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Russell and Kripke on Quantifier Scope

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The measure of an idea is to be found in its consequences; the rest is, largely, aesthetics. In recent critical comments on Russell's theory of descriptions, Saul Kripke alleges some interesting consequences that are destructive to the theory. (Kripke [2011] p. 225-254) Here I confine myself to two: first, that on Russell's original statement problematic intermediate scope readings are introduced, via Russell's use of "wished to know"; and, second, that Russell's, admittedly, clever joke about yachts, connected with quantifier scope, misfires because Russell's theory entails the implausible idea that in order to meaningfully assert "The size I thought your yacht was is greater than the size your yacht is" the guest must have "had an exact idea of the size of the yacht." (Kripke [2011] p. 240).

It is here argued that the first objection, and in particular Kripke's proposal for emending Russell's statement, is unwarranted owing to syntactical properties of the sentence Russell, actually, employs. Kripke's second objection, I shall maintain, fails to take into account Russell's theory of quantity and magnitude, a theory that strengthens Russell's position with respect to the type of objection Kripke raises. I begin with a brief statement of Russell's theory, one that accords with the presentation Kripke, himself, provides. I, then, proceed to examine his objections.

I

We have it that on Russell's theory a definite description occurring in a sentence takes the form 'The ϕ ψ's' and is to be analyzed as involving a binary quantifier:

$[\text{the } x](\phi x, \psi x)$.

Definite descriptions, according to Russell, can be eliminated by making use, among other things, of the existential quantifier, although he states his original theory, mainly, in terms of universal quantification. Kripke states Russell's proposal for the elimination of descriptions in the following way (with slight notational differences),

$(\exists x)((y)(\phi x \equiv y = x) \,\&\, \psi x)$

It will expedite the discussion to quote the passage Kripke cites in making his proposed change in Russell's statement of the theory. Let's begin with the quotation from Russell Kripke cites.

> I shall therefore state three puzzles which a theory as to denoting ought to be able to solve and I shall show later that my theory solves them.
>
> (1) If *a* is identical with *b*, whatever is true of the one is true of the other...Now George IV wished to know whether Scott was the author of *Waverley*. Hence we may substitute *Scott* for *the author of "Waverley,"* and thereby prove that George IV wished to know whether Scott was Scott. Yet an interest in the law of identity can hardly be attributed to the first gentleman of Europe. (Russell [1905] p. 47)

Kripke proposes that Russell's sentence, 'George IV wished to know whether Scott was the author of *Waverley*', be replaced because it contains a double embedding introduced by 'wished to know' subject to problematic intermediate scope readings. He proposes that we substitute for it 'George IV wished that he should know whether Scott was the author of *Waverley*'; but, because Russell intended a single embedding, this sentence, itself, should be replaced by 'George IV wondered whether Scott was the author of *Waverley*'. I will argue that neither Russell's original sentence, nor Kripke's proposed first substitution, carries the possibility of an intermediate scope reading. This will provide the occasion not only of responding to Kripke's criticism but also of exploring issues connecting grammar and the transcription of ordinary sentences into a canonical language.

The substitution Kripke first proposes is somewhat puzzling since it does not seem to avoid the *apparent* possibility of an intermediate scope reading. It will be argued here that in the case of Russell's original sentence 'wished to know' does not in fact introduce a double embedding of the sort required to sustain the criticism. This only appears to be the case because it contains two verbs. I will argue, however, that this is not a sufficient condition for a possible intermediate scope reading.

Russell applies his theory of descriptions to the sentence

(**A**) George IV wished to know whether Scott was the author of *Waverley*.

Given this sentence, what was George IV's interest? There are, at least, two possibilities depending on whether we adopt a *primary,* or *secondary,* scope reading of the description occurring in the sentence.

(*primary)*

'One and only one man wrote *Waverley*, and George IV wished to know whether Scott was that man'.

(*secondary*)

'George IV wished to know whether one and only one man wrote *Waverley* and Scott was that man'.

In the case of the primary reading, George IV need not be wondering about anyone he would, himself, describe in any such terms as 'the author of *Waverley*'. As Russell notes, George IV may be interested in whether some man he saw at a distance was Scott, a man who might just happen to be the author of *Waverley*. George IV may have had no interest in who the author of *Waverley* is, only in whether the man he saw at a distance was Scott. As a statistical fact, it is doubtful that this would be the preferred reading.

Consider Kripke's example of a sentence, (1), that possesses primary, secondary, and intermediate scope readings.

(1) Hoover charged that the Berrigans planned to kidnap a high official.

There are three possible interpretations of this sentence.

(*primary)*

(2) ($\exists x$)(*x* is a high official and Hoover charged that the Berrigans planned to kidnap *x*)

(*intermediate*)

(3) Hoover charged that ($\exists x$)(*x* is a high official that the Berrigans planned to kidnap *x*)

(*secondary*)

(4) Hoover charged that the Berrigans planned to kidnap ($\exists x$)(*x* is a high official)

Kripke elaborates: (3) reports that there was a particular official the Berrigans intended to kidnap but it doesn't tell us whether Hoover knew who that official was. (2) reports that, indeed, Hoover knew who it was, and (4) suggests that the Berrigans didn't know yet who their victim would be. For our purposes, what is important to note is that the subject falling within the domain of the verbs, 'charged' and 'planned', (viz. 'the Berrigans') is not the same as the subject of the matrix sentence, viz. 'Hoover'. I think this is a relevant consideration. That is, I believe scope properties may depend (although not necessarily) in part on the syntactical relations of matrix and embedded subject positions. Consider the sentence (1') containing an anaphoric pronoun referentially coindexed with the matrix subject 'Hoover'.

(1') Hoover_1 said that the he_1 planned to kidnap a high official.

Are there, then, three possible interpretations of this sentence?

(*primary)*

(2′) $(\exists x)$(x is a high official and Hoover$_1$ said that he$_1$ planned to kidnap x)

(*intermediate*)

(3′) Hoover$_1$ said that $(\exists x)$(x is a high official and that he$_1$ planned to kidnap x)

(*secondary*)

(4′) Hoover$_1$ said that he$_1$ planned to kidnap $(\exists x)$(x is a high official)

If we take what is being asserted as to what Hoover knew as indicative of scope relations then we can say this: In the case of (2′) Hoover may not have, even, known that who he intended to kidnap was a high official; while (3′) suggests that he did know; and (4′) is such that he may have planned to kidnap some high official without knowing, yet, who it would be. Unlike (3), (3′) appears to rule out Hoover's not knowing who the potential victim was, so there appears to be a contrast. Since Hoover said (or claimed, etc) that there was some particular high official *he* planned to kidnap, then he must have known who it was. If what we have suggested proves anything, it proves that the semantic interpretation of scope relations in a logical transcription depends on grammatical features of the original sentence. Under "grammatical" we include anaphoric relations. Let's explore a bit more the dependence of scope interpretation on anaphoric relations.

Kripke begins by proposing that we replace (A) with (1′).

(1′) George IV wished that he should know whether Scott was the author of *Waverley*.

He points out, first, that this sentence introduces the "newer problem" of *de se* (his precise meaning remains unclear); second, it involves a double embedding. But since Russell intends a single embedding he changes it to 'George IV wondered whether Scott was the author of *Waverley*'.

Consider two sentences (5a, b): one containing an overt pronominal in the domain of an attitude verb, 'wish'; the other lacking an overt, morphologically realized pronominal, but requiring some grammatical subject for a "complete functional complex" (the expression comes from Chomsky).

5a. George wished that he should know whether p.

b. George wished to know whether p.

In order to render more vivid the point I want to make, I will introduce another pair, (6a) and (6b). Keep in mind that all contexts are extensional according Russell.

6a. George wished that he could sink the ball.

b. George wished to sink the ball.

It will be argued that the double embedding, *if* that in fact is what it is, entailed by 'wished to know' is either irrelevant or it is the case that double embeddings do not, *necessarily*, introduce the possibility of intermediate scope readings. If this is correct then Kripke's initial proposal lacks logical motivation.

Notice that (5b) and (6b) lack an overt subject in their sentential complements. Making use of a familiar tool from linguistic theory let's recast (6b) as

6c. George wished PRO to sink the ball.

Now we have a representation of the subject of the embedded sentential complement, viz. 'PRO'. Next, consider the relation of 'PRO' to 'George'. The reference of 'PRO' is *controlled* by 'George'. It is not, merely, that the two just happen to be coreferential. No noncoindexed expression will preserve grammaticality; nor in this construction is noncoreference a possibility. 'PRO' is not "free."

In addition there is at least one contrast to be drawn between (6a) and (6c). The subject of the sentential complement of (6a) is not controlled; that is, the position occupied by 'he' need not be coindexed with George. For example, (6d) is perfectly grammatical.

6d. George$_i$ wished that John$_j$ could sink the ball ($i \neq j$).

Suppose that the correct reading of (6a) implies that George only cares that the ball be sunk; that is, he doesn't really care who does it; he will try; but that he be the one who tries is unimportant. Such a reading is not possible in the case of (6c). The reason is that 'PRO' is controlled by 'George' and, so, what he wishes is that he, *himself*, sink the ball. If I am right, (6c) is the only rendering of our report of George's wish that allows for such contrast. For now, we put aside the larger issue (maybe) connected with the so-called problem of *de se*.

Kripke's proposal is to eliminate the apparent double embedding in the sentence with which we began, viz. (A); this in accordance with what he takes to be Russell's intention. The point to be made here in response to Kripke is that, because the subject of 'to know' ('PRO') is controlled by 'George IV', (7) allows no possible intermediate scope reading.

(7) George IV wished PRO to know whether Scott was the author of *Waverley*.

Even where PRO control is not at issue, e.g. (8),

(8) George IV wished he knew whether Scott was the author of *Waverley*,

an intermediate scope reading is not available, where 'George IV' and 'he' are understood as referring to the same person. Consider the available possible readings of scope where PRO control is not involved, including the intermediate scope reading to be rejected.

(*primary*)

(9) One and only one man wrote *Waverley* and George IV wished that he knew whether Scott was he.

(*intermediate*)

(10) George IV wished that one and only one man wrote *Waverley* and that he knew whether Scott was he.

(*secondary*)

(11) George IV wished that he knew whether one and only one man wrote *Waverley* and that Scott was he.

Obviously, (10) is not an intermediate scope reading of (8), at all, since (8) says nothing about who George IV *wishes* wrote *Waverley*. I conclude that Kripke's criticism of Russell's original formulation based as it is on an implicit problematic intermediate scope reading of (A) is unwarranted. Next, we turn to a second criticism Kripke makes of Russell's theory.

II

Kripke says that a famous joke Russell uses to introduce scope distinctions misfires; and, more importantly, that this reveals a shortcoming in the way in which he presents his theory. According to Kripke, "the joke is wrong." (Kripke [2011] p. 240) I will argue that there is nothing wrong with Russell's joke. I begin by quoting the passage cited by Kripke. Next I identify an entire class of sentences which is such that if Kripke were right in his interpretation of Russell's theory it would supply numerous counterexamples. However, once Russell's account of such sentences is examined, the problem Kripke identifies is seen to be no problem at all. What we will discover is that, while Kripke decries the fact that many have erred owing to a reluctance to actually read Russell's paper, Kripke's error follows from a failure to take into account a much neglected component of Russell's philosophy of mathematics. The passage Kripke cites contains the joke Kripke targets.

> When we say..."So-and-so" or "So-and-so is true," etc., the "so-and-so" must be a proposition. Suppose now that "so-and-so" contains a denoting phrase. We may either eliminate this denoting phrase from the subordinate proposition "so-and-so," or from the whole proposition in which 'so-and-so' is a mere constituent. Different propositions

> result according to which we do. I have heard of a touchy owner of a yacht to whom a guest, on first seeing it, remarked, "I thought your yacht was larger than it is," and the owner replied, "No, my yacht is not larger than it is". (Russell [1905] p. 52)

Russell avers that there are two possible ways of interpreting the guest's remark but that only one makes sense. The one that makes no sense is "I thought the size of your yacht was greater than the size of your yacht." The one that does make sense is "The size I thought your yacht was is greater than the size it is." More exactly, Kripke claims that that this "implies" that there was a unique size the guest thought the yacht was. (Kripke [2011] p. 240) However there is some question as to whether this fact, if it is a fact, follows from Russell's *analysis* of definite descriptions, or whether it is an implication of pragmatics or context. What is in the guest's mind can hardly follow from the *analysis* alone. If it does not follow from Russell's analysis, then it is doubtful that it can be used against Russell's theory. Since Kripke does not elaborate on how what is in the mind of the guest follows from Russell's theory it will be difficult to see how he escapes the charge that he has neglected to take into account Russell' theory in its totality.

Russell's response to Kripke's assertion that the guest must have had a "unique size," or "exact idea," in mind would most likely be to point out, first and foremost, that his theory of descriptions, which Kripke claims to be wrong on the basis of this example, depends on his logical atomism and that this fundamental principle is violated as long as such an "exact idea" of the yacht's size is expressed by any such thing as '35 yards'.

In the first place, this expression makes use of reference to cardinal numbers and being logical constructions such numbers cannot occur in the final analysis of sentences containing definite descriptions and, therefore, will not be included in a logical description of what is in the guest's mind following a complete analysis based on Russell's theory. To retain such expressions would violate Russell's commitment to logical atomism. Secondly, the use of 'yards' entails a convention, one depending on 'volition', and volition is not among the analysans of Russell's analysis. The plausible alternative to Kripke's characterization is to say that on Russell's theory any idea the guest may be required to have in mind must get there the way other ideas get in the mind upon a complete analysis into logical atoms: acquaintance. In what follows, I will suggest that Russell was aware of the need for this approach to magnitudes and in particular to sentences containing reference to them. The difficulty Kripke claims to have identified as refuting Russell's theory occurs only in cases of one type of sentence, a type Russell had by 1905 closely scrutinized. Examples of this type include the following.

I thought the candy was sweeter than it is.

I thought the temperature was warmer than it is.

I thought the redness of the apple was greater than it is.

I thought the resemblance of lions and tigers was greater than it is.

I thought the difference between maroon and blue was greater than it is.

In each case the same objection can be raised: Russell's theory requires that a unique numerical value be assigned to a magnitude if the uniqueness condition is to be satisfied. By 'magnitude' we mean the "dimensions" of temperature, redness, resemblance, difference in color between maroon and blue, etc. To be sure, Kripke has expressed awareness that one plausible approach to the problem, e.g. the case of yacht size, is to speak of "intervals" of size, but he is dismissive and appears to be unaware of what Russell had said of relevance to the topic. (Kripke [2011] p. 240) But even had he pursued the matter in the direction of comparatives, he would probably have missed Russell's way out. The reason is that comparatives are not really at the root of the problem, although they provide the key to unlocking the puzzle; for the issue is not one of comparatives so much as one of magnitudes, where by a *magnitude* Russell means "anything that can be greater or less." (Russell [1903] p. 159). What is crucial is not so much the relation of comparatives to magnitudes, but the relation of number to magnitudes. Before proceeding, I digress momentarily and entertain a possible scenario that underscores the importance of examining the circumstance that may cause us to prefer a wide scope over a narrow scope reading of descriptions.

Consider, then the following situation. Tom and a guest are on a bridge looking at yachts. Tom points one out, and the guest acknowledges, that he sees it. Tom remarks, after being sure that the guest knows which yacht he is talking about, "Are you acquainted with Joe's yacht?" To which the guest says "Yes." Then Tom says, "Then you think Joe's yacht is bigger than Bob's." The guest replies "Yes." But as it turns out Joe and Bob are one and the same person. On an extensional reading, then, the following sentence is true:

(13) The guest thought Joe's yacht is bigger than Joe's yacht.

This is nonsense. If so, the identity of Bob and Joe forces a wide scope reading, confirming the link between reference and syntax. But what is of immediate interest is that if Kripke is right, Russell's theory requires that in order that the guest thinks that the yacht was bigger than it is he must have had some unique size *in mind*, one computed according to some formula supplied, perhaps, by the yacht club. Is this a plausible reading of Russell's theory? We now return to the issue of magnitudes in Russell's philosophy of mathematics in order to discount this, rather, exotic demand on the theory.

Just as in the case of the wide scope reading of our original sentence, (A) -- where George IV was acquainted with Waverley at a distance, but did not know, but did wonder, whether it was

Scott -- the guest may have been acquainted with "how big" the yacht was without knowing, or believing, that its size was of such and such a numerical value computed by a certain complex, agreed upon, formula. In the case of the magnitudes mentioned in our examples of comparative sentences it is typically the case that we are acquainted with specific magnitudes and that we can think about the magnitude with which we are acquainted without measurement. In fact, using a formula to arrive at such a magnitude would require (as we pointed out earlier) treating the magnitude's value as a logical construction, rather than an object of acquaintance. What Kripke misses is, precisely, this fact, a fact that did not escape the attention of Russell. For Russell it is a mistake to believe, as Kripke apparently does, that magnitudes referred to by definite descriptions must be thought about in terms of numerical measures such being 35 yards. All that is essential to Russell's theory is that the guest have in mind an idea of what may *have* an exact value, viz. the size of the yacht, not that he have in mind an exact idea of what that exact value *is*.

Russell, it will be recalled, on a number of occasions was very explicit in maintaining that his objective was to provide "a theory of quantity that does not depend on number." (Russell [1903] p. 158). Russell notes that when we take numbers as the measure of magnitudes, and then speak of comparisons between magnitudes in terms of these numerical measures, we may get conflicting answers to the same question when we dispense with the numerical measures and judge the magnitudes as objects of acquaintance instead. Russell remarks,

> But I do not think that it can be shown generally that, if A, B, C, D be the numbers measuring four magnitudes, and A - B = C – D, then *the* difference of the magnitudes are equal. It would seem, for instance, that *the* difference between one inch and two inches is greater than that between 1001 inches and 1002 inches. (Russell [1903] p. 179; italics added - srb)

The point Russell makes here has persisted and has been applied elsewhere in the treatment of magnitudes resisting numerical representation, such as the controversy surrounding the concept of utility in economics. Kenneth Arrow, e.g. raises a point based on this sort of Russellian consideration when he remarks,

> Does it make sense to say that an increase of temperature from 0^0 to 1^0 is just as intense as an increase in temperature from 100^0 to 101^0? No more than it can be said that there is any meaning in comparing marginal utilities at different levels of well-being. (Arrow [1951] p. 10)

Setting aside problems related to regarding utility as a magnitude of definite intensities, Kripke is wrong to believe that having a belief about *the* size of the yacht in question would require having "an exact idea of the size of the yacht" on Russell's theory. (Kripke [2011] p. 240) The

reason he is wrong is that we are in the domain of magnitudes, and magnitudes, in particular the comparison of magnitudes, do not require any such numerically represented "exact idea" in order that we may have particular magnitudes in mind; and this is common among those magnitudes we have mentioned in the class of comparative sentences that invites Kripke's criticism. This is more obviously the case in the case of magnitudes lacking cardinal significance.

Kripke's criticism is centered on understanding a comparison of magnitudes as a matter of arithmetic computation, whereas Russell's "way out" demands that we acknowledge the difference between arithmetic and quantitative judgments. (Russell [1903] pp. 159; 179) We may be acquainted with *particular* instances of magnitudes where no quantitative judgment is possible. Some magnitudes, such as size, *can* possess cardinal significance; but they need not; and this is the point to be made in defense of Russell. We conclude by raising one other point against Kripke's attack on Russell.

Conventions may be set regarding how the size of a yacht is to be determined. So much is to be expected where yachts and, therefore, status is involved. But what about tug boats? Kripke alleges that Russell's is wrong and his joke a failure because he didn't take the facts of size measurement into account. (Kripke [2011] p. 240). However, the formula for computing the size of a boat is interest relative and a matter of convention. Thus, one formula might be used when determining the cost of mooring a boat over the winter; while another may be used in determining the charges to be levied on passing through a canal. So the numerical value assigned to the size of a tug boat, based on computation, and the size of a yacht need not, as a matter of expert testimony, be determined in the same way. Suppose at some time one formula was used. On the basis of this formula George's yacht is the biggest yacht in the harbor. Suppose that the formula is changed and it turns out, much to my surprise, that George's yacht is smaller than other yachts. I remark to George "I thought your yacht was bigger than it is; that is, I thought that George's boat was so big that a change in formula would not affect the ranking. I was wrong. How big a yacht is, therefore, i.e., *the* size of the yacht, can be talked about independently of formulas or numerical measurements. There are additional and, perhaps, more important considerations.

Suppose the formula for the size of a yacht differs from the formula for computing the size of a battleship. I say,

"George believes that the battleship is bigger than the yacht."

Neglecting Russell's theory of magnitudes and accepting Kripke's interpretation would, almost certainly, lead us to believe this is nonsense and that Russsell's theory must, surely, collapse. However, once Russell's theory of magnitudes is taken into account and along with it his commitment to atomism and the role acquaintance plays in preserving this principle, then this

seems like a plausible scenario; and the statement seems meaningful, even commonplace *on a Russellian theory of descriptions.* Much the same can be said regarding other comparative constructions that raise the same problem Kripke believes he has found with Russell's theory. We *can* describe *the* relative sizes as we see them, without any reliance on expert testimony in making a precise determination, according to one formula, scale, etc. and, given Russell's theory of magnitudes, our acquaintance can be with particular magnitudes in the absence of numerical measures. Russell in reply to Kripke would point out that the guest in the case of his joke *does* have in mind the *idea* of a specific magnitude, one given in experience, at some point, and not requiring a complex mathematical formula in order for it to *be* thought about with the help of an exact idea.

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