Prior Analytics* about the terms. A definition of the universal and the singular terms is given only in the *De Interpretatione*, where a term is called universal if it is of such a nature as to be predicated of many subjects, e.g. 'man'; a term which does not have this property is called singular, e.g. 'Callias'.[2] Aristotle forgets that a non-universal term need not be singular, for it may be empty, like the term 'goat-stag' cited by himself a few chapters before.[3]

In building up his logic Aristotle did not take notice either of singular or of empty terms. In the first chapters of the *Prior Analytics*, containing the systematic exposition of his syllogistic, only universal terms are mentioned. Alexander justly remarks that the very definition of the premiss given by Aristotle has application to universal terms alone and is not suitable to individual or singular.[4] It is evident that the terms of universal and particular premisses must be universal. Aristotle certainly would not accept as meaningful expressions like 'All Calliases are men' or 'Some Calliases are men', if there were only one Callias. The same must be said about the terms of indefinite premisses: they, too, are universal. This follows both from the name Aristotle has chosen for them and from the examples he gives. A man who is

I willingly agree with E. Kapp, who says (op. cit., p. 29) that these two different meanings of the word *ὅρος* 'are entirely independent of one another and were never mixed up by Aristotle himself. But unfortunately no less a scholar than Carl Prantl . . . based his picture of Aristotle's logic on this homonymy . . . he identified the empty syllogistic *horos* ("term") with the metaphysical correlate of *horos* in the sense of definition ("Begriff" in Prantl's German). The result was a disastrous confusion.'

[1] *An. pr.* i. 1, 24^{a}17 (continuation of the text quoted in p. 3, n. 3) *οὗτος δὲ ἢ καθόλου ἢ ἐν μέρει ἢ ἀδιόριστος. λέγω δὲ καθόλου μὲν τὸ παντὶ ἢ μηδενὶ ὑπάρχειν, ἐν μέρει δὲ τὸ τινὶ ἢ μὴ τινὶ ἢ μὴ παντὶ ὑπάρχειν, ἀδιόριστον δὲ τὸ ὑπάρχειν ἢ μὴ ὑπάρχειν ἄνευ τοῦ καθόλου ἢ κατὰ μέρος, οἷον τὸ τῶν ἐναντίων εἶναι τὴν αὐτὴν ἐπιστήμην ἢ τὸ τὴν ἡδονὴν μὴ εἶναι ἀγαθόν.*

[2] *De int.* 7, 17^{a}39 *λέγω δὲ καθόλου μὲν ὃ ἐπὶ πλειόνων πέφυκε κατηγορεῖσθαι, καθ' ἕκαστον δὲ ὃ μή, οἷον ἄνθρωπος μὲν τῶν καθόλου, Καλλίας δὲ τῶν καθ' ἕκαστον.*

[3] Ibid. 1, 16^{a}16 *τραγέλαφος.*

[4] Alexander 100. 11 *κατὰ γὰρ αἰσθητοῦ καὶ ἑνὸς κατ' ἀριθμὸν οὐκέθ' ἁρμόζει τὸ κατὰ παντὸς οὐδὲ ὁ διορισμὸς ὅλως· ὁ γὰρ διορισμὸς τῶν προτάσεων ἐπὶ τῶν καθόλου χώραν ἔχει· τὰ δὲ ἄτομα οὐ καθόλου.* Cf. ibid. 65. 26.

undecided whether it is true to say 'No pleasure is good' or only 'Some pleasure is not good', may say without defining the quantity of the subject: 'Pleasure is not good.' But in this last sentence 'pleasure' is still a universal term as it was in the two previous sentences. Throughout the whole systematic exposition of his syllogistic Aristotle in practice treats indefinite premisses like particulars without explicitly stating their equivalence.[1] This was done only by Alexander.[2]

Indefinite premisses are of no importance in the Aristotelian system of logic. No logical thesis, whether a law of conversion or a syllogism, is formulated by Aristotle with this kind of premiss. It was but right that they should be dropped by later logicians, who retained only four kinds of premiss, well known to every student of traditional logic, viz. the universal affirmative, the universal negative, the particular affirmative, and the particular negative. In this fourfold division there is no place left for singular premisses.[3]

§ 3. *Why singular terms were omitted by Aristotle*

There is an interesting chapter in the *Prior Analytics* where Aristotle divides all things into three classes. Some, he says, are such that they cannot be predicated truly of anything at all, like Cleon and Callias and the individual and sensible, but other things may be predicated of them, e.g. man or animal. Some other things, and these are the second class, are themselves predicated of others but nothing prior is predicated of them. For this class of things no example is given, but it is clear that Aristotle means what is most universal, like being, *τὸ ὄν*. To the third class belong those things that may be predicated of others and others of them, e.g. man of Callias and animal of man, and as a rule, concludes Aristotle, arguments and inquiries are concerned with this class of things.[4]

[1] See, for example, *An. pr.* i. 4, 26^{a}29 *ὁ γὰρ αὐτὸς ἔσται συλλογισμὸς ἀδιορίστου τε καὶ ἐν μέρει ληφθέντος*, or 7, 29^{a}27 *δῆλον δὲ καὶ ὅτι τὸ ἀδιόριστον ἀντὶ τοῦ κατηγορικοῦ τοῦ ἐν μέρει τιθέμενον τὸν αὐτὸν ποιήσει συλλογισμὸν ἐν ἅπασι τοῖς σχήμασιν.*

[2] Alexander 30. 29 *περὶ δὲ τῶν ἀδιορίστων* (scil. *τῆς τῶν ἀδιορίστων ἀντιστροφῆς*) *οὐ λέγει, ὅτι μηδὲ χρήσιμοι πρὸς συλλογισμούς εἰσιν αὗται, καὶ ὅτι ἴσον ταῖς ἐπὶ μέρους δύνανται.*

[3] Arguments on behalf of the thesis that singular propositions may be regarded as forming a sub-class of universals—see, for example, J. N. Keynes, *Formal Logic*, London (1906), p. 102—are in my opinion entirely wrong.

[4] *An. pr.* i. 27, 43^{a}25–43 *ἁπάντων δὴ τῶν ὄντων τὰ μέν ἐστι τοιαῦτα ὥστε κατὰ μηδενὸς ἄλλου κατηγορεῖσθαι ἀληθῶς καθόλου (οἷον Κλέων καὶ Καλλίας καὶ τὸ καθ'*

There are some inexactitudes in this passage that must first be corrected. It is not correct to say that a thing may be predicated of another thing. Things cannot be predicated, because a predicate is a part of a proposition and a proposition is a series of spoken or written words having a certain meaning. The term 'Callias' may be predicated of another term, but never the thing Callias. The given classification is not a division of things but a division of terms.

It is further not correct to say that individual or singular terms, like 'Callias', cannot be truly predicated of anything else. Aristotle himself gives examples of true propositions with a singular predicate, as 'That white object is Socrates' or 'That which approaches is Callias',[1] saying that such propositions are 'incidentally' true. There are other examples of this kind which are not merely incidentally true, as 'Socrates is Socrates' or 'Sophroniscus was the father of Socrates'.

A third inexactitude concerns the conclusion drawn by Aristotle from this classification of terms. It is not true that our arguments and inquiries deal as a rule with such universal terms as may be predicated of others and others of them. It is plain that individual terms are as important as universal, not only in everyday life but also in scientific researches. This is the greatest defect of the Aristotelian logic, that singular terms and propositions have no place in it. What was the cause?

There is an opinion among philosophers that Aristotle constructed his system of logic under the influence of Plato's philosophy; for it was Plato who believed that the object of true knowledge must be stable and capable of a precise definition, which is of the universal and not of the singular. I cannot agree with this opinion. It has no confirmation in the text of the *Prior Analytics*. This purely logical work is entirely exempt from any philosophic contamination; so is the passage cited above. The argument that our inquiries are concerned with universal terms as a rule is a practical one, and though it is very weak and

ἕκαστον καὶ αἰσθητόν), κατὰ δὲ τούτων ἄλλα (καὶ γὰρ ἄνθρωπος καὶ ζῷον ἑκάτερος τούτων ἐστί)· τὰ δ' αὐτὰ μὲν κατ' ἄλλων κατηγορεῖται, κατὰ δὲ τούτων ἄλλα πρότερον οὐ κατηγορεῖται· τὰ δὲ καὶ αὐτὰ ἄλλων καὶ αὐτῶν ἕτερα, οἷον ἄνθρωπος Καλλίου καὶ ἀνθρώπου ζῷον. . . . καὶ σχεδὸν οἱ λόγοι καὶ αἱ σκέψεις εἰσὶ μάλιστα περὶ τούτων.

[1] *An. pr.* i. 27, 43[a]33 τῶν γὰρ αἰσθητῶν σχεδὸν ἕκαστόν ἐστι τοιοῦτον ὥστε μὴ κατηγορεῖσθαι κατὰ μηδενός, πλὴν ὡς κατὰ συμβεβηκός· φαμὲν γάρ ποτε τὸ λευκὸν ἐκεῖνο Σωκράτην εἶναι καὶ τὸ προσιὸν Καλλίαν.

Aristotle must have felt its weakness, yet it is not corroborated by any philosophical argument borrowed from Plato.

There is, however, another remarkable point that may throw some light on our problem. Aristotle emphasizes that a singular term is not suited to be a predicate of a true proposition, as a most universal term is not suited to be a subject of such a proposition. The first assertion, as we have already seen, is not generally true, and the second also seems to be false. But it does not matter whether these assertions are true or false. It suffices to know that Aristotle regarded them as true and that he eliminated from his system just those kinds of terms which in his opinion were not suited to be both subjects and predicates of true propositions. And here, as I see it, lies the chief point of our problem. It is essential for the Aristotelian syllogistic that the same term may be used as a subject and as a predicate without any restriction. In all three syllogistic figures known to Aristotle there exists one term which occurs once as a subject and then again as a predicate: in the first figure it is the middle term, in the second figure the major term, and in the third figure the minor term. In the fourth figure all three terms occur at the same time as subjects and as predicates. Syllogistic as conceived by Aristotle requires terms to be homogeneous with respect to their possible positions as subjects and predicates. This seems to be the true reason why singular terms were omitted by Aristotle.

§ 4. *Variables*

In Aristotle's systematic exposition of his syllogistic no examples are given of syllogisms with concrete terms. Only non-valid combinations of premisses are exemplified through such terms, which are of course universal, like 'animal', 'man', 'horse'. In valid syllogisms all terms are represented by letters, i.e. by variables, e.g. 'If *R* belongs to all *S* and *P* belongs to some *S*, then *P* belongs to some *R*'.[1]

The introduction of variables into logic is one of Aristotle's greatest inventions. It is almost incredible that till now, as far as I know, no one philosopher or philologist has drawn attention to

[1] Ibid. i. 6, 28[b]7 εἰ γὰρ τὸ μὲν *P* παντὶ τῷ *Σ* τὸ δὲ *Π* τινί, ἀνάγκη τὸ *Π* τινὶ τῷ *P* ὑπάρχειν. This is a mood of the third figure, called later Disamis, with transposed premisses.

this most important fact.[1] I venture to say that they must all have been bad mathematicians, for every mathematician knows that the introduction of variables into arithmetic began a new epoch in that science. It seems that Aristotle regarded his invention as entirely plain and requiring no explanation, for there is nowhere in his logical works any mention of variables. It was Alexander who first said explicitly that Aristotle presents his doctrine in letters, *στοιχεῖα*, in order to show that we get the conclusion not in consequence of the matter of the premisses, but in consequence of their form and combination; the letters are marks of universality and show that such a conclusion will follow always and for any term we may choose.[2] There is another commentator, John Philoponus, who is also fully aware of the significance and importance of variables. He says that Aristotle, after showing by examples how every premiss may be converted, states some universal rules of conversion taking letters instead of terms. For a universal sentence is disproved by one example in which it is false, but is proved either by going through all particulars (which is an endless and impossible operation) or by stating an evident universal rule. Such a rule is given here by Aristotle in letters, and the reader is allowed to substitute (*ὑποβάλλειν*) for the letters any concrete terms he wants.[3]

We know already that only universal terms may be substituted for the variables. In an example quoted above,[4] Aristotle performs such a substitution, saying: 'Let *A* be deciduous, *B*—broad-leaved plant, *C*—vine.' This is the only kind of substitution we meet in the *Prior Analytics*. Aristotle never substitutes for a variable *A* another variable *B*, although he is perfectly aware that the same syllogistic mood may be formulated with different

[1] I am glad to learn that Sir David Ross in his edition of the *Analytics*, p. 29, emphasizes that by using variables Aristotle became the founder of formal logic.

[2] Alexander 53. 28 *ἐπὶ στοιχείων τὴν διδασκαλίαν ποιεῖται ὑπὲρ τοῦ ἐνδείξασθαι ἡμῖν, ὅτι οὐ παρὰ τὴν ὕλην γίνεται τὰ συμπεράσματα ἀλλὰ παρὰ τὸ σχῆμα καὶ τὴν τοιαύτην τῶν προτάσεων συμπλοκήν καὶ τὸν τρόπον· οὐ γὰρ ὅτι ἤδε ἡ ὕλη, συνάγεται συλλογιστικῶς τόδε, ἀλλ' ὅτι ἡ συζυγία τοιαύτη. τὰ οὖν στοιχεῖα τοῦ καθόλου καὶ ἀεὶ καὶ ἐπὶ παντὸς τοῦ ληφθέντος τοιοῦτον ἔσεσθαι τὸ συμπέρασμα δεικτικά ἐστιν.*

[3] Philoponus 46. 25 *δείξας ὅπως ἑκάστη τῶν προτάσεων ἀντιστρέφει διὰ παραδειγμάτων . . . καθολικοὺς κανόνας παραδίδωσι τὰ στοιχεῖα παραλαμβάνων ἀντὶ τῶν ὅρων . . . τὸν μὲν γὰρ καθόλου λόγον ἐλέγχει μὲν καὶ ἓν παράδειγμα, ὡς ἤδη εἴρηται, κατασκευάζει δὲ ἢ ἡ διὰ πάντων τῶν κατὰ μέρος διέξοδος, ὅπερ ἐστὶν ἄπειρον καὶ ἀδύνατον, ἢ ἡ διὰ καθολικοῦ κανόνος πίστις· ὅπερ ποιεῖ νῦν διὰ τῶν στοιχείων διδοὺς ἑκάστῳ, ὥσπερ εἴρηται, ἐπ' ἐξουσίας χρῆσθαι καὶ ὑποβάλλειν ἀντὶ τῶν στοιχείων οἵας ἂν βούληται ὕλης ὅρους.*

[4] See p. 2, n.

variables. The mood Disamis, for instance, cited at the beginning of this section, is formulated with the letters *R*, *S*, *P*; elsewhere it is formulated with *C*, *B*, *A*.[1] It is evident that the validity of a syllogism does not depend on the shape of the variables used in its formulation: Aristotle knows that without saying it. It is again Alexander who states this fact explicitly.[2]

There is no passage in the *Prior Analytics* where two different variables are identified. Even where the same term is substituted for two variables, these two variables are not identified. In Book II of the *Prior Analytics* Aristotle discusses the problem whether a syllogism can be made out of opposite premisses. This can be done, he states, in the second and third figure. Let *B* and *C*, he continues, both stand for 'science' and *A* for 'medicine'. If one assumes that 'All medicine is science' and that 'No medicine is science', he has assumed that '*B* belongs to all *A*' and '*C* belongs to no *A*', so that 'Some science is not science'.[3] The syllogistic mood to which this refers runs thus: 'If *B* belongs to all *A* and *C* belongs to no *A*, then *C* does not belong to some *B*.'[4] In order to get from this mood a syllogism with opposite premisses, it suffices to identify the variables *B* and *C*, i.e. to substitute *B* for *C*. We get by this substitution: 'If *B* belongs to all *A* and *B* belongs to no *A*, then *B* does not belong to some *B*.' The heavy roundabout way by means of concrete terms, such as 'science' and 'medicine', is quite unnecessary. It seems that the straight way in this problem, i.e. the way by identifying variables, was not seen by Aristotle.

Aristotle knows that sentences like 'Some science is not science' cannot be true.[5] The generalization of such sentences 'Some *A* is not *A*' (i.e. '*A* does not belong to some *A*') also must be false. It is not very probable that Aristotle knew this formula; it is

[1] *An. pr.* ii. 7, 59[a]17 *εἰ γὰρ τὸ Γ παντὶ τῷ Β, τὸ δὲ Α τινὶ τῷ Β, ἀνάγκη τὸ Α τινὶ τῷ Γ ὑπάρχειν.*

[2] Alexander 380. 2 *οὐ γὰρ παρὰ τὸ τὸ μὲν Α αὐτῶν εἶναι τὸ δὲ Β ἢ Γ ἡ συναγωγή· τὸ γὰρ αὐτὸ γίνεται, κἂν ἄλλοις ἀντὶ τούτων χρησώμεθα.*

[3] *An. pr.* ii. 15, 64[a]23 *ἔστω γὰρ ἐπιστήμη ἐφ' οὗ τὸ Β καὶ Γ, ἰατρικὴ δ' ἐφ' οὗ Α. εἰ οὖν λάβοι πᾶσαν ἰατρικὴν ἐπιστήμην καὶ μηδεμίαν ἰατρικὴν ἐπιστήμην, τὸ Β παντὶ τῷ Α εἴληφε καὶ τὸ Γ οὐδενί, ὥστ' ἔσται τις ἐπιστήμη οὐκ ἐπιστήμη.*

[4] This syllogism is a mood of the third figure, called later Felapton, with transposed premisses. In the systematic exposition of the syllogistic it is formulated with the letters *R*, *S*, *P*. See ibid. i. 6, 28[a]26 *ἂν τὸ μὲν Ρ παντὶ τῷ Σ, τὸ δὲ Π μηδενὶ ὑπάρχῃ, ἔσται συλλογισμὸς ὅτι τὸ Π τινὶ τῷ Ρ οὐχ ὑπάρξει ἐξ ἀνάγκης.*

[5] Ibid. ii. 15, 64[b]7 *φανερὸν δὲ καὶ ὅτι ἐκ ψευδῶν μὲν ἔστιν ἀληθὲς συλλογίσασθαι, . . . , ἐκ δὲ τῶν ἀντικειμένων οὐκ ἔστιν· ἀεὶ γὰρ ἐναντίος ὁ συλλογισμὸς γίνεται τῷ πράγματι.*

Alexander again who saw the falsity and applied this fact to prove the law of conversion of the universal negative premiss. The proof he gives proceeds by *reductio ad absurdum*: If the premiss '*A* belongs to no *B*' is not convertible, let us suppose that *B* belongs to some *A*. From these two premisses we get by a syllogism of the first figure the absurd conclusion: '*A* does not belong to some *A*.'[1] It is obvious that Alexander has in mind the mood of the first figure called later Ferio: 'If *A* belongs to no *B* and *B* belongs to some *C*, then *A* does not belong to some *C*',[2] and that in this mood he identifies the variables *A* and *C*, substituting *A* for *C*. This is perhaps the neatest example of an argument by substitution derived from an ancient source.

§ 5. *Syllogistic necessity*

The first Aristotelian syllogism, called later Barbara, may be represented, as we have already seen,[3] in the form of the following implication:

If *A* is predicated of all *B*
and *B* is predicated of all *C*,
then *A* is predicated of all *C*.

But there is still a difference between this formulation and the genuine Greek text. The premisses are the same in the English version as in the Greek, but the exact translation of the conclusion would be '*A* must be predicated of all *C*'. This word 'must' (*ἀνάγκη*) is the sign of the so-called 'syllogistic necessity'. It is used by Aristotle in almost all implications which contain variables and represent logical laws, i.e. laws of conversion or syllogisms.[4]

There are, however, some syllogisms where this word is omitted; take, for instance, this Aristotelian form of the mood Barbara: 'If *A* belongs to all *B* and *C* belongs to all *A*, then *C* belongs to all *B*.'[5] Since it was possible to omit the word in some syllogisms, it must be possible to eliminate it entirely from all syllogisms. Let us see, therefore, what the word means and why it is used by Aristotle.

[1] Alexander 34. 15 *ἔνεστι δὲ καὶ διὰ συλλογισμοῦ δεῖξαι διὰ τοῦ πρώτου σχήματος γινομένου, ὡς καὶ αὐτὸς προσχρῆται τῇ εἰς ἀδύνατον ἀπαγωγῇ· εἰ γάρ τις μὴ λέγοι ἀντιστρέφειν τὴν καθόλου ἀποφατικήν, κείσθω τὸ A μηδενὶ τῷ B· εἰ δὲ μὴ ἀντιστρέφει, ἔστω τὸ B τινὶ τῷ A· γίνεται ἐν πρώτῳ σχήματι τὸ A τινὶ τῷ A μὴ ὑπάρχον, ὅπερ ἄτοπον.*

[2] *An. pr.* i. 4, 26[a]25 *εἰ τὸ μὲν A μηδενὶ τῷ B ὑπάρχει, τὸ δὲ B τινὶ τῷ Γ, ἀνάγκη τὸ A τινὶ τῷ Γ μὴ ὑπάρχειν.* [3] See p. 3, n. 2. [4] See p. 7, n.; p. 9, nn. 1, 4; above, n. 2.

[5] *An. pr.* ii. 11, 61[b]34 *εἰ γὰρ τὸ A παντὶ τῷ B καὶ τὸ Γ παντὶ τῷ A, τὸ Γ παντὶ τῷ B.*

The problem appears simple, and is settled implicitly by Aristotle himself incidentally in his treatment of the laws of conversion, when he says: 'If *A* belongs to some *B*, it is necessary that *B* should belong to some *A*; but if *A* does not belong to some *B*, it is not necessary that *B* should not belong to some *A*.' For if *A* stands for 'man' and *B* for 'animal', it is true that some animal is not man, but it is not true that some man is not animal, because all men are animals.[1] We see from this example that Aristotle uses the sign of necessity in the consequent of a true implication in order to emphasize that the implication is true for all values of variables occurring in the implication. We may therefore say 'If *A* belongs to some *B*, it is necessary that *B* should belong to some *A*', because it is true that 'For all *A* and for all *B*, if *A* belongs to some *B*, then *B* belongs to some *A*'. But we cannot say 'If *A* does not belong to some *B*, it is necessary that *B* should not belong to some *A*', because it is not true that 'For all *A* and for all *B*, if *A* does not belong to some *B*, then *B* does not belong to some *A*'. There exist, as we have seen, values for *A* and *B* that verify the antecedent of the last implication, but do not verify its consequent. In modern formal logic expressions like 'for all *A*' or 'for all *B*', where *A* and *B* are variables, are called universal quantifiers. The Aristotelian sign of syllogistic necessity represents a universal quantifier and may be omitted, since a universal quantifier may be omitted when it stands at the head of a true formula.

This, of course, is all known to students of modern formal logic, but some fifty years ago it was certainly not known to philosophers. It is not strange, therefore, that one of them, Heinrich Maier, has chosen our problem as the basis of what is, in my opinion, a bad philosophical speculation. He states:[2] 'The conclusion follows from the premisses with necessary consequence. This consequence arises from the syllogistic principle and its necessity reveals very properly the synthetic power of the function of reasoning.' I do not understand this last sentence, because

[1] Ibid. i. 2, 25[a]20–6 *εἰ γὰρ τὸ A τινὶ τῷ B, καὶ τὸ B τινὶ τῷ A ἀνάγκη ὑπάρχειν... εἰ δέ γε τὸ A τινὶ τῷ B μὴ ὑπάρχει, οὐκ ἀνάγκη καὶ τὸ B τινὶ τῷ A μὴ ὑπάρχειν, οἷον εἰ τὸ μὲν B ἐστὶ ζῷον, τὸ δὲ A ἄνθρωπος. ἄνθρωπος μὲν γὰρ οὐ παντὶ ζῴῳ, ζῷον δὲ παντὶ ἀνθρώπῳ ὑπάρχει.*

[2] H. Maier, *Die Syllogistik des Aristoteles*, vol. ii *b*, Tübingen (1900), p. 236: 'Aus den Prämissen folgt mit notwendiger Konsequenz der Schlußsatz. Diese Konsequenz entspringt dem syllogistischen Prinzip, und die Notwendigkeit, die ihr anhaftet, bekundet recht eigentlich die synthetische Kraft der Schlußfunktion.'

I cannot grasp the meaning of the words 'the synthetic power of the function of reasoning'. Moreover, I am not sure what is meant by 'the syllogistic principle', as I do not know whether any such principle exists at all. 'On the ground of both premisses [Maier continues his speculations[1]] which I think and express, I must also think and express the conclusion by virtue of a compulsion lying in my thinking.' This sentence I can certainly understand, but it is manifestly false. You may easily see its falsehood if you think and pronounce the premisses of a syllogism, e.g. 'All *A* is *C*' and 'Some *B* is not *C*', without pronouncing the conclusion which follows from them.

§ 6. *What is formal logic?*

'It is usual to say that logic is formal, in so far as it is concerned merely with the form of thought, that is with our manner of thinking irrespective of the particular objects about which we are thinking.' This is a quotation from the well-known text-book of formal logic by Keynes.[2] And here is another quotation, from the *History of Philosophy* by Father Copleston: 'The Aristotelian Logic is often termed formal logic. Inasmuch as the Logic of Aristotle is an analysis of the forms of thought—this is an apt characterization.'[3]

In both quotations I read the expression 'form of thought', which I do not understand. Thought is a psychical phenomenon and psychical phenomena have no extension. What is meant by the form of an object which has no extension? The expression 'form of thought' is inexact and it seems to me that this inexactitude arose from a wrong conception of logic. If you believe indeed that logic is the science of the laws of thought, you will be disposed to think that formal logic is an investigation of the forms of thought.

It is not true, however, that logic is the science of the laws of thought. It is not the object of logic to investigate how we are thinking actually or how we ought to think. The first task belongs to psychology, the second to a practical art of a similar kind to mnemonics. Logic has no more to do with thinking than mathematics has. You must think, of course, when you have to carry

[1] Op. cit., p. 237: 'Auf Grund der beiden Prämissen, die ich denke und ausspreche, muß ich kraft eines in meinem Denken liegenden Zwangs auch den Schlußsatz denken und aussprechen.'

[2] Op. cit., p. 2.

[3] Op. cit., p. 277.

out an inference or a proof, as you must think, too, when you have to solve a mathematical problem. But the laws of logic do not concern your thoughts in a greater degree than do those of mathematics. What is called 'psychologism' in logic is a mark of the decay of logic in modern philosophy. For this decay Aristotle is by no means responsible. Throughout the whole *Prior Analytics*, where the theory of the syllogism is systematically exposed, there exists not one psychological term. Aristotle knows with an intuitive sureness what belongs to logic, and among the logical problems treated by him there is no problem connected with a psychical phenomenon such as thinking.

What is therefore, according to Aristotle, the object of logic, and why is his logic called formal? The answer to this question is not given by Aristotle himself but by his followers, the Peripatetics.

There was a dispute among the philosophical schools of Ancient Greece about the relation of logic to philosophy. The Stoics contended that logic was a part of philosophy, the Peripatetics said that it was only an instrument of philosophy, and the Platonists were of the opinion that logic was equally a part and an instrument of philosophy. The dispute itself is of no great interest or importance, because the solution of the disputed problem seems to be for the most part a matter of convention. But an argument of the Peripatetics, preserved by Ammonius in his commentary on the *Prior Analytics*, deserves our attention.

Ammonius agrees with the Platonists and says: If you take syllogisms with concrete terms, as Plato does in proving syllogistically that the soul is immortal, then you treat logic as a part of philosophy; but if you take syllogisms as pure rules stated in letters, e.g. '*A* is predicated of all *B*, *B* of all *C*, therefore *A* is predicated of all *C*', as do the Peripatetics following Aristotle, then you treat logic as an instrument of philosophy.[1]

[1] Ammonius 10. 36 *κατὰ γὰρ Πλάτωνα καὶ τὸν ἀληθῆ λόγον οὔτε μέρος ἐστίν* (scil. *ἡ λογική*), *ὡς οἱ Στωϊκοί φασιν καὶ τινὲς τῶν Πλατωνικῶν, οὔτε μόνως ὄργανον, ὡς οἱ ἐκ τοῦ Περιπάτου φασίν, ἀλλὰ καὶ μέρος ἐστὶν καὶ ὄργανον φιλοσοφίας· ἐὰν μὲν γὰρ μετὰ τῶν πραγμάτων λάβῃς τοὺς λόγους, μέρος ἐστίν, ἐὰν δὲ ψιλοὺς τοὺς κανόνας ἄνευ τῶν πραγμάτων, ὄργανον. ὥστε καλῶς οἱ ἐκ τοῦ Περιπάτου τὰ παρὰ Ἀριστοτέλει ἀφορῶντες ὄργανον αὐτήν φασιν· ψιλοὺς γὰρ κανόνας παραδίδωσιν, οὐ πράγματα λαμβάνων ὑποκείμενα ἀλλὰ τοῖς στοιχείοις τοὺς κανόνας ἐφαρμόζων· οἷον τὸ Α κατὰ παντὸς τοῦ Β, τὸ Β κατὰ παντὸς τοῦ Γ, τὸ Α ἄρα κατὰ παντὸς τοῦ Γ.* The syllogistic proof of the thesis that the soul is immortal is given a few lines farther on (11. 10): *ἡ ψυχὴ αὐτοκίνητον, τοῦτο δὲ ἀεικίνητον, τοῦτο δὲ ἀθάνατον, ἡ ψυχὴ ἄρα ἀθάνατον.*

It is important to learn from this passage that according to the Peripatetics, who followed Aristotle, only syllogistic laws stated in variables belong to logic, and not their applications to concrete terms. The concrete terms, i.e. the values of the variables, are called the matter, ὕλη, of the syllogism. If you remove all concrete terms from a syllogism, replacing them by letters, you have removed the matter of the syllogism and what remains is called its form. Let us see of what elements this form consists.

To the form of the syllogism belong, besides the number and the disposition of the variables, the so-called logical constants. Two of them, the conjunctions 'and' and 'if', are auxiliary expressions and form part, as we shall see later, of a logical system which is more fundamental than that of Aristotle. The remaining four constants, viz. 'to belong to all', 'to belong to none', 'to belong to some' and 'to not-belong to some',[1] are characteristic of Aristotelian logic. These constants represent relations between universal terms. The medieval logicians denoted them by *A*, *E*, *I*, and *O* respectively. The whole Aristotelian theory of the syllogism is built up on these four expressions with the help of the conjunctions 'and' and 'if'. We may say therefore: The logic of Aristotle is a theory of the relations *A*, *E*, *I*, and *O* in the field of universal terms.

It is obvious that such a theory has nothing more in common with our thinking than, for instance, the theory of the relations of greater and less in the field of numbers. There are, indeed, some similarities between these two theories. Compare, for example, the syllogism Barbara:

If *a* belongs to all *b*
and *b* belongs to all *c*,
then *a* belongs to all *c*,

with the following arithmetical law:

If *a* is greater than *b*
and *b* is greater than *c*,
then *a* is greater than *c*.

There are, of course, differences between these two laws: the range of variables is not the same, and the relations are different.

[1] *ὑπάρχειν παντί, ὑπάρχειν οὐδενί, ὑπάρχειν τινί, οὐχ ὑπάρχειν τινί = ὑπάρχειν οὐ παντί.* Instead of *ὑπάρχειν* Aristotle sometimes uses the verb *κατηγορεῖσθαι*. Syllogisms in concrete terms are formulated with *εἶναι*. See p. 2, n.; p. 3, n. 1, and the next section (7).

But both relations, although different and occurring between different terms, have one property in common: they are both transitive, i.e. they are particular cases of the formula:

If *a* has the relation *R* to *b*
and *b* has the relation *R* to *c*,
then *a* has the relation *R* to *c*.

It is a curious thing that this very fact was observed by the logicians of the later school of the Stoics. Arguments like 'the first is greater than the second, the second is greater than the third, therefore the first is greater than the third' were called by the Stoics, as Alexander declares, 'non-methodically conclusive' and were not treated as syllogisms in the sense of their logic. Nevertheless, the Stoics regarded such arguments as similar (*ὅμοιοι*) to categorical syllogisms.[1] This observation of the Stoics, which Alexander tries to confute without producing convincing counter-arguments, corroborates the supposition that the logic of Aristotle was conceived as a theory of special relations, like a mathematical theory.

§ 7. *What is formalism?*

Formal logic and formalistic logic are two different things. The Aristotelian logic is formal without being formalistic, whereas the logic of the Stoics is both formal and formalistic. Let us explain what in modern formal logic is meant by 'formalism'.

Modern formal logic strives to attain the greatest possible exactness. This aim can be reached only by means of a precise language built up of stable, visually perceptible signs. Such a language is indispensable for any science. Our own thoughts not formed in words are for ourselves almost inapprehensible and the thoughts of other people, when not bearing an external shape, could be accessible only to a clairvoyant. Every scientific truth, in order to be perceived and verified, must be put into an external form intelligible to everybody. All these statements seem incontestably true. Modern formal logic gives therefore the utmost

[1] Alexander 21. 30 *οἱ ἀμεθόδως περαίνοντες λόγοι παρὰ τοῖς Στωϊκοῖς, οἷον 'τὸ πρῶτον τοῦ δευτέρου μεῖζον, τὸ δὲ δεύτερον τοῦ τρίτου, τὸ ἄρα πρῶτον τοῦ τρίτου μεῖζον.'* Ibid. 345. 13 *τοιοῦτοί εἰσι καὶ οὓς λέγουσιν οἱ νεώτεροι* (i.e. *Στωϊκοί*) *ἀμεθόδως περαίνοντας. οὓς ὅτι μὲν μὴ λέγουσι συλλογιστικῶς συνάγειν, ὑγιῶς λέγουσι... ὅτι δὲ ἡγοῦνται ὁμοίους αὐτοὺς εἶναι τοῖς κατηγορικοῖς συλλογισμοῖς... τοῦ παντὸς διαμαρτάνουσιν.*

attention to precision of language. What is called formalism is the consequence of this tendency. In order to understand what it is, let us analyse the following example.

There exists in logic a rule of inference, called formerly *modus ponens* and now the rule of detachment. According to this rule, if an implication of the form 'If α, then β' is asserted and the antecedent of this implication is asserted too, we are allowed to assert its consequent β. In order to be able to apply this rule we must know that the proposition α, asserted separately, expresses 'the same' thought as the antecedent α of the implication, since only in this case are we allowed to perform the inference. We can state this only in the case where these two α's have exactly the same external form. For we cannot directly grasp the thoughts expressed by these α's, and a necessary, although not sufficient, condition for identifying two thoughts is the external equality of their expressions. When, for instance, asserting the implication 'If all philosophers are men, then all philosophers are mortal' you would also assert as second premiss the sentence 'Every philosopher is a man', you could not get from these premisses the conclusion 'All philosophers are mortal', because you would have no guarantee that the sentence 'Every philosopher is a man' represents the same thought as the sentence 'All philosophers are men'. It would be necessary to confirm by means of a definition that 'Every *A* is *B*' means the same as 'All *A*'s are *B*'s'; on the ground of this definition replace the sentence 'Every philosopher is a man' by the sentence 'All philosophers are men', and only then will it be possible to get the conclusion. By this example you can easily comprehend the meaning of formalism. Formalism requires that the same thought should always be expressed by means of exactly the same series of words ordered in exactly the same manner. When a proof is formed according to this principle, we are able to control its validity on the basis of its external form only, without referring to the meaning of the terms used in the proof. In order to get the conclusion β from the premisses 'If α, then β' and α, we need not know either what α or what β really means; it suffices to notice that the two α's contained in the premisses have the same external form.

Aristotle and his followers, the Peripatetics, were not formalists. As we have already seen, Aristotle is not scrupulously exact in formulating his theses. The most striking case of this inexactitude is the structural discrepancy between the abstract and concrete forms of the syllogisms. Take as an example the syllogism with opposite premisses quoted above, in our section 4.[1] Let *B* and *C* be 'science' and *A* 'medicine'. Aristotle states:

In variables:	In concrete terms:
If *B* belongs to all *A*	If all medicine is science
and *C* belongs to no *A*,	and no medicine is science,
then *C* does not belong to some *B*.[2]	then some science is not science.

The difference of corresponding premisses, of which the two syllogisms consist, is evident. Take, for instance, the first premiss. To the formula '*B* belongs to all *A*' would correspond the sentence 'Science belongs to all medicine', and to the sentence 'All medicine is science' would correspond the formula 'All *A* is *B*'. The sentence in concrete terms, given by Aristotle, cannot be regarded as a substitution of the abstract formula accepted by him. What is the cause of this difference?

Alexander gives three explanations of this problem:[3] the first may be omitted as unimportant, the last is a philosophical one and is, in my opinion, wrong; only the second deserves our attention. According to this explanation, in formulae with the verb 'to be predicated of something' and, we may add, with the verb 'to belong to something', the subject and the predicate are better distinguishable (*γνωριμώτεροι*) than, we may add again, in formulae with the verb 'to be'. In fact, in formulae with 'to be' the subject as well as the predicate is used in the nominative; in formulae preferred by Aristotle only the predicate is in the nominative, and the subject is either in the genitive or in the dative and therefore can be more easily distinguished from the predicate. Very instructive, too, is the final remark of Alexander, from which it follows that to say 'Virtue is predicated of all justice' instead of the customary 'All justice is virtue' was felt in Ancient Greek to be as artificial as in modern languages.

[1] See p. 9, n. 3.

[2] The conclusion in variables is dropped in the Greek text.

[3] Alexander 54. 21 *χρῆται δὲ τῷ κατὰ παντὸς καὶ τῷ κατὰ μηδενὸς ἐν τῇ διδασκαλίᾳ, ὅτι διὰ τούτων γνώριμος ἡ συναγωγὴ τῶν λόγων, καὶ ὅτι οὕτως λεγομένων γνωριμώτερος ὅ τε κατηγορούμενος καὶ ὁ ὑποκείμενος, καὶ ὅτι πρῶτον τῇ φύσει τὸ κατὰ παντὸς τοῦ ἐν ὅλῳ αὐτῷ, ὡς προείρηται. ἡ μέντοι χρῆσις ἡ συλλογιστικὴ ἐν τῇ συνηθείᾳ ἀνάπαλιν ἔχει· οὐ γὰρ ἡ ἀρετὴ λέγεται κατὰ πάσης δικαιοσύνης, ἀλλ' ἀνάπαλιν πᾶσα δικαιοσυνὴ ἀρετή. διὸ καὶ δεῖ κατ' ἀμφοτέρας τὰς ἐκφορὰς γυμνάζειν ἑαυτούς, ἵνα τῇ τε χρήσει παρακολουθεῖν δυνώμεθα καὶ τῇ διδασκαλίᾳ.*

There are still more cases of inexactitude in Aristotelian logic. Aristotle constantly uses different phrases for the same thoughts. I shall give only a few examples of this kind. He begins his syllogistic with the words '*A* is predicated of all *B*', but shortly he changes these words into the phrase '*A* belongs to all *B*', which seems to be regular. The words 'is predicated' and 'belongs' are frequently omitted, sometimes even the important sign of the quantity 'all' is dropped. Besides the form '*A* belongs to some *B*' there are forms which may be translated '*A* belongs to some of the *B*'s'. The premisses of the syllogism are combined by means of different conjunctions. Syllogistic necessity is expressed in different ways and is sometimes entirely omitted.[1] Although these inexactitudes have no bad consequences for the system, they contribute in no way to its clearness or simplicity.

This procedure of Aristotle is probably not accidental, but seems to derive from some preconceptions. Aristotle says occasionally that we ought to exchange equivalent terms, words for words and phrases for phrases.[2] Commenting on this passage, Alexander declares that the essence of the syllogism depends not on words but on their meanings.[3] This statement, which is manifestly directed against the Stoics, can be understood thus: the syllogism does not change its essence, i.e. it remains a syllogism, if some of its expressions are replaced by other equivalent expressions, e.g. if the expression 'to be predicated of all' is replaced by the equivalent expression 'to belong to all'. The Stoics were of a directly opposite opinion. They would say that the essence of the syllogism depends on words, but not on their meanings. If therefore the words are changed, the syllogism ceases to exist. This is

[1] The phrase *τὸ A κατὰ παντὸς τοῦ B* (*κατηγορεῖται* is twice omitted) is used in the mood Barbara (see p. 3, n. 2), *τὸ A παντὶ τῷ B* (*ὑπάρχει* is altogether omitted) is used in another formulation of the same mood (see p. 10, n. 5). The phrase *τὸ A τινὶ τῶν B* appears in the laws of conversion; elsewhere, e.g. in the mood Disamis, we have *τὸ A τινὶ τῷ B* (see p. 9, n. 1). The logically important word *παντί* is altogether omitted in a formulation of the mood Barbara (see p. 2, n.). The conjunction 'and' is for the most part denoted by *μέν . . . δέ* (see, for example, p. 7, n. or p. 10, n. 2), sometimes by *καί* (see p. 3, n. 2; p. 10, n. 5). Syllogistic necessity is as a rule expressed by *ἀνάγκη ὑπάρχειν* (see p. 7, n. or p. 9, n. 1), in the mood Felapton it is denoted by *ὑπάρξει ἐξ ἀνάγκης* (see p. 9, n. 4). In one case it is dropped (see p. 10, n. 5).

[2] *An. pr.* i. 39, $49^{b}3$ *δεῖ δὲ καὶ μεταλαμβάνειν ἃ τὸ αὐτὸ δύναται, ὀνόματα ἀντ' ὀνομάτων καὶ λόγους ἀντὶ λόγων.*

[3] Alexander 372. 29 *οὐκ ἐν ταῖς λέξεσιν ὁ συλλογισμὸς τὸ εἶναι ἔχει, ἀλλ' ἐν τοῖς σημαινομένοις.*

illustrated by Alexander with an example from the logic of the Stoics. The rule of inference called *modus ponens*:

If α, then β;
but α;
therefore β,

is the first 'indemonstrable' syllogism of the Stoics. Both the Stoics and the Peripatetics seem mistakenly to regard the phrases 'If α, then β' and 'α entails β' as having the same meaning. But if, in the syllogism given above, you replace the premiss 'If α, then β' by 'α entails β', saying:

α entails β;
but α;
therefore β,

you get according to the Stoics a valid rule of inference, but not a syllogism. The logic of the Stoics is formalistic.[1]

[1] Alexander 373. 28 *Ἀριστοτέλης μὲν οὖν οὕτως περὶ τῶν κατὰ τὰς λέξεις μεταλήψεων φέρεται* (see p. 18, n. 2). *οἱ δὲ νεώτεροι* (i.e. *οἱ Στωϊκοί*), *ταῖς λέξεσιν ἐπακολουθοῦντες οὐκέτι δὲ τοῖς σημαινομένοις, οὐ ταὐτόν φασι γίνεσθαι ἐν ταῖς εἰς τὰς ἰσοδυναμούσας λέξεις μεταλήψεσι τῶν ὅρων· ταὐτὸν γὰρ σημαίνοντος τοῦ 'εἰ τὸ A τὸ B' τῷ 'ἀκολουθεῖ τῷ A τὸ B', συλλογιστικὸν μὲν λόγον φασὶν εἶναι τοιαύτης ληφθείσης τῆς λέξεως 'εἰ τὸ A τὸ B, τὸ δὲ A, τὸ ἄρα B', οὐκέτι δὲ συλλογιστικὸν ἀλλὰ περαντικὸν τὸ 'ἀκολουθεῖ τῷ A τὸ B, τὸ δὲ A, τὸ ἄρα B'.*

CHAPTER II

THESES OF THE SYSTEM

§ 8. *Theses and rules of inference*

THE Aristotelian theory of the syllogism is a system of true propositions concerning the constants *A*, *E*, *I*, and *O*. True propositions of a deductive system I call theses. Almost all theses of the Aristotelian logic are implications, i.e. propositions of the form 'If α, then β'. There are known only two theses of this logic not beginning with 'if', viz. the so-called laws of identity: '*A* belongs to all *A*' or 'All *A* is *A*', and '*A* belongs to some *A*' or 'Some *A* is *A*'. Neither of these laws was explicitly stated by Aristotle, but they were known to the Peripatetics.[1]

The implications belonging to the system are either laws of conversion (and laws of the square of opposition not mentioned in the *Prior Analytics*) or syllogisms. The laws of conversion are simple implications, for instance: 'If *A* belongs to all *B*, then *B* belongs to some *A*.'[2] The antecedent of this implication is the premiss '*A* belongs to all *B*', the consequent is '*B* belongs to some *A*'. This implication is regarded as true for all values of the variables *A* and *B*.

All Aristotelian syllogisms are implications of the type 'If α and β, then γ', where α and β are the two premisses and γ is the conclusion. The conjunction of the premisses 'α and β' is the antecedent, the conclusion γ is the consequent. As an example take the following formulation of the mood Barbara:

If *A* belongs to all *B*
and *B* belongs to all *C*,
then *A* belongs to all *C*.

In this example α means the premiss '*A* belongs to all *B*', β the premiss '*B* belongs to all *C*', and γ the conclusion '*A* belongs to all *C*'. This implication is also regarded as true for all values of the variables *A*, *B*, and *C*.

[1] Cf. p. 9, n. 5, p. 10, n. 1. In the passage quoted in the latter note Alexander says that the proposition '*A* does not belong to some *A*' is absurd. That means that the contradictory proposition '*A* belongs to all *A*' is true.

[2] *An. pr.* i. 2, 25^{a}17 *εἰ δὲ παντὶ τὸ Α τῷ Β, καὶ τὸ Β τινὶ τῷ Α ὑπάρξει.*

It must be said emphatically that no syllogism is formulated by Aristotle as an inference with the word 'therefore' (*ἄρα*), as is done in the traditional logic. Syllogisms of the form:

All *B* is *A*;
all *C* is *B*;
therefore
all *C* is *A*

are not Aristotelian. We do not meet them until Alexander.[1] This transference of the Aristotelian syllogisms from the implicational form into the inferential is probably due to the influence of the Stoics.

The difference between the Aristotelian and the traditional syllogism is fundamental. The Aristotelian syllogism as an implication is a proposition, and as a proposition must be either true or false. The traditional syllogism is not a proposition, but a set of propositions which are not unified so as to form one single proposition. The two premisses written usually in two different lines are stated without a conjunction, and the connexion of these loose premisses with the conclusion by means of 'therefore' does not give a new compound proposition. The famous Cartesian principle, 'Cogito, ergo sum', is not a true principle, because it is not a proposition. It is an inference, or, according to a scholastic terminology, a consequence. Inferences and consequences, not being propositions, are neither true nor false, as truth and falsity belong only to propositions. They may be valid or not. The same has to be said of the traditional syllogism. Not being a proposition the traditional syllogism is neither true nor false; it can be valid or invalid. The traditional syllogism is either an inference, when stated in concrete terms, or a rule of inference, when stated in variables. The sense of such a rule may be explained by the example given above: When you put such values for *A*, *B*, and *C* that the premisses '*A* belongs to all *B*' and '*B* belongs to all *C*' are true, then you must accept as true the conclusion '*A* belongs to all *C*'.

If you find a book or an article where no difference is made between the Aristotelian and the traditional syllogism, you may

[1] In Alexander 47. 9 we find a syllogism in concrete terms with *ἄρα*: *πᾶν ζῷον οὐσία ἐστί, πᾶν ζῷον ἔμψυχόν ἐστι, τὶς ἄρα οὐσία ἔμψυχός ἐστιν.* At 382. 18 we have a complex syllogism in four variable terms with *ἄρα*: *τὸ Α παντὶ τῷ Β, τὸ Β παντὶ τῷ Γ, τὸ Α οὐδενὶ τῷ Δ, τὸ ἄρα Δ οὐδενὶ τῷ Γ.*

be sure that the author is either ignorant of logic or has never seen the Greek text of the *Organon*. Scholars like Waitz, the modern editor and commentator of the *Organon*, Trendelenburg, the compiler of the *Elementa logices Aristoteleae*, Prantl, the historian of logic, all knew the Greek text of the *Organon* well, but nevertheless they did not see the difference between the Aristotelian and the traditional syllogism. Only Maier seems to have felt for a moment that something is wrong here, when he asks for permission to replace the Aristotelian syllogism by the more familiar and more convenient form of the later logic; immediately afterwards he quotes the mood Barbara in its usual traditional form, neglecting differences he has seen between this form and that of Aristotle, and does not even say what differences he has seen.[1] When we realize that the difference between a thesis and a rule of inference is from the standpoint of logic a fundamental one, we must agree that an exposition of Aristotelian logic which disregards it cannot be sound. We have to this day no genuine exposition of Aristotelian logic.

It is always easy to deduce from an implicational thesis the corresponding rule of inference. Let us suppose that an implicational proposition 'If α, then β' is true: if α is true, we can always get β by detachment, so that the rule 'α therefore β' is valid. When the antecedent of an implicational thesis is a conjunction, as in the Aristotelian syllogisms, we must first change the conjunctional form 'If α and β, then γ' into the purely implicational form 'If α, then if β, then γ'. A moment of reflection is sufficient to convince ourselves that this transformation is correct. Supposing now that α and β are true premisses of a syllogism, we get the conclusion γ, applying the rule of detachment twice to the purely implicational form of the syllogism. If, therefore, an Aristotelian syllogism of the form 'If α and β, then γ' is true, the corresponding traditional mood of the form 'α, β, therefore γ' is valid. But conversely, it seems impossible to deduce the corresponding Aristotelian syllogism from a valid traditional mood by known logical rules.

[1] Maier, op. cit., vol. ii *a*, p. 74, n. 2: 'Es ist vielleicht gestattet, hier und im Folgenden die geläufigere Darstellungsform der späteren Logik, die zugleich leichter zu handhaben ist, an die Stelle der aristotelischen zu setzen.' The mood Barbara is quoted ibid., p. 75, thus:

alles B ist A
alles C ist B
alles C ist A

where the stroke replaces the word 'therefore'.

§ 9. *The syllogistic figures*

There are some controversial problems connected with the Aristotelian logic that are of historical interest without having any great logical importance. Among these is the problem of the syllogistic figures. The division of the syllogisms into figures has, in my opinion, only a practical aim: we want to be sure that no true syllogistic mood is omitted.

Aristotle divided the syllogistic moods into three figures. The shortest and clearest description of these figures is to be found not in the systematic part of the *Prior Analytics* but in the later chapters of that work. If we want, Aristotle says, to prove *A* of *B* syllogistically, we must take something common in relation to both, and this is possible in three ways: by predicating either *A* of *C* and *C* of *B*, or *C* of both, or both of *C*. These are the figures of which we have spoken, and it is clear that every syllogism must be made in one or other of these figures.[1]

It follows from this that *A* is the predicate and *B* the subject of the conclusion we have to prove syllogistically. *A* is called, as we shall see later, the major term and *B* the minor; *C* is the middle term. The position of the middle term as subject or predicate of the premisses is the principle by which Aristotle divides the syllogistic moods into figures. Aristotle says explicitly that we shall recognize the figure by the position of the middle term.[2] In the first figure the middle term is the subject of the major term and the predicate of the minor term, in the second figure it is the predicate, and in the last figure the subject, of both the other terms. Aristotle, however, is mistaken when he says that every syllogism must be in one of these three figures. There is a fourth possibility, viz. that the middle term is the predicate of the major term and the subject of the minor term. Moods of this kind are now spoken of as belonging to the fourth figure.

In the above passage Aristotle has overlooked this fourth

[1] *An. pr.* i. 23, 40^{b}30 *εἰ δὴ δέοι τὸ A κατὰ τοῦ B συλλογίσασθαι ἢ ὑπάρχον ἢ μὴ ὑπάρχον, ἀνάγκη λαβεῖν τι κατά τινος.* 41^{a}13 *εἰ οὖν ἀνάγκη μέν τι λαβεῖν πρὸς ἄμφω κοινόν, τοῦτο δ' ἐνδέχεται τριχῶς (ἢ γὰρ τὸ A τοῦ Γ καὶ τὸ Γ τοῦ B κατηγορήσαντας, ἢ τὸ Γ κατ' ἀμφοῖν, ἢ ἄμφω κατὰ τοῦ Γ), ταῦτα δ' ἐστὶ τὰ εἰρημένα σχήματα, φανερὸν ὅτι πάντα συλλογισμὸν ἀνάγκη γίνεσθαι διὰ τούτων τινὸς τῶν σχημάτων.*

[2] Ibid. 32, 47^{b}13 *τῇ τοῦ μέσου θέσει γνωριοῦμεν τὸ σχῆμα.*

possibility, although a few chapters farther on he himself gives a proof by a syllogism in the fourth figure. It is the same problem again: we have to prove *A* of *E* syllogistically, where *A* is the major term and *E* the minor. Aristotle gives practical indications how to solve this problem. We must construct a list of universal propositions having the terms *A* and *E* as subjects or predicates. In this list we shall have four types of universal affirmative proposition (I omit the negative propositions), '*B* belongs to all *A*', '*A* belongs to all *C*', '*Z* belongs to all *E*', and '*E* belongs to all *H*'. Each of the letters *B*, *C*, *Z*, and *H* represents any term fulfilling the above conditions. When we find among the *C*'s a term identical with a term among the *Z*'s, we get two premisses with a common term, say *Z*: '*A* belongs to all *Z*' and '*Z* belongs to all *E*', and the proposition '*A* belongs to all *E*' is proved in the mood Barbara. Let us now suppose that we cannot prove the universal proposition '*A* belongs to all *E*', as the *C*'s and *Z*'s have no common term, but we want at least to prove the particular proposition '*A* belongs to some *E*'. We can prove it in two different ways: if there is a term among the *C*'s identical with a term among the *H*'s, say *H*, we get the mood Darapti of the third figure: '*A* belongs to all *H*', '*E* belongs to all *H*', therefore '*A* must belong to some *E*'. But there is still another way when we find among the *H*'s a term identical with a term among the *B*'s, say *B*; we then get a syllogism with the premisses '*E* belongs to all *B*' and '*B* belongs to all *A*', from which we deduce the proposition '*A* belongs to some *E*' by converting the conclusion '*E* belongs to all *A*' obtained from these premisses by the mood Barbara.[1]

This last syllogism: 'If *E* belongs to all *B* and *B* belongs to all *A*, then *A* belongs to some *E*', is a mood neither of the first figure nor of the second or third. It is a syllogism where the middle term

[1] *An. pr.* i. 28, 44[a]12–35 *ἔστω γὰρ τὰ μὲν ἐπόμενα τῷ A ἐφ' ὧν B, οἷς δ' αὐτὸ ἕπεται, ἐφ' ὧν Γ . . .· πάλιν δὲ τῷ E τὰ μὲν ὑπάρχοντα, ἐφ' οἷς Z, οἷς δ' αὐτὸ ἕπεται, ἐφ' οἷς H . . . . εἰ μὲν οὖν ταὐτό τι ἔσται τῶν Γ τινὶ τῶν Z, ἀνάγκη τὸ A παντὶ τῷ E ὑπάρχειν· τὸ μὲν γὰρ Z παντὶ τῷ E, τῷ δὲ Γ παντὶ τὸ A, ὥστε παντὶ τῷ E τὸ A. εἰ δὲ τὸ Γ καὶ τὸ H ταὐτόν, ἀνάγκη τινὶ τῶν E τὸ A ὑπάρχειν· τῷ μὲν γὰρ Γ τὸ A, τῷ δὲ H τὸ E παντὶ ἀκολουθεῖ. . . . εἰ δὲ τῷ H τὸ B ταὐτόν, ἀντεστραμμένος ἔσται συλλογισμός· τὸ μὲν γὰρ E τῷ A ὑπάρξει παντί, τὸ γὰρ B τῷ A, τὸ δὲ E τῷ B (ταὐτὸ γὰρ ἦν τῷ H)· τὸ δὲ A τῷ E παντὶ μὲν οὐκ ἀνάγκη ὑπάρχειν, τινὶ δ' ἀνάγκη διὰ τὸ ἀντιστρέφειν τὴν καθόλου κατηγορίαν τῇ κατὰ μέρος.* I read *τὴν καθόλου κατηγορίαν τῇ* with codex B (see Waitz, i. 196; the footnote in Bekker to 44[a]34 seems to be a misprint) and Alexander 306. 16 against *τῇ καθόλου κατηγορίᾳ τὴν* in Bekker and Waitz. I am glad to see that this reading is also accepted by Sir David Ross.

B is the predicate of the major term *A* and the subject of the minor term *E*. It is the mood Bramantip of the fourth figure. Nevertheless it is as valid as any other Aristotelian mood. Aristotle calls it a 'converted syllogism' (*ἀντεστραμμένος συλλογισμός*) because he proves this mood by converting the conclusion of the mood Barbara. There are two other moods, Camestres of the second figure and Disamis of the third, which Aristotle proves in the same manner, by converting the conclusion of moods of the first figure. Let us consider the proof of Disamis: 'If *R* belongs to all *S* and *P* belongs to some *S*, then *P* belongs to some *R*'. As the second premiss can be converted into '*S* belongs to some *P*', we get by the mood Darii the conclusion '*R* belongs to some *P*'. By converting this conclusion into '*P* belongs to some *R*' we get the proof of Disamis. Aristotle here applies the conversion to the conclusion of the mood Darii, which gives another syllogism of the fourth figure called Dimaris: 'If *R* belongs to all *S* and *S* belongs to some *P*, then *P* belongs to some *R*.'[1]

All these deductions are logically correct, and so are the moods obtained by their means. Aristotle knows, indeed, that besides the fourteen moods of the first, second, and third figures established by him systematically in the early chapters of the *Prior Analytics* there are still other true syllogisms. Two of them are quoted by him at the end of this systematic exposition. It is evident, he says, that in all the figures, whenever a syllogism does not result, if both the terms are affirmative or negative nothing necessary follows at all, but if one is affirmative, the other negative, and if the negative is stated universally, a syllogism always results linking the minor to the major term, e.g. if *A* belongs to all or some *B*, and *B* belongs to no *C*; for if the premisses are converted it is necessary that *C* does not belong to some *A*.[2] From the second premiss

[1] *An. pr.* i. 6, 28[b]7 *εἰ γὰρ τὸ μὲν P παντὶ τῷ Σ τὸ δὲ Π τινί, ἀνάγκη τὸ Π τινὶ τῷ P ὑπάρχειν. ἐπεὶ γὰρ ἀντιστρέφει τὸ καταφατικόν, ὑπάρξει τὸ Σ τινὶ τῷ Π, ὥστ' ἐπεὶ τὸ μὲν P παντὶ τῷ Σ, τὸ δὲ Σ τινὶ τῷ Π, καὶ τὸ P τινὶ τῷ Π ὑπάρξει· ὥστε τὸ Π τινὶ τῷ P.* This passage refutes the assertion of Friedrich Solmsen that Aristotle was not willing to apply the procedure of conversion to the conclusion. *Die Entstehung der aristotelischen Logik und Rhetorik*, Berlin (1929), p. 55: 'Die Umkehrung dringt in die conclusio ein, in der Aristoteles sie nicht kennen wollte.'

[2] *An. pr.* i. 7, 29[a]19 *δῆλον δὲ καὶ ὅτι ἐν ἅπασι τοῖς σχήμασιν, ὅταν μὴ γίνηται συλλογισμός, κατηγορικῶν μὲν ἢ στερητικῶν ἀμφοτέρων ὄντων τῶν ὅρων οὐδὲν ὅλως γίνεται ἀναγκαῖον, κατηγορικοῦ δὲ καὶ στερητικοῦ, καθόλου ληφθέντος τοῦ στερητικοῦ, ἀεὶ γίνεται συλλογισμὸς τοῦ ἐλάττονος ἄκρου πρὸς τὸ μεῖζον, οἷον εἰ τὸ μὲν A παντὶ τῷ B ἢ τινί, τὸ δὲ B μηδενὶ τῷ Γ· ἀντιστρεφομένων γὰρ τῶν προτάσεων ἀνάγκη τὸ Γ τινὶ τῷ A μὴ ὑπάρχειν.*

given here by Aristotle we get by conversion the proposition '*C* belongs to no *B*', from the first premiss we get '*B* belongs to some *A*', and from these two propositions results, according to the mood Ferio of the first figure, the conclusion '*C* does not belong to some *A*'. Two new syllogistic moods are thus proved, called later Fesapo and Fresison:

If *A* belongs to all *B*	If *A* belongs to some *B*
and *B* belongs to no *C*,	and *B* belongs to no *C*,
then *C* does not belong to some *A*.	then *C* does not belong to some *A*.

Aristotle calls the minor term *C* and the major term *A* because he treats the premisses from the point of view of the first figure. He says, therefore, that from the given premisses a conclusion results in which the minor term is predicated of the major.

Three other syllogisms belonging to the fourth figure are mentioned by Aristotle at the beginning of Book II of the *Prior Analytics*. Aristotle states here that all universal syllogisms (i.e. syllogisms with a universal conclusion) give more than one result, and of particular syllogisms the affirmative yield more than one, the negative yield only one conclusion. For all premisses are convertible except the particular negative; and the conclusion states something about something. Consequently all syllogisms except the particular negative yield more than one conclusion, e.g. if *A* has been proved to belong to all or to some *B*, then *B* must belong to some *A*; and if *A* has been proved to belong to no *B*, then *B* belongs to no *A*. This is a different conclusion from the former. But if *A* does not belong to some *B*, it is not necessary that *B* should not belong to some *A*, for it may possibly belong to all *A*.[1]

We see from this passage that Aristotle knows the moods of the fourth figure, called later Bramantip, Camenes, and Dimaris, and that he gets them by conversion of the conclusion of the moods Barbara, Celarent, and Darii. The conclusion of a syllogism is a proposition stating something about something, i.e. a premiss, and therefore the laws of conversion can be applied to it.

[1] *An. pr.* ii. 1, 53[a]4 *οἱ μὲν καθόλου* (scil. *συλλογισμοὶ*) *πάντες ἀεὶ πλείω συλλογίζονται, τῶν δ' ἐν μέρει οἱ μὲν κατηγορικοὶ πλείω, οἱ δ' ἀποφατικοὶ τὸ συμπέρασμα μόνον. αἱ μὲν γὰρ ἄλλαι προτάσεις ἀντιστρέφουσιν, ἡ δὲ στερητικὴ οὐκ ἀντιστρέφει· τὸ δὲ συμπέρασμα τὶ κατά τινός ἐστιν. ὥσθ' οἱ μὲν ἄλλοι συλλογισμοὶ πλείω συλλογίζονται, οἷον εἰ τὸ A δέδεικται παντὶ τῷ B ἢ τινί, καὶ τὸ B τινὶ τῷ A ἀναγκαῖον ὑπάρχειν· καὶ εἰ μηδενὶ τῷ B τὸ A, οὐδὲ τὸ B οὐδενὶ τῷ A. τοῦτο δ' ἕτερον τοῦ ἔμπροσθεν. εἰ δὲ τινὶ μὴ ὑπάρχει, οὐκ ἀνάγκη καὶ τὸ B τινὶ τῷ A μὴ ὑπάρχειν· ἐνδέχεται γὰρ παντὶ ὑπάρχειν.*

It is important that propositions of the type '*A* belongs to no *B*' and '*B* belongs to no *A*' are regarded by Aristotle as different.

It follows from these facts that Aristotle knows and accepts all the moods of the fourth figure. This must be emphasized against the opinion of some philosophers that he rejected these moods. Such a rejection would be a logical error which cannot be imputed to Aristotle. His only mistake is the omission of these moods in the systematic division of the syllogisms. We do not know why he did so. Philosophical reasons, as we shall see later, must be excluded. The most probable explanation is given, in my opinion, by Bocheński,[1] who supposes that Book I, chapter 7 and Book II, chapter 1 of the *Prior Analytics*, where these new moods are mentioned, were composed by Aristotle later than the systematic exposition of chapters 4–6 of Book I. This hypothesis seems to me the more probable, as there are many other points in the *Prior Analytics* suggesting that the contents of this work grew during its composition. Aristotle did not have time to draw up systematically all the new discoveries he had made, and left the continuation of his logical work to his pupil Theophrastus. Theophrastus, indeed, found for the moods of the fourth figure which are 'homeless' in Aristotle's system a place among the moods of the first figure.[2] For this purpose he had to introduce a slight modification into the Aristotelian definition of the first figure. Instead of saying that in the first figure the middle term is the subject of the major and the predicate of the minor, as Aristotle does,[3] he said generally that in the first figure the middle term is the subject of one premiss and the predicate of another. Alexander repeats this definition, which probably comes from Theophrastus, and seems not to see that it differs from the Aristotelian description of the first figure.[4] The correction of

[1] I. M. Bocheński, O.P., *La Logique de Théophraste*, Collectanea Friburgensia, Nouvelle Série, fasc. xxxii, Fribourg en Suisse (1947), p. 59.

[2] Alexander 69. 27 *Θεόφραστος δὲ προστίθησιν ἄλλους πέντε τοῖς τέσσαρσι τούτοις οὐκέτι τελείους οὐδ' ἀναποδείκτους ὄντας, ὧν μνημονεύει καὶ ὁ Ἀριστοτέλης, τῶν μὲν ἐν τούτῳ τῷ βιβλίῳ προελθών, τῶν δὲ ἐν τῷ μετὰ τοῦτο τῷ δευτέρῳ κατ' ἀρχάς.* Cf. ibid. 110. 12.

[3] Cf. p. 23, n. 1.

[4] Alexander 258. 17 (ad i. 23) *ἡ δὲ τοῦ μέσου σχέσις πρὸς τά, ὧν λαμβάνεται μέσον, τριχῶς γίνεται (ἢ γὰρ ἐν μέσῳ τίθεται αὐτῶν τῷ μὲν ὑποκείμενος αὐτῶν τοῦ δὲ κατηγορούμενος, ἢ ἀμφοτέρων κατηγορεῖται, ἢ ἀμφοτέροις ὑπόκειται).* Ibid. 349. 5 (ad i. 32) *ἂν μὲν γὰρ ὁ μέσος ἐν ἀμφοτέραις ὢν ταῖς προτάσεσιν οὕτως ᾖ ὡς τοῦ μὲν κατηγορεῖσθαι αὐτῶν τῷ δὲ ὑποκεῖσθαι, πρῶτον ἔσται σχῆμα.*

Theophrastus is as good a solution of the problem of the syllogistic figures as the addition of a new figure.

§ 10. *The major, middle, and minor terms*

There is still another error committed by Aristotle in the *Prior Analytics*, with more serious consequences. It concerns the definition of the major, minor, and middle terms as given in his characterization of the first figure. This begins with the words: 'Whenever three terms are so related to one another that the last is contained in the middle and the middle is contained or not in the first, the extremes must form a perfect syllogism.' This is how he begins; in the next sentence he explains what he means by the middle term: 'I call that term the middle which is itself contained in another and contains another in itself, which by position also becomes the middle.'[1] Aristotle then investigates the syllogistic forms of the first figure with universal premisses without using the expressions 'major term' and 'minor term'. These expressions occur for the first time when he comes to the moods of the first figure with particular premisses. Here we find the following explanations: 'I call that term the major in which the middle term is contained and that term the minor which comes under the middle.'[2] These explanations of the major and the minor term, like that of the middle term, are expressed quite generally. It would seem that Aristotle intends to apply them to all moods of the first figure.[3] If he thought, however, that they are capable of covering all cases, he was mistaken.

In fact these explanations can be applied only to syllogisms of the mood Barbara with concrete terms and true premisses, e.g.:

(1) If all birds are animals
and all crows are birds,
then all crows are animals.

In this syllogism there is a term, 'bird', which is itself contained in another term, 'animal', and contains in itself a third term,

[1] *An. pr.* i. 4, $25^{b}32$ ὅταν οὖν ὅροι τρεῖς οὕτως ἔχωσι πρὸς ἀλλήλους ὥστε τὸν ἔσχατον ἐν ὅλῳ εἶναι τῷ μέσῳ καὶ τὸν μέσον ἐν ὅλῳ τῷ πρώτῳ ἢ εἶναι ἢ μὴ εἶναι, ἀνάγκη τῶν ἄκρων εἶναι συλλογισμὸν τέλειον. καλῶ δὲ μέσον μὲν ὃ καὶ αὐτὸ ἐν ἄλλῳ καὶ ἄλλο ἐν τούτῳ ἐστίν, ὃ καὶ τῇ θέσει γίνεται μέσον.

[2] Ibid., $26^{a}21$ λέγω δὲ μεῖζον μὲν ἄκρον ἐν ᾧ τὸ μέσον ἐστίν, ἔλαττον δὲ τὸ ὑπὸ τὸ μέσον ὄν.

[3] Maier, op. cit., vol. ii *a*, pp. 49, 55, really treats them as definitions valid for all the moods of the first figure.

'crow'. According to the given explanation 'bird' would be the middle term. Consequently 'animal' would be the major term and 'crow' the minor term. It is evident that the major term is so called because it is the largest in extent, as the minor term is the smallest.

We know, however, that syllogisms with concrete terms are only applications of logical laws, but do not belong to logic themselves. The mood Barbara as a logical law must be stated with variables:

(2) If all *B* is *A*
and all *C* is *B*,
then all *C* is *A*.

To this logical law the given explanations are not applicable, because it is not possible to determine extensional relations between variables. It may be said that *B* is the subject in the first premiss and the predicate in the second, but it cannot be stated that *B* is contained in *A* or that it contains *C*; for the syllogism (2) is true for all values of the variables *A*, *B*, and *C*, even for those which do not verify its premisses. Take 'bird' for *A*, 'crow' for *B*, and 'animal' for *C*: you get a true syllogism:

(3) If all crows are birds
and all animals are crows,
then all animals are birds.

The extensional relations of the terms 'crow', 'bird', and 'animal' are of course independent of syllogistic moods and remain the same in syllogism (3) as they were in (1). But the term 'bird' is no longer the middle term in (3) as it was in (1); 'crow' is the middle term in (3) because it occurs in both premisses, and the middle term must be common to both premisses. This is the definition of the middle term accepted by Aristotle for all figures.[1] This general definition is incompatible with the special explanation given by Aristotle for the first figure. The special explanation of the middle term is obviously wrong. It is evident also that the explanations of the major and minor terms which Aristotle gives for the first figure are wrong, too.

Aristotle does not give a definition of the major and minor terms valid for all figures; but practically he treats the predicate

[1] *An. pr.* i. 32, $47^{a}38$ μέσον δὲ θετέον τῶν ὅρων τὸν ἐν ἀμφοτέραις ταῖς προτάσεσι λεγόμενον· ἀνάγκη γὰρ τὸ μέσον ἐν ἀμφοτέραις ὑπάρχειν ἐν ἅπασι τοῖς σχήμασιν.

of the conclusion as the major term and the subject of the conclusion as the minor term. It is easy to see how misleading this terminology is: in syllogism (3) the major term 'bird' is smaller in extension than the minor term 'animal'. If the reader feels a difficulty in accepting syllogism (3) because of its false minor, he may read 'some animals' instead of 'all animals'. The syllogism:

(4) If all crows are birds
and some animals are crows,
then some animals are birds

is a valid syllogism of the mood Darii with true premisses. And here again, as in syllogism (3), the largest term 'animal' is the minor term; 'bird', middle in extension, is the major term; and the smallest term, 'crow', is the middle term.

The difficulties we have already met are still greater when we take as examples syllogisms with negative premisses, e.g. the mood Celarent:

If no *B* is *A*
and all *C* is *B*,
then no *C* is *A*.

B is the middle term; but does it fulfil the conditions laid down by Aristotle for the middle term of the first figure? Certainly not. And which of the terms, *C* or *A*, is the major and which is the minor? How can we compare these terms with respect to their extension? There is no positive answer to these last questions, as they spring from a mistaken origin.[1]

§ 11. *The history of an error*

The faulty definition of the major and the minor terms, given by Aristotle for the first figure, and the misleading terminology he adopts, were already in antiquity a source of difficulty. The problem arose in the case of the second figure. All the moods of this figure have a negative conclusion and the first two moods, called later Cesare and Camestres, yield a universal negative conclusion. From the premisses '*M* belongs to all *N*' and '*M* belongs to no *X*' follows the conclusion '*X* belongs to no *N*', and by conversion of this result we get a second conclusion, '*N* belongs to no *X*'. In both syllogisms *M* is the middle term; but how are we to decide which of the two remaining terms, *N* and *X*, is the major term and which is the minor? Do major and minor terms exist 'by nature' (*φύσει*) or only 'by convention' (*θέσει*)?[1]

Such problems, according to Alexander, were raised by the later Peripatetics. They saw that in universal affirmative premisses there can be a major term by nature, because in such premisses the predicate is larger in extension (*ἐπὶ πλέον*) than the subject, but the same is not true in universal negative premisses.[2] We cannot know, for instance, which of the terms 'bird' or 'man' is major, because it is equally true that 'no bird is a man' and that 'no man is a bird'. Herminus, the teacher of Alexander, tried to answer this question by modifying the meaning of the expression 'major term'. He says that of two such terms, 'bird' and 'man', that is the major which in a systematic classification of the animals is nearer to the common genus 'animal'. In our example it is the term 'bird'.[3] Alexander is right when he rejects this theory and its further elaboration given by Herminus, but he also rejects the opinion that the major term is the predicate of the conclusion. The major term, he says, would not be fixed in this case, as the universal negative premiss is convertible, and what till now has been a major term instantly becomes a minor, and it would depend upon us to make the same term major and minor.[4] His own solution is based on the assumption that when we are forming a syllogism we are choosing premisses for a given problem

[1] We have no guarantee, as Keynes (op. cit., p. 286) justly remarks, that the major term will be the largest in extension and the minor the smallest, when one of the premisses is negative or particular. Thus, Keynes continues, 'the syllogism—No *M* is *P*, All *S* is *M*, therefore, No *S* is *P*—yields as one case [here there follows a diagram representing three circles *M*, *P*, and *S*, a large *S* included in a larger *M*, outside of them a small *P*] where the major term may be the smallest in extent, and the middle the largest.' Keynes forgets that it is not the same to draw a small circle *P* outside of a large circle *S* and to maintain that the term *P* is smaller in extent than the term *S*. Terms can be compared with respect to their extent only in the case when one of them is contained in the other.

[1] Alexander 72. 17 *ζητεῖται, εἰ φύσει ἐν δευτέρῳ σχήματι μείζων τίς ἐστι καὶ ἐλάττων ἄκρος, καὶ τίνι οὗτος κριθήσεται.*

[2] Ibid. 72. 24 *ἐπὶ μὲν γὰρ τῶν καταφατικῶν μείζων ὁ κατηγορούμενος καθόλου, ὅτι καὶ ἐπὶ πλέον· διὰ τούτου γὰρ οὐδὲ ἀντιστρέφει· ὥστε φύσει αὐτῷ τὸ μείζονα εἶναι ὑπάρχει. ἐπὶ δὲ τῶν καθόλου ἀποφατικῶν οὐκέτι τοῦτο ἀληθές.*

[3] Ibid. 27 *Ἑρμῖνος οἴεται, ἐν δευτέρῳ σχήματι τὸν μείζονα ἄκρον εἶναι . . . τὸν ἐγγύτερον τοῦ κοινοῦ γένους αὐτῶν (ἂν γὰρ ὦσιν οἱ ἄκροι ὄρνεον καὶ ἄνθρωπος, ἐγγυτέρω τοῦ κοινοῦ γένους αὐτῶν, τοῦ ζῴου, τὸ ὄρνεον τοῦ ἀνθρώπου καὶ ἐν τῇ πρώτῃ διαιρέσει, διὸ καὶ μείζων ἄκρος τὸ ὄρνεον).*

[4] Ibid. 75. 10 *ἀλλ' οὐδὲ ἁπλῶς πάλιν ῥητέον μείζονα τὸν ἐν τῷ συμπεράσματι τοῦ συλλογισμοῦ κατηγορούμενον, ὡς δοκεῖ τισιν· οὐδὲ γὰρ οὗτος δῆλος· ἄλλοτε γὰρ ἄλλος ἔσται καὶ οὐχ ὡρισμένος τῷ ἀντιστρέφειν τὴν καθόλου ἀποφατικήν, καὶ ὁ τέως μείζων αὖθις ἐλάττων, καὶ ἐφ' ἡμῖν ἔσται τὸν αὐτὸν καὶ μείζω καὶ ἐλάττω ποιεῖν.*

conceived as the conclusion. The predicate of this conclusion is the major term, and it does not matter whether we afterwards convert this conclusion or not: in the problem as first given the major term was and remains the predicate.[1] Alexander forgets that when we are forming a syllogism we are not always choosing premisses for a given conclusion, but sometimes we are deducing new conclusions from given premisses.

The problem was settled only after Alexander. What John Philoponus writes on the subject deserves to be regarded as classic. According to him we may define the major and the minor term either for the first figure alone or for all the three figures together. In the first figure the major term is the predicate of the middle and the minor is the subject of the middle. Such a definition cannot be given for the other two figures because the relations of the extremes to the middle term are in the other figures the same. We must therefore accept as a common rule for all figures that the major term is the predicate of the conclusion and the minor term is the subject of the conclusion.[2] That this rule is only a convention follows from another passage of Philoponus, where we read that the universal moods of the second figure have a major and a minor term only by convention, but not by nature.[3]

§ 12. *The order of the premisses*

Around the Aristotelian logic arose some queer philosophical prejudices which cannot be explained rationally. One of them is directed against the fourth figure, disclosing sometimes a strange aversion to it, another is the odd opinion that in all syllogisms the major premiss should be stated first.

[1] Alexander 75. 26 *τὸν δὴ ἐν τῷ προκειμένῳ προβλήματι εἰς τὴν δεῖξιν κατηγορούμενον τοῦτο θετέον μείζονα· καὶ γὰρ εἰ ἀντιστρέφει καὶ διὰ τοῦτο γίνεται ὁ αὐτὸς καὶ ὑποκείμενος, ἀλλ' ἔν γε τῷ ἡμῖν εἰς τὸ δεῖξαι προκειμένῳ κατηγορούμενος ἦν τε καὶ μένει.*

[2] Philoponus 67. 19 *ἴδωμεν πρότερον καὶ τίς ἐστι μείζων ὅρος καὶ τίς ἐλάττων. τοῦτο δὲ δυνατὸν μὲν καὶ κοινῶς ἐπὶ τῶν τριῶν σχημάτων διορίσασθαι καὶ ἰδίᾳ ἐπὶ τοῦ πρώτου. καὶ ἰδίᾳ μὲν ἐπὶ τοῦ πρώτου σχήματος μείζων ὅρος ἐστὶν ὁ τοῦ μέσου κατηγορούμενος, ἐλάττων δὲ ὁ τῷ μέσῳ ὑποκείμενος. καὶ τοῦτο μὲν ἰδιαζόντως ἐπὶ τοῦ πρώτου λέγομεν, ἐπειδὴ ὁ μέσος ἐν τῷ πρώτῳ τοῦ μὲν κατηγορεῖται τῷ δὲ ὑπόκειται. ἀλλ' ἐπειδὴ κατ' οὐδέτερον τῶν ἄλλων σχημάτων διάφορον ἔχουσι σχέσιν οἱ ἄκροι πρὸς τὸν μέσον, δῆλον ὅτι οὐκέτι ἁρμόσει ἡμῖν οὗτος ὁ προσδιορισμὸς ἐπ' ἐκείνων. χρηστέον οὖν κοινῷ κανόνι ἐπὶ τῶν τριῶν σχημάτων τούτῳ, ὅτι μείζων ἐστὶν ὅρος ὁ ἐν τῷ συμπεράσματι κατηγορούμενος, ἐλάττων δὲ ὁ ἐν τῷ συμπεράσματι ὑποκείμενος.*

[3] Ibid. 87. 10 *τὸ δὲ μεῖζον ἄκρον ἐν τούτῳ τῷ σχήματι τῶν δύο προτάσεων καθόλου οὐσῶν οὐκ ἔστι φύσει ἀλλὰ θέσει.*

From the standpoint of logic the order of the premisses in the Aristotelian syllogisms is arbitrary, because the premisses of the syllogism form a conjunction and the members of a conjunction are commutable. It is only a convention that the major premiss is stated first. Nevertheless, some philosophers, like Waitz or Maier, maintain that the order of the premisses is fixed. Waitz censures Apuleius for having changed this order,[1] and Maier rejects Trendelenburg's opinion that Aristotle does not tie it down.[2] No arguments are given in either case.

I do not know who is the author of the opinion that the order of the premisses is fixed. Certainly it is not Aristotle. Although Aristotle has not given a definition of the major and minor terms valid for all the three figures, it is always easy to determine which term and which premiss are regarded by him as the major and which as the minor. Aristotle, in his systematic exposition of the syllogistic, uses different letters to denote different terms; for each figure he puts them in alphabetical order (*θέσις*) and says explicitly which term is denoted by a given letter. We have thus for the first figure the letters *A*, *B*, *C*; *A* is the major term, *B* the middle, and *C* the minor.[3] For the second figure we have the letters *M*, *N*, *X*, where *M* is the middle term, *N* the major, and *X* the minor.[4] For the third figure we have the letters *P*, *R*, *S*, where *P* is the major term, *R* the minor, and *S* the middle.[5]

[1] Waitz, op. cit., vol. i, p. 380: 'Appuleius in hunc errorem se induci passus est, ut propositionum ordinem immutaverit.'

[2] Maier, op. cit., vol. ii *a*, p. 63: 'Darnach is Trendelenburg's Auffassung, dass Aristoteles die Folge der Prämissen frei lasse, falsch. Die Folge der Prämissen ist vielmehr festgelegt.' It is not clear to me what reasons he refers to by *darnach*.

[3] This follows from the definition given by Aristotle for the first figure; see p. 28, n. 1. Cf. Alexander 54. 12 *ἔστω γὰρ μείζων μὲν ἄκρος τὸ A, μέσος δὲ ὅρος τὸ B, ἐλάττων δὲ ἄκρος τὸ Γ.*

[4] *An. pr.* i. 5, 26^{b}34 *ὅταν δὲ τὸ αὐτὸ τῷ μὲν παντὶ τῷ δὲ μηδενὶ ὑπάρχῃ, ἢ ἑκατέρῳ παντὶ ἢ μηδενί, τὸ μὲν σχῆμα τὸ τοιοῦτον καλῶ δεύτερον, μέσον δὲ ἐν αὐτῷ λέγω τὸ κατηγορούμενον ἀμφοῖν, ἄκρα δὲ καθ' ὧν λέγεται τοῦτο, μεῖζον δὲ ἄκρον τὸ πρὸς τῷ μέσῳ κείμενον, ἔλαττον δὲ τὸ πορρωτέρω τοῦ μέσου. τίθεται δὲ τὸ μέσον ἔξω μὲν τῶν ἄκρων, πρῶτον δὲ τῇ θέσει.* Cf. Alexander 78. 1 *χρῆται γὰρ στοιχείοις οὐ τοῖς A, B, Γ, οἷς ἐν τῷ πρώτῳ σχήματι, ἀλλὰ τοῖς M, N, Ξ, μέσον μὲν λαμβάνων τὸ M τὸ ἀμφοτέρων κατηγορούμενον καὶ τὴν πρώτην ἔχον τάξιν ἐν τῇ καταγραφῇ, μείζονα δὲ ἄκρον τὸ N ἐφεξῆς κείμενον μετὰ τὸν μέσον, ἔσχατον δὲ καὶ ἐλάττονα τὸ Ξ.*

[5] *An. pr.* i. 6, 28^{a}10 *ἐὰν δὲ τῷ αὐτῷ τὸ μὲν παντὶ τὸ δὲ μηδενὶ ὑπάρχῃ, ἢ ἄμφω παντὶ ἢ μηδενί, τὸ μὲν σχῆμα τὸ τοιοῦτον καλῶ τρίτον, μέσον δ' ἐν αὐτῷ λέγω καθ' οὗ ἄμφω τὰ κατηγορούμενα, ἄκρα δὲ τὰ κατηγορούμενα, μεῖζον δ' ἄκρον τὸ πορρώτερον τοῦ μέσου, ἔλαττον δὲ τὸ ἐγγύτερον. τίθεται δὲ τὸ μέσον ἔξω μὲν τῶν ἄκρων, ἔσχατον δὲ τῇ θέσει.* Cf. Alexander 98. 20 *ἐπὶ τούτου τοῦ σχήματος πάλιν χρῆται στοιχείοις*

Aristotle states the major premiss first in all the moods of the first and the second figure, and in two moods of the third figure, Darapti and Ferison.[1] In the remaining moods of the third figure, Felapton, Disamis, Datisi, and Bocardo, the minor premiss is stated first.[2] The most conspicuous example is the mood Datisi. This mood is formulated in the same chapter twice; in both formulations the letters are the same, but the premisses are inverted. The first formulation runs: 'If *R* belongs to some *S*, and *P* to all *S*, *P* must belong to some *R*.'[3] The first premiss of this syllogism is the minor premiss, for it contains the minor term *R*. The second formulation reads: 'If *P* belongs to all *S*, and *R* to some *S*, then *P* will belong to some *R*.'[4] The first premiss of this second syllogism is the major premiss, as it contains the major term *P*. Attention must be called to the fact that this second formulation is given only occasionally, while the standard formula of this mood, belonging to the systematic exposition, is enunciated with transposed premisses.

In Book II of the *Prior Analytics* we meet other moods with transposed premisses, as Darii,[5] Camestres,[6] Baroco.[7] Even Barbara, the main syllogism, is occasionally quoted by Aristotle with the minor premiss first.[8] I can hardly understand, in view of these examples, how some philosophers knowing the Greek text of the *Organon* could have formed and maintained the opinion that the order of the premisses is fixed and the major premiss must be stated first. It seems that philosophical prejudices may sometimes destroy not only common sense but also the faculty of seeing facts as they are.

§ 13. *Errors of some modern commentators*

The story of the fourth figure may serve as another example to

τοῖς Π, P, Σ, καὶ ἔστιν αὐτῷ τοῦ μὲν μείζονος ἄκρου σημαντικὸν τὸ Π, τοῦ δὲ ἐλάττονος καὶ ὀφείλοντος ὑποκεῖσθαι ἐν τῷ γινομένῳ συμπεράσματι τὸ P, τοῦ δὲ μέσου τὸ Σ.

[1] See, for instance, p. 3, n. 2 (Barbara) and p. 10, n. 2 (Ferio).

[2] See p. 9, n. 4 (Felapton), and p. 7, n. (Disamis).

[3] *An. pr.* i. 6, 28^{b}12 εἰ τὸ μὲν P τινὶ τῷ Σ τὸ δὲ Π παντὶ ὑπάρχει, ἀνάγκη τὸ Π τινὶ τῷ P ὑπάρχειν.

[4] Ibid. 28^{b}26 εἰ γὰρ παντὶ τὸ Π τῷ Σ ὑπάρχει, τὸ δὲ P τινὶ τῷ Σ, καὶ τὸ Π τινὶ τῷ P ὑπάρξει.

[5] Ibid. ii. 11, 61^{b}41 εἰ γὰρ τὸ A τινὶ τῷ B, τὸ δὲ Γ παντὶ τῷ A, τινὶ τῷ B τὸ Γ ὑπάρξει.

[6] Ibid. ii. 8, 60^{a}3 εἰ τὸ A μηδενὶ τῷ Γ, τῷ δὲ B παντί, οὐδενὶ τῷ Γ τὸ B.

[7] Ibid. 60^{a}5 εἰ γὰρ τὸ A τινὶ τῷ Γ μὴ ὑπάρχει, τῷ δὲ B παντί, τὸ B τινὶ τῷ Γ οὐχ ὑπάρξει.

[8] See p. 10, n. 5.

show how strange philosophical prejudices sometimes are. Carl Prantl, the well-known historian of logic, begins his consideration of this figure with the following words: 'The question why silly playthings, as, for instance, the so-called Galenian fourth figure, are not to be found in Aristotle, is one we do not put at all; it plainly cannot be our task to declare at every step of the Aristotelian logic that this or that nonsense does not occur in it.'[1] Prantl does not see that Aristotle knows and accepts the moods of the so-called Galenian fourth figure and that it would be a logical error not to regard these moods as valid. But let us go farther. Commenting upon the passage where Aristotle speaks of the two moods later called Fesapo and Fresison,[2] Prantl first states these moods as rules of inference:

All *B* is *A*	Some *B* is *A*
No *C* is *B*	No *C* is *B*
Some *A* is not *C*	Some *A* is not *C*

—he does not, of course, see the difference between the Aristotelian and the traditional syllogism—and then he says: 'By transposition of the major premiss and the minor it becomes possible for the act of reasoning to begin'; and further: 'Such kinds of reasoning are, of course, not properly valid, because the premisses ordered as they were before the transposition are simply nothing for the syllogism.'[3] This passage reveals, in my opinion, Prantl's entire ignorance of logic. He seems not to understand that Aristotle proves the validity of these moods not by transposing the premisses, i.e. by inverting their order, but by converting them, i.e. by changing the places of their subjects and predicates.

[1] Carl Prantl, *Geschichte der Logik im Abendlande*, vol. i, p. 272: 'Die Frage aber, warum einfältige Spielereien, wie z. B. die sog. Galenische vierte Figur, sich bei Aristoteles nicht finden, werfen wir natürlich gar nicht auf; . . . wir können selbstverständlicher Weise nicht die Aufgabe haben, bei jedem Schritte der aristotelischen Logik eigens anzugeben, dass dieser oder jener Unsinn sich bei Aristoteles nicht finde.'

[2] See p. 25, n. 2.

[3] Prantl, op. cit., vol. i, p. 276:

'Alles B ist A	Einiges B ist A
Kein C ist B	Kein C ist B
Einiges A ist nicht C	Einiges A ist nicht C

woselbst durch Vertauschung des Untersatzes mit dem Obersatze es möglich wird, dass die Thätigkeit des Schliessens beginne; . . . natürlich aber sind solches keine eigenen berechtigten Schlussweisen, denn in solcher Anordnung vor der Vornahme der Vertauschung sind die Prämissen eben einfach nichts für den Syllogismus.'

Moreover, it is out of place to say that, two premisses being given, the act of reasoning begins when one premiss is stated first, but no syllogism results when the other precedes. From the standpoint of logic Prantl's work is useless.

The same may be said of Heinrich Maier's work. His treatise on the syllogistic figures generally and the fourth figure in particular is in my opinion one of the most obscure chapters of his laborious but unfortunate book.[1] Maier writes that two opinions of the criterion for the syllogistic figures stand opposed to each other: one (especially Ueberweg) sees this criterion in the position of the middle term as subject or predicate, the other (especially Trendelenburg) sees it in the extensional relations of the middle term to the extremes. It is not yet settled, Maier says, which of these opinions is right.[2] He adopts the second as his own, relying on Aristotle's characterization of the first figure. We know already that this characterization is logically untenable. Maier not only accepts it, but modifies the Aristotelian characterizations of the two other figures according to the first. Aristotle describes the second figure somewhat carelessly as follows: 'Whenever the same term belongs to all of one subject and to none of the other, or to all of each subject, or to none of either, I call such a figure the second; by "middle term" in it I mean that which is predicated of both subjects, by "extremes" the terms of which this is said.'[3] Maier remarks: 'When we reflect that the expressions "*B* is included in *A*", "*A* belongs to *B*", and "*A* is predicated of *B*" are interchangeable, then we may put this characterization according to the description of the first figure in the following words.'[4] Maier commits here his first error: it is not true that the three expressions he quotes can be exchanged for each other. Aristotle states explicitly: 'To say that one term is included in another is the same as to say that the other is predicated of all of the first.'[5] The expression '*B* is included in *A*' means, therefore,

[1] See Maier, op. cit., vol. ii *a*, 'Die drei Figuren', pp. 47–71, and vol. ii *b*, 'Ergänzung durch eine 4. Figur mit zwei Formen', pp. 261–9.

[2] Op. cit., vol. ii *a*, p. 48, n. 1.

[3] See the Greek text on p. 33, n. 4.

[4] Op. cit., vol. ii *a*, p. 49: 'Erwägt man nämlich, dass die Ausdrücke "B liegt im Umfang von A", "A kommt dem Begriff B zu" und "A wird von B ausgesagt" mit einander vertauscht werden können, so lässt sich die Charakteristik der zweiten Figur, welche der Beschreibung der ersten parallel gedacht ist, auch so fassen.'

[5] *An. pr.* i. 1, 24^b26 *τὸ δὲ ἐν ὅλῳ εἶναι ἕτερον ἑτέρῳ καὶ τὸ κατὰ παντὸς κατηγορεῖσθαι θατέρου θάτερον ταὐτόν ἐστιν.*

the same as '*A* is predicated of all *B*' or '*A* belongs to all *B*', but does not mean '*A* is predicated of *B*' or '*A* belongs to *B*'. With this first error is connected a second: Maier maintains that the negative premiss also has the external form of subordination of one term to another, like the affirmative universal premiss.[1] What is here meant by 'external form'? When *A* belongs to all *B*, then *B* is subordinated to *A*, and the external form of this relation is just the proposition '*A* belongs to all *B*'. But in a negative premiss, e.g. '*A* belongs to no *B*', the subordination of terms does not exist, nor does its form. Maier's assertion is logically nonsense.

Let us now quote Maier's description of the second figure. It runs thus: 'Whenever of two terms one is included, and the other is not included, in the same third term, or both are included in it, or neither of them, we have the second figure before us. The middle term is that which includes both remaining terms, and the extremes are the terms which are included in the middle.'[2] This would-be characterization of the second figure is again logically nonsense. Take the following example: Two premisses are given: '*A* belongs to all *B*' and '*C* belongs to no *A*'. If *A* belongs to all *B*, then *B* is included in *A*, and if *C* belongs to no *A*, it is not included in *A*. We have therefore two terms, *B* and *C*, one of which, *B*, is included, and the other, *C*, is not included in the same third term *A*. According to Maier's description we should have the second figure before us. What we have, however, is not the second figure, but only two premisses '*A* belongs to all *B*' and '*C* belongs to no *A*', from which we can get by the mood Celarent of the first figure the conclusion '*C* belongs to no *B*', and by the mood Camenes of the fourth figure the conclusion '*B* belongs to no *C*'.

The peak, however, of logical absurdity Maier attains by his assertion that there exists a fourth syllogistic figure consisting of only two moods, Fesapo and Fresison. He supports this assertion by the following argument: 'The Aristotelian doctrine overlooks one possible position of the middle term. This term may be less

[1] Op. cit., vol. ii *a*, p. 60, n. 1: 'auch der negative syllogistische Satz hat wenigstens die äussere Form der Subordination.' Cf. also ibid., p. 50.

[2] Ibid., p. 49: 'Wenn im Umfang eines und desselben Begriffes der eine der beiden übrigen Begriffe liegt, der andere nicht liegt, oder aber beide liegen oder endlich beide nicht liegen, so haben wir die zweite Figur vor uns. Mittelbegriff ist derjenige Begriff, in dessen Umfang die beiden übrigen, äußere Begriffe aber diejenigen, die im Umfang des mittleren liegen.'

general than the major and more general than the minor, it may secondly be more general, and thirdly less general, than the extremes, but it may be also more general than the major term and at the same time less general than the minor.'[1] When we remind ourselves that according to Maier the major term is always more general than the minor,[2] and that the relation 'more general than' is transitive, we cannot avoid the strange consequence of his argument that the middle term of his fourth figure should be at the same time more and less general than the minor term. From the standpoint of logic Maier's work is useless.

§ 14. *The four Galenian figures*

In almost every text-book of logic you may find the remark that the inventor of the fourth figure was Galen, a Greek physician and philosopher living in Rome in the second century A.D. The source of this remark is suspect. We do not find it either in the extant works of Galen or in the works of the Greek commentators (including Philoponus). According to Prantl the medieval logicians received the information from Averroes, who says that the fourth figure was mentioned by Galen.[3] To this vague information we may add two late Greek fragments found in the nineteenth century, and also very vague. One of them was published in 1844 by Mynas in the preface to his edition of Galen's *Introduction to Dialectic*, and republished by Kalbfleisch in 1897. This fragment of unknown authorship tells us that some later scholars transformed the moods added by Theophrastus and Eudemus to the first figure into a new fourth figure, referring to Galen as the father of this doctrine.[4] The other Greek fragment was found by Prantl in a logical work

[1] Op. cit., vol. ii *b*, p. 264: 'Die aristotelische Lehre läßt eine mögliche Stellung des Mittelbegriffs unbeachtet. Dieser kann specieller als der Ober- und allgemeiner als der Unterbegriff, er kann ferner allgemeiner, er kann drittens specieller als die beiden äußeren Begriffe: aber er kann auch allgemeiner als der Ober- und zugleich specieller als der Unterbegriff sein.'

[2] Ibid., vol. ii *a*, p. 56: 'Oberbegriff ist stets, wie in der 1. Figur ausdrücklich festgestellt ist, der allgemeinere, Unterbegriff der weniger allgemeine.'

[3] Prantl, i. 571, n. 99, quotes Averroes in a Latin translation edited in Venice (1553): 'Et ex hoc planum, quod figura quarta, de qua meminit Galenus, non est syllogismus super quem cadat naturaliter cogitatio.' Cf. also Prantl, ii. 390, n. 322.

[4] K. Kalbfleisch, *Über Galens Einleitung in die Logik*, 23. Supplementband der Jahrbücher für klassische Philologie, Leipzig (1897), p. 707: *Θεόφραστος δὲ καὶ Εὔδημος καί τινας ἑτέρας συζυγίας παρὰ τὰς ἐκτεθείσας τῷ Ἀριστοτέλει προστεθήκασι τῷ πρώτῳ σχήματι . . ., ἃς καὶ τέταρτον ἀποτελεῖν σχῆμα τῶν νεωτέρων ᾠήθησάν τινες ὡς πρὸς πατέρα τὴν δόξαν τὸν Γαληνὸν ἀναφέροντες.*

of Ioannes Italus (eleventh century A.D.). This author says sarcastically that Galen maintained the existence of a fourth figure in opposition to Aristotle, and, thinking that he would appear cleverer than the old logical commentators, fell very far short.[1] That is all. In view of such a weak basis of sources, Ueberweg suspected a misunderstanding in the matter, and Heinrich Scholz writes in his *History of Logic* that Galen is probably not responsible for the fourth figure.[2]

For fifty years there has existed a Greek scholium in print which clears up the whole matter in an entirely unexpected way. Although printed, it seems to be unknown. Maximilian Wallies, one of the Berlin editors of the Greek commentaries on Aristotle, published in 1899 the extant fragments of Ammonius' commentary on the *Prior Analytics*, and has inserted in the preface a scholium of an unknown author found in the same codex as that in which the fragments of Ammonius are preserved. The scholium is entitled 'On all the kinds of syllogism', and begins thus:

'There are three kinds of syllogism: the categorical, the hypothetical, and the syllogism *κατὰ πρόσληψιν*. Of the categorical there are two kinds: the simple and the compound. Of the simple syllogism there are three kinds: the first, the second, and the third figure. Of the compound syllogism there are four kinds: the first, the second, the third, and the fourth figure. For Aristotle says that there are only three figures, because he looks at the simple syllogisms, consisting of three terms. Galen, however, says in his *Apodeictic* that there are four figures, because he looks at the compound syllogisms consisting of four terms, as he has found many such syllogisms in Plato's dialogues.'[3]

The unknown scholiast further gives us some explanations, from

[1] Prantl, ii. 302, n. 112: *τὰ δὲ σχήματα τῶν συλλογισμῶν ταῦτα· ὁ Γαληνὸς δὲ καὶ τέταρτον ἐπὶ τούτοις ἔφασκεν εἶναι, ἐναντίως πρὸς τὸν Σταγειρίτην φερόμενος, ὃς λαμπρότερον ἀναφανῆναι οἰόμενος τῶν τὴν λογικὴν πραγματείαν ἐξηγουμένων παλαιῶν ὡς πορρωτάτω εὐθέως ἐκπέπτωκε.*

[2] Fr. Ueberweg, *System der Logik*, Bonn (1882), 341. Cf. also Kalbfleisch, op. cit., p. 699; H. Scholz, *Geschichte der Logik*, Berlin (1931), p. 36.

[3] M. Wallies, *Ammonii in Aristotelis Analyticorum Priorum librum I Commentarium*, Berlin (1899), p. ix: *Περὶ τῶν εἰδῶν πάντων τοῦ συλλογισμοῦ. τρία εἴδη ἐστὶ τοῦ [ἁπλοῦ] συλλογισμοῦ· τὸ κατηγορικόν, τὸ ὑποθετικόν, τὸ κατὰ πρόσληψιν. τοῦ δὲ κατηγορικοῦ δύο ἐστὶν εἴδη· ἁπλοῦν, σύνθετον. καὶ τοῦ μὲν ἁπλοῦ τρία ἐστὶν εἴδη· πρῶτον σχῆμα, δεύτερον σχῆμα, τρίτον σχῆμα. τοῦ δὲ συνθέτου τέσσαρά ἐστιν εἴδη· πρῶτον σχῆμα, δεύτερον σχῆμα, τρίτον, τέταρτον σχῆμα. Ἀριστοτέλης μὲν γὰρ τρία τὰ σχήματά φησιν πρὸς τοὺς ἁπλοῦς συλλογισμοὺς ἀποβλέπων τοὺς ἐκ τριῶν ὅρων συγκειμένους. Γαληνὸς δ' ἐν τῇ οἰκείᾳ Ἀποδεικτικῇ δ τὰ σχήματα λέγει πρὸς τοὺς συνθέτους συλλογισμοὺς ἀποβλέπων τοὺς ἐκ δ ὅρων συγκειμένους πολλοὺς τοιούτους εὑρὼν ἐν τοῖς Πλάτωνος διαλόγοις.*

which we can gather how Galen may have found these four figures. Compound syllogisms consisting of four terms may be formed by combinations of the three figures I, II, and III of simple syllogisms in nine different ways: I to I, I to II, I to III, II to II, II to I, II to III, III to III, III to I, III to II. Two of these combinations, viz. II to II and III to III, do not give syllogisms at all, and of the remaining combinations II to I gives the same figure as I to II, III to I the same as I to III, and III to II the same as II to III. We get thus only four figures, I to I, I to II, I to III, and II to III.[1] Examples are given, of which three are taken from Plato's dialogues, two from the *Alcibiades*, and one from the *Republic*.

This precise and minute account must be explained and examined. Compound syllogisms of four terms have three premisses and two middle terms, say *B* and *C*, which form the premiss *B–C* or *C–B*. Let us call this the middle premiss. *B* forms together with *A*, the subject of the conclusion, the minor premiss, and *C* forms together with *D*, the predicate of the conclusion, the major premiss. We thus obtain the following eight combinations (in all the premisses the first term is the subject, the second the predicate):

Figure	*Minor*	*Middle*	*Major*	*Conclusion*	
	Premiss				
F1	*A–B*	*B–C*	*C–D*	*A–D*	I to I
F2	*A–B*	*B–C*	*D–C*	*A–D*	I to II
F3	*A–B*	*C–B*	*C–D*	*A–D*	II to III
F4	*A–B*	*C–B*	*D–C*	*A–D*	II to I
F5	*B–A*	*B–C*	*C–D*	*A–D*	III to I
F6	*B–A*	*B–C*	*D–C*	*A–D*	III to II
F7	*B–A*	*C–B*	*C–D*	*A–D*	I to III
F8	*B–A*	*C–B*	*D–C*	*A–D*	I to I

If we adopt the principle of Theophrastus that in the first

[1] Wallies, op. cit., pp. ix–xi: *ὁ κατηγορικὸς συλλογισμὸς ἁπλοῦς, ὡς Ἀριστοτέλης· σχῆμα Α Β Γ. σύνθετος, ὡς Γαληνός· Α πρὸς Α, Α πρὸς Β, Α πρὸς Γ, Β πρὸς Β, Β πρὸς Α, Β πρὸς Γ, Γ πρὸς Γ, Γ πρὸς Α, Γ πρὸς Β.*

συλλογιστικόν· *Α πρὸς Α,* *Α πρὸς Β,* *Α πρὸς Γ,* *Β πρὸς Γ.*
Α *Β* *Γ* *Δ*

ἀσυλλόγιστον· Β πρὸς Β, Γ πρὸς Γ, (οὐ γὰρ γίνεται συλλογισμὸς οὔτε ἐκ δύο ἀποφατικῶν οὔτε ἐκ δύο μερικῶν)·

Β πρὸς Α, *Γ πρὸς Α,* *Γ πρὸς Β,*
Β *Γ* *Δ*

οἱ αὐτοί εἰσιν τοῖς συλλογισμοῖς ὡς ὑπογέγραπται.

Aristotelian figure the middle term is the subject of one premiss—it does not matter of which, the major or the minor—and the predicate of another, and define by this principle which figure is formed by the minor and middle premisses on the one hand, and by the middle and major premisses on the other, we get the combinations of figures shown in the last column. Thus, for instance, in the compound figure F2 the minor premiss together with the middle forms the figure I, as the middle term *B* is the predicate of the first premiss and the subject of the second, and the middle premiss together with the major forms the figure II, as the middle term *C* is the predicate of both premisses. This was probably how Galen has got his four figures. Looking at the last column we see at once that, as Galen held, the combinations II to II and III to III do not exist, not for the reason, as the scholiast mistakenly says, that no conclusion results either from two negative or two particular premisses, but because no term can occur in the premisses three times. It is obvious also that if we extend the principle of Theophrastus to compound syllogisms and include in the same figure all the moods that from the same combination of premisses yield either the conclusion *A–D* or the conclusion *D–A*, we get as Galen does the same figure from the combination I to II as from the combination II to I. For, interchanging in figure F4 the letters *B* and *C* as well as the letters *A* and *D*, we get the scheme:

F4 *D–C* *B–C* *A–B* *D–A*,

and as the order of the premisses is irrelevant we see that the conclusion *D–A* results in F4 from the same premisses as *A–D* in F2. For the same reason figure F1 does not differ from figure F8, F3 from F6, or F5 from F7. It is possible, therefore, to divide the compound syllogisms of four terms into four figures.

The scholium edited by Wallies explains all historical problems connected with the alleged invention of the fourth figure by Galen. Galen divided syllogisms into four figures, but these were the compound syllogisms of four terms, not the simple syllogisms of Aristotle. The fourth figure of the Aristotelian syllogisms was invented by someone else, probably very late, perhaps not before the sixth century A.D. This unknown scholar must have heard something about the four figures of Galen, but he either did not understand them or did not have Galen's text at hand. Being in

opposition to Aristotle and to the whole school of the Peripatetics, he eagerly seized the occasion to back up his opinion by the authority of an illustrious name.

REMARK. The problem of compound syllogisms raised by Galen has considerable interest from the systematic point of view. Investigating the number of valid moods of the syllogisms consisting of three premisses, I have found that there are forty-four valid moods, the figures F1, F2, F4, F5, F6, and F7 having six moods each, and figure F8 eight. Figure F3 is empty. It has no valid moods, for it is not possible to find premisses of the form *A–B*, *C–B*, *C–D* such that a conclusion of the form *A–D* would follow from them. This result, if known, would certainly be startling for students of the traditional logic. Mr. C. A. Meredith, who attended my lectures delivered on this subject in 1949 at University College, Dublin, has found some general formulae concerning the number of figures and valid moods for syllogisms of n terms, including expressions of 1 and 2 terms. I publish these formulae here with his kind permission:

Number of terms	n
Number of figures	2^{n-1}
Number of figures with valid moods .	$\frac{1}{2}(n^2-n+2)$
Number of valid moods . . .	$n(3n-1)$

For all n every non-empty figure has 6 valid moods, except one that has $2n$ valid moods.

Examples:

Number of terms	1, 2, 3, 4,..., 10
Number of figures	1, 2, 4, 8,..., 512
Number of figures with valid moods .	1, 2, 4, 7,..., 46
Number of valid moods . . .	2, 10, 24, 44,..., 290

It is obvious that for large n's the number of figures with valid moods is comparatively small against the number of all figures. For $n = 10$ we have 46 against 512 respectively, i.e. 466 figures are empty. For $n = 1$ there is only 1 figure, *A–A*, with 2 valid moods, i.e. the laws of identity. For $n = 2$ there are 2 figures:

	Premiss	*Conclusion*
F1	*A–B*	*A–B*
F2	*B–A*	*A–B*

with 10 valid moods, 6 in F1 (viz. four substitutions of the propositional law of identity, e.g. 'if all *A* is *B*, then all *A* is *B*', and two laws of subordination), and 4 moods in F2 (viz. four laws of conversion).

CHAPTER III

THE SYSTEM

§ 15. *Perfect and imperfect syllogisms*

IN the introductory chapter to the syllogistic Aristotle divides all syllogisms into perfect and imperfect. 'I call that a perfect syllogism', he says, 'which needs nothing other than what has been stated to make the necessity evident; a syllogism is imperfect, if it needs either one or more components which are necessary by the terms set down, but have not been stated by the premisses.'[1] This passage needs translation into logical terminology. Every Aristotelian syllogism is a true implication, the antecedent of which is the joint premisses and the consequent the conclusion. What Aristotle says means, therefore, that in a perfect syllogism the connexion between the antecedent and the consequent is evident of itself without an additional proposition. Perfect syllogisms are self-evident statements which do not possess and do not need a demonstration; they are indemonstrable, *ἀναπόδεικτοι*.[2] Indemonstrable true statements of a deductive system are now called axioms. The perfect syllogisms, therefore, are the axioms of the syllogistic. On the other hand, the imperfect syllogisms are not self-evident; they must be proved by means of one or more propositions which result from the premisses, but are different from them.

Aristotle knows that not all true propositions are demonstrable.[3] He says that a proposition of the form '*A* belongs to *B*' is demonstrable if there exists a middle term, i.e. a term which forms with *A* and *B* true premisses of a valid syllogism having the above proposition as the conclusion. If such a middle term does

[1] *An. pr.* i. 1, 24b22 *τέλειον μὲν οὖν καλῶ συλλογισμὸν τὸν μηδενὸς ἄλλου προσδεόμενον παρὰ τὰ εἰλημμένα πρὸς τὸ φανῆναι τὸ ἀναγκαῖον, ἀτελῆ δὲ τὸν προσδεόμενον ἢ ἑνὸς ἢ πλειόνων, ἃ ἔστι μὲν ἀναγκαῖα διὰ τῶν ὑποκειμένων ὅρων, οὐ μὴν εἴληπται διὰ πρότασεων.*

[2] Commenting upon the above passage Alexander uses the expression *ἀναπόδεικτος*, 24. 2: *ἑνὸς μὲν οὖν προσδέονται οἱ ἀτελεῖς συλλογισμοὶ οἱ μιᾶς ἀντιστροφῆς δεόμενοι πρὸς τὸ ἀναχθῆναι εἴς τινα τῶν ἐν τῷ πρώτῳ σχήματι τῶν τελείων καὶ ἀναποδείκτων, πλειόνων δὲ ὅσοι διὰ δύο ἀντιστροφῶν εἰς ἐκείνων τινὰ ἀνάγονται.* Cf. also p. 27, n. 2.

[3] *An. post.* i. 3, 72b18 *ἡμεῖς δέ φαμεν οὔτε πᾶσαν ἐπιστήμην ἀποδεικτικὴν εἶναι, ἀλλὰ τὴν τῶν ἀμέσων ἀναπόδεικτον.*